AD NUMBER

AD254472

NEW LIMITATION CHANGE

TO

Approved for public release, distribution unlimited

FROM

Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; Feb 1961. Other requests shall be referred to Office of Naval Research, 800 North Quincy Street, Arlington, VA 22217-5660.

AUTHORITY

ONR ltr, 15 Jun 1977

Interim Report

I-A2049-12

AN ANALYTICAL AND EXPERIMENTAL STUDY OF THE STABILITY OF EXTERNALLY-PRESSURIZED, GAS-LUBRICATED THRUST BEARINGS

by

L. Licht and H. G. Elrod, Jr.

February 1961

Prepared under

Contract Nonr-2342(00)
Task NR 061 - 113

Jointly Supported by

DEPARTMENT OF DEFENSE
ATOMIC ENERGY COMMISSION
MARITIME ADMINISTRATION
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Administered by

OFFICE OF NAVAL RESEARCH
Department of the Navy

COPY NO.

THE FRANKLIN INSTITUT
LABORATORIES FOR RESEARCH AND DEVELOPME
PHILADELPHIA PENNSYLVAI

THE FRANKLIN INSTITUTE • *Laboratories for Research and Development*

Interim Report I-A2049-12

AN ANALYTICAL AND EXPERIMENTAL STUDY OF THE STABILITY OF EXTERNALLY-PRESSURIZED, GAS-LUBRICATED THRUST BEARINGS

by

I. Licht and H. G. Elrod, Jr.

Prepared under

Contract Nonr 2342(00)
Task NR 061-113

Jointly Supported by

DEPARTMENT OF DEFENSE
ATOMIC ENERGY COMMISSION
MARITIME ADMINISTRATION
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Administered by

OFFICE OF NAVAL RESEARCH
Department of the Navy

FOREWORD

This report deals with both the theoretical and experimental phases of an investigation of the stability of externally-pressurized, gas-lubricated bearings.

The analysis, presented in Interim Report No. I-A2049-4, has been extended and complemented by additional material in Appendices 2 through 7 and is included in this report for the sake of completeness. Detailed descriptions of apparatus, instruments and experiments are given and are supplemented by 65 figures, containing graphs drawings and photographic records.

The theoretical predictions of the present and of earlier analyses are compared with experimental results, areas of further research are suggested and recommendations are made for utilizing the results of the present investigation for design purposes.

ACKNOWLEDGEMENT

This research was performed under contract administered by the Office of Naval Research and supported jointly by the Department of Defense, Atomic Energy Commission and Maritime Administration. The authors are indebted to Captain W. T. Sawyer and Mr. S. W. Doroff, Office of Naval Research, for their assistance and the cooperation received in all phases of this investigation.

Professor D. D. Fuller, whose counsel greatly contributed to the success of this undertaking, initiated and coordinated the research in gas lubrication at The Franklin Institute. — The Franklin Institute Laboratories, under the direction of Mr. F. L. Jackson, provided ideal conditions and atmosphere in which to carry out this project and the Friction and Lubrication Branch, headed by Mr. W. S. Shugarts, cooperated wholeheartedly in all phases of this investigation. The authors wish to thank Miss S. Goff, Mr. S. Malanoski and Mr. C. Stevenson, who shared in the laborious numerical calculations, and Mr. L. McGinn and Mr. W. Ruegner, who were responsible for the programming of the computer. The meticulously detailed set of engineering drawings of the experimental apparatus is due to Mr. J. Kuehn. Mr. C. Bognar installed the apparatus and assisted in the course of the experiments.

The apparatus was manufactured by the Peter J. Salmon Company, Glenside, Pa., and the bearing surfaces were lapped by Dr. F. C. Stein of the Stein Seal Company, Philadelphia. Dr. B. Sternlicht, General Electric Laboratories, Schenectady, New York and Dr. W. Gross, International Business Machines Laboratories, San Jose, California and Dr. S. Ausman, Autonetics, Downey, California, provided useful technical information and offered constructive criticism.

To these people and many others not mentioned specifically, who contributed to the success of this work, the authors owe their gratitude.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

LIST OF APPENDICES

LIST OF FIGURES

LIST OF FIGURES (Continued)

LIST OF FIGURES (Continued)

APPENDIX FIGURES

I. ABSTRACT

The subject of this investigation is the stability of externally-pressurized, gas-lubricated bearings. An analysis is made which treats the flow within the gas film on a distributed, rather than on a lumped parameter basis. Rectangular and circular bearing configurations are considered and the effect of several parameters on stability is investigated. Results of the present analysis are compared and contrasted with those obtained from earlier, simplified theories. The experimental program pursued here is guided by the analysis. An apparatus is designed, constructed and instrumented to provide for the variation of governing parameters and for the accurate determination of their critical values. Comparison of theory and experiment is made and the validity of theoretical assumptions, relative to the experimental model, is discussed.

II. INTRODUCTION

1. Instabilities of Gas-Lubricated Bearings and Related Problems – Review of Pertinent Literature

Recent advances in space, computer and nuclear technology have precipitated a renewed interest in gas lubrication. Gas bearings are being successfully incorporated in systems preceding the use of conventional lubricants and rolling elements. To quote contrasting examples of application – gas bearings have been used for small turbine rotors, driven and supported by hot exhaust gases, and also at very low temperatures in gas liquefaction machines. They have been employed as a method of gyroscope support in inertial guidance devices, as sensing elements in computers and as low-friction guides in accelerometers, dynamometers, vibration tables and torque measuring instruments. In nuclear reactors, gas bearings have provided a solution for sealed and isolated systems, in which radioactive gases furnish the only available lubrication medium.

Gas bearings may be broadly classified into two types:

a. Self-acting bearings, in which the load capacity is induced through the relative motion of surfaces.

b. Externally-pressurized bearings, which rely for their load capacity on an external pressure source.

One major difficulty encountered in applications of gas bearings is their susceptibility to self-excited vibrations. In unstable, self-acting bearings, the center of the rotating journal may either describe a closed path within the clearance circle, or follow a spiral path until surface contact and destruction take place. This type of instability, generally referred to in literature as "whirl", occurs also with incompressible lubricants, but appears to be particularly troublesome with gases. It has been the subject of numerous theoretical and experimental investigations [36, 37, 38, 39, 40, 41, 42, 43]†, but its mechanism is not yet fully understood.

The subject of the present investigation is an instability, associated with externally-pressurized bearings and sometimes referred to as "air" or "pneumatic hammer". It is frequently encountered in valves, regulators and other elements of pneumatic and hydraulic control systems [8, 9, 12, 13, 14, 15, 16], and is closely allied to such seemingly unrelated phenomena as self-excited vibrations of fuel injectors [17, 57] and pressure reducers [52], axial auto-oscillations of steam-cushioned turbine rotors [57] and violent pressure fluctuations in fan-ventilated chambers [18, 57]. The "pneumatic hammer" in externally-pressurized bearings is not associated with the steady, relative motion of surfaces, nor greatly influenced by it at moderate rotational speeds. If, however, the amplitude of vibration becomes excessively large in relation to the clearance, surfaces may contact and scoring, or complete destruction ensue. Moreover, even relatively minute auto-oscillations cannot be tolerated in applications involving sensitive instruments, or inertial guidance devices. On the other hand, unless the selection of geometrical and

† Numbers in brackets refer to bibliography.

fluid parameters is governed by at least a qualitative understanding of the physical mechanism of instability, it is probable that the bearing will be unstable in all, or part of the contemplated region of operation.

The phenomenon was observed in externally-pressurized thrust and journal bearings by Vaught [34], Wildmann [33], Gumbel [19], Pigott and Macks [25], Fischer, Cherubim and Fuller [35], Robinson and Sterry [29], Graneek and Kerr [30], Allen, Stokes and Whitley [32] and many others. Rathe [28] established empirical criteria for specific bearing configurations used in inertial guidance. A number of investigators have speculated on likely causes of instability and have suggested remedial measures, based on observation, or on intuition [26, 30, 32, 33]. The first, rational explanations of the phenomenon in gas lubrication were given independently by Licht, Fuller and Sternlicht [5], who considered the instability of a simple, air-lubricated thrust bearing, and by Richardson [2, 3], who concentrated on the dynamics of externally-pressurized, air-lubricated journal bearings, assuming the initial eccentricity to be small and motion to be limited to a single degree of freedom. More recently, Licht [7] extended the analysis to multiple-pad, journal bearings, assuming plane motion of the journal, initially in an eccentric equilibrium position. Roudebush [6] considered the stability of a circular thrust bearing and obtained digital computer solutions for the nonlinear, lumped, fluid-dynamic equations. His conclusions were in agreement with those based on linearized analyses, although computer solutions were necessarily limited to a number of specific cases.

The analyses referred to in the preceding paragraphs were of the lumped-parameter type and relied on arbitrary, a priori assumptions with regard to variation of pressure profile and flow in the bearing clearance. The objective in making these assumptions was to permit a simple formulation of the problem and to avoid the use of partial differential equations in the mathematical description of the fluid-dynamic phenomena occurring in the bearing clearance.

2. Purpose and Outline of the Present Investigation

Because of the large number of parameters which influence the stability of externally-pressurized bearings, and because of the complexity introduced through nonlinearity, the present investigation is restricted to a perturbation analysis. The limitation that perturbation analyses give necessary conditions of instability, not sufficient conditions of stability, is recognized, but linearization has frequently provided a good first approximation in the solution of fundamental problems of fluid dynamics. More specifically, analog and digital computer studies of pneumatic controls indicate that results obtained from linearized equations often compare very favorably with exact solutions [10, 52]. Some encouragement may also be derived from the fact that analog solutions of non-linear equations for a bearing using an incompressible lubricant, but geometrically identical with the model proposed in this investigation, were in good agreement with results obtained from their linearized equivalents [11].

Results of previous experiments, reported in reference [4], agreed only qualitatively with theoretical predictions of simplified analyses [4, 5]. It was felt that, although the simplified analyses explained the basic aspects of the physical mechanism of "pneumatic hammer" in gas bearings, the degree of simplification in formulating the problem was excessive. Richardson [2, 3] recognized that dynamic changes in pressure-profile depended on time rates of boundary

motion. In fact, a priori assumptions with regard to pressure-profile variation and mass continuity appear to have little justification, particularly for configurations in which the surface corresponding to the slot constitutes a large fraction of the total bearing area. Equally questionable are the results of simplified analyses, according to which stability is not influenced by the bearing mass [3, 5].†

In the theoretical part of the present investigation, while retaining the method of small perturbations and a number of simplifying assumptions, the nonsteady flow in the bearing clearance is treated on a continuous, rather than on a lumped basis. The fluid-dynamic phenomenon in the slot is described by a partial differential equation. The latter is required to satisfy the boundary condition at the outer bearing perimeter and to be compatible with the continuity equation for a centrally located, variable-volume recess, and also with the equation of motion of the bearing mass. Stability analyses are made for axial motion of parallel-surface thrust bearings. Rectangular and circular configurations†† are considered and stability boundaries are obtained to show the effect of several, important bearing parameters and to show how the results compare with those obtained from lumped-parameter theories.

In the experimental part of the present investigation, the bearing is circular and identical in geometry to the theoretical model. The surface bounded by the perimeter of a centrally-located, adjustable-volume recess, constitutes a small fraction of the total bearing area, in order to hinder the escape of fluid and magnify the trapping and squeezing effect in the slot. Provision is also made to vary the bearing mass without altering the load, in order to observe the influence of mass on stability.

---

† See also Appendix 2.

†† Schematics, drawings and photographs of actual thrust plates are shown in Figures 1, 7, 8, 13, 19 and 20.

III. ANALYSIS[†]

1. Derivation of Characteristic Equations

In what follows, equations are simultaneously developed for the rectangular and circular configurations shown in Figures 1a and 1b. Equations applicable to both geometries are denoted with common numbers. Those denoted by the letter "a" refer to the rectangular geometry, while those denoted by the letter "b" refer to the circular geometry.

Equations (1) through (4) are those developed by G. I. Taylor and P. G. Saffman [1], who investigated compressibility effects at low Reynolds number. One case they considered consisted of two parallel discs, separated by a narrow air gap, when rotation was accompanied by small amplitude, forced vibration in the axial direction. By contrast, the present investigation concerns itself with the onset of self-excited oscillations. It differs also because, in the absence of rotation, fluid flow is induced by means of an external pressure source connected to an internal, variable-volume reservoir.

It is assumed that forces due to inertia are small compared with those due to viscous stresses, and that the pressure gradient in the transverse direction of the slot is negligible. The amplitude of oscillations in the anticipated frequency band width can be considered arbitrarily small in comparison with the magnitude of the gap width. The mean flow velocity in the annulus can then be obtained from the reduced Navier-Stokes equation and written as:

$$\bar{u} = -\frac{H^2}{12\mu}\frac{\partial P}{\partial x} \tag{1a}$$

$$\bar{u} = -\frac{H^2}{12\mu}\frac{\partial P}{\partial r} \tag{1b}$$

Since the dimension of the annulus separating the metallic surfaces is small in comparison with the linear dimension of the slot, it is reasonable to assume that conductivity prevents any appreciable variation in temperature and that the relation

$$\frac{P}{\rho} = \text{constant} \tag{2}$$

represents the thermodynamic process to a sufficient degree of approximation.

[†] Material contained in this section was presented at the 23rd National Conference of the Applied Mechanics Division, ASME, June 1960, and was published under the title "A Study of the Stability of Externally Pressurized Gas Bearings", by Likhi and H. Elrod, Jr., Journal of Applied Mechanics, Vol. 27, No. 2, June 1960, pp. 250–258.

The continuity equation then becomes:

$$\frac{\partial}{\partial t}(\rho H) + \frac{\partial}{\partial x}(\rho H \bar{u}) = 0 \tag{3a}$$

$$\frac{\partial}{\partial t}(\rho H) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho H \bar{u}) = 0 \tag{3b}$$

Substitution of equations (1) and (2) in the continuity equation (3) yields a dynamic lubrication equation:

$$\frac{\partial}{\partial x} P\left(\frac{\partial P}{\partial x}\right) = \frac{12\mu}{H^3}\frac{\partial}{\partial t}(PH) \tag{4a}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rP\frac{\partial P}{\partial r}\right) = \frac{12\mu}{H^3}\frac{\partial}{\partial t}(PH) \tag{4b}$$

The basic assumptions and the derivation of these equations are given in Appendix 6.

Equation (4) must satisfy the boundary condition

$$P(R, t) = P_{at} \tag{5b}$$

$$P(X, t) = P_{at} \tag{5b}$$

and be compatible with the equation of motion†:

$$m\frac{d^2H}{dt^2} = 2X_r(P_r - P_{ro}) + 2\int_{X_r}^{X}(P - P_o)dx + F(t) \tag{6a}$$

$$m\frac{d^2H}{dt^2} = \pi R_r^2(P_r - P_{ro}) + 2\pi\int_{R_r}^{R}(P - P_o)rdr + F(t) \tag{6b}$$

in which $F(t)$ is the excitation function.

---

† The effect of external damping is negligibly small

In addition, the continuity equation of the variable-volume recess is as follows:

$$\frac{dM_r}{dt} = W_1 - W_2 \tag{7}$$

In expressing the quantities on the right hand side of equation (7), it is assumed that the flow can be treated as quasi-steady. Theoretical and experimental investigations of pulsating flow through orifices were conducted by Schultz-Grunow [46], who discussed the conditions under which instantaneous flow rates could be evaluated from steady-flow equations and gave criteria for their applicability. To express the left hand side of equation (7), it is necessary to describe an extremely complex process, associated with simultaneous charging and discharging of a variable-volume container. In the present analysis it will be assumed that fluid properties throughout the recess volume are uniform at any instant of time and that the process may be adequately described by the pressure-density relation P/ρ^{γ} = constant. The quasi-static charging and discharging of an adiabatic container, for example, may be considered as isentropic to the first degree of approximation, perfect mixing of entering gas being assumed. Experiments conducted by Shearer [9] and Skinner and Wagner [56] also indicate that, for relatively rapid rates of charging and discharging, the assumption P/ρ^{γ} = constant is reasonable. The effect of wave propagation, heat transfer and irreversibilities, associated with the processes of charging and discharging of fixed and variable volume containers, cannot always be neglected [51, 53, 54, 55]. However, assumptions identical with those made in implementing equation (7) have been frequently and successfully employed in related problems, in the field of pneumatic controls and regulators [9, 14, 52]. Consequently, the quantities appearing in equation (7) are expressed as follows:

$$W_1 = 2\left(\frac{\pi d_s^2}{4}\right)\left[\frac{\gamma}{RT_o}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}(P_s P_r - P_r^2)\right]^{\frac{1}{2}} \tag{8}$$†

$$W_2 = -\frac{H^3}{6\mu}\left(\rho\frac{\partial P}{\partial x}\right)_{X_r} \tag{9a}$$

$$W_2 = -\frac{\pi H^3}{6\mu}\left(r\rho\frac{\partial P}{\partial r}\right)_{R_r} \tag{9b}$$

† The flow through the supply nozzle can be approximated closely by a quadrant of an ellipse whenever P_r/P_s is considerably above the critical-pressure ratio. This is Fliegner's approximation, and is convenient for computational purposes.

$$\begin{cases} M_r = 2X_r(\delta + H)\rho_r & (10a) \\ M_r = \pi R_r^2(\delta + H)\rho_r & (10b) \end{cases}$$

while the pressure-density relation for the recess volume is taken as:

$$\left(\frac{\rho_r}{\rho_{ro}}\right)^{\gamma} = \frac{P_r}{P_{ro}} \tag{11}$$

In the next step the perturbation quantities

$$\begin{aligned} P - P_o &= p \\ P_r - P_{ro} &= p_r \\ H - H_o &= h \end{aligned} \tag{12}$$

and their derivatives are introduced. Neglecting their squares and cross products as quantities of higher order, we obtain instead of equations (4), (6) and (7) a set of corresponding expressions.

The nonlinear, partial differential equation (4) is replaced by one which is linear, but contains variable coefficients.

Thus in terms of perturbation quantities we have:

$$\begin{cases} \dfrac{\partial^2 p}{\partial x^2} + \left(\dfrac{2P_o'}{P_o}\right)\dfrac{\partial p}{\partial x} + \left(\dfrac{P_o''}{P_o}\right)p = \dfrac{12\mu}{H_o^2}\left(\dfrac{1}{P_o}\dfrac{\partial p}{\partial t} + \dfrac{1}{H_o}\dfrac{dh}{dt}\right) & (13a) \\ \dfrac{\partial^2 p}{\partial r^2} + \left(\dfrac{2P_o'}{P_o} + \dfrac{1}{r}\right)\dfrac{\partial p}{\partial r} + \left(\dfrac{P_o''}{P_o} + \dfrac{1}{r}\dfrac{P_o'}{P_o}\right)p = \dfrac{12\mu}{H_o^2}\left(\dfrac{1}{P_o}\dfrac{\partial p}{\partial t} + \dfrac{1}{H_o}\dfrac{dh}{dt}\right) & (13b) \end{cases}$$

From equation (6) we obtain,

$$\left\{ m \frac{d^2h}{dt^2} = 2X_r p_r + 2\int_{X_r}^{X} p\,dx + f(t) \right. \tag{14a}$$

$$\left\{ m \frac{d^2h}{dt^2} = \pi R_r^2 p_r + 2\pi \int_{R_r}^{R} pr\,dr + f(t) \right. \tag{14b}$$

in which $f(t)$ is the disturbance.

Equation (7) reduces to

$$\left\{ K_1 \frac{dh}{dt} - K_3 h = (K_3 + K_5) p_r + K_6 \frac{\partial p_r}{\partial x} - K_2 \frac{\partial p_r}{\partial t} \right. \tag{15a}$$

$$\left\{ C_1 \frac{dh}{dt} - C_3 h = (C_3 + C_5) p_r + C_6 \frac{\partial p_r}{\partial r} - C_2 \frac{\partial p_r}{\partial t} \right. \tag{15b}$$

in which the coefficients K_i and C_i represent the quantities listed in the following:

$$\left.\begin{aligned}
K_1 &= \left(\frac{\partial M_r}{\partial H}\right)_o = \frac{2X_r P_{ro}}{RT_o} \\
K_2 &= \left(\frac{\partial M_r}{\partial P_r}\right)_o = \frac{2X_r H_o}{\gamma RT_o}\left(\frac{\dot{o}}{H_o} + 1\right) \\
K_3 &= \left(\frac{\partial W_1}{\partial P_r}\right)_o = \frac{W_o}{2P_{ro}}\left(\frac{\frac{P_a}{P_{ro}} - 2}{\frac{P_a}{P_{ro}} - 1}\right) \\
K_4 &= -\left(\frac{\partial W_2}{\partial H}\right)_{\substack{o \\ x = X_r}} = -\frac{W_o}{H_o} \\
K_5 &= -\left(\frac{\partial W_2}{\partial P}\right)_{\substack{o \\ x = X_r}} = -\frac{W_o}{P_{ro}} \\
K_6 &= -\left(\frac{\partial W_2}{\dot{o}\left(\frac{\partial P}{\partial x}\right)}\right)_{\substack{o \\ x = X_r}} = -\frac{2P_{ro} W_o}{A_o}
\end{aligned}\right\} \tag{16a}$$

in which

$$P_o(x) = \left[P_{ro}^2 + A_o(x - X_r)\right]^{1/2}$$

$$A_o = -\frac{P_{ro}^2 - P_{st}^2}{X - X_r}$$

$$C_1 = \left(\frac{\partial M_r}{\partial H}\right)_o = \frac{\pi R_r^2 P_{ro}}{RT_o}$$

$$C_2 = \left(\frac{\partial M_r}{\partial P_r}\right)_o = \frac{\pi R_r^2 H_o}{\gamma RT_o}\left(\frac{\delta}{H_o}+1\right)$$

$$C_3 = \left(\frac{\partial W_1}{\partial P_r}\right)_o = \frac{W_o}{2P_{ro}}\left(\frac{\frac{P_s}{P_{ro}}-2}{\frac{P_s}{P_{ro}}-1}\right)$$

$$C_4 = -\left(\frac{\partial W_2}{\partial H}\right)_{\substack{o \\ r=R_r}} = -\frac{3W_o}{H_o}$$

$$C_5 = -\left(\frac{\partial W_2}{\partial P}\right)_{\substack{o \\ r=R_r}} = -\frac{W_o}{P_{ro}}$$

$$C_6 = -\left(\frac{\partial W_2}{\partial\left(\frac{\partial P}{\partial r}\right)}\right)_{r=R_r} = -\frac{2R_r P_{ro} W_o}{B_o}$$

(16b)

in which

$$P_o(r) = \left(P_{ro}^2 + B_o \log_e \frac{r}{R_r}\right)^{1/2}$$

$$B_o = -\frac{P_{ro}^2 - P_{at}^2}{\log_e \frac{R}{R_r}}$$

Solutions of equation (13) must satisfy equations (14) and (15) as well as the boundary condition

$$\begin{cases} p(X, t) = 0 & (17a) \\ p(R, t) = 0 & (17b) \end{cases}$$

The first space derivative in equation (13) can be eliminated by means of the following substitution:

$$\begin{cases} p = \frac{v}{P_o} = \frac{v}{z^{1/2}} & (13a) \\ p = \frac{v}{r^{1/2} P_o} & (18b) \end{cases}$$

Introducing the new variable v into equations (13), (14) and (15) we take the Laplace transform under the assumption of initial rest conditions.

Then with

$$\mathcal{L}_t(v) \equiv \bar{V} = \int_o^\infty v e^{-st}\,dt$$

$$\mathcal{L}_t(h) \equiv \bar{h} = \int_o^\infty h e^{-st}\,dt$$

$$\mathcal{L}_t(f) \equiv \bar{f} = \int_o^\infty f e^{-st}\,dt$$

equation (13) transforms to:

$$\bar{V}'' - \frac{\epsilon s}{z^{1/2}}\bar{V} = \epsilon s z^{1/2}\frac{\bar{h}}{H_o} \tag{19a}$$

$$\bar{V}'' + \left(\frac{1}{4r^2} - \frac{\eta s}{P_o}\right)\bar{V} = \eta s r^{1/2} P_o \frac{\bar{h}}{H_o} \tag{19b}$$

in which

$$\epsilon = \frac{12\mu}{H_o^2 A_o^2} = \frac{12\mu}{H_o^2}\left(\frac{X - X_r}{P_{ro}^2 - P_{at}^2}\right)^2 \tag{20a}$$

$$\eta = \frac{12\mu}{H_o^2} \tag{20b}$$

Equation (14) transforms to:

$$ms^2\frac{\bar{h}}{H_o} - \frac{2X_r}{H_o P_{ro}}(\bar{V})_{P_{ro}^2} + \frac{2(X - X_r)}{H_o(P_{ro}^2 - P_{at}^2)}\int_{P_{ro}^2}^{P_{at}^2}\frac{\bar{V}}{z^{1/2}}\,dz = \bar{f}(s) \tag{21a}$$

$$ms^2\frac{\bar{h}}{H_o} - \frac{\pi R_r^{3/2}(\bar{V})_{R_r}}{H_o P_{ro}} - \frac{2\pi}{H_o}\int_{R_r}^{R}\frac{\bar{V}r^{1/2}}{P_o}\,dr = \bar{f}(s) \tag{21b}$$

Equation (15) transforms to:

$$\left(sK_1 - K_4\right)\frac{\bar{h}}{H_o} + \left(sK_2 - K_3\right)\frac{(\bar{V})_{P_{ro}^2}}{H_o P_{ro}} - \frac{2P_{ro}}{H_o} K_5 \left(\frac{\partial \bar{V}}{\partial z}\right)_{P_{ro}^2} = 0 \tag{22a}$$

$$\left(sC_1 - C_4\right)\frac{\bar{h}}{H_o} + \left(sC_2 - C_3 + C_5 \frac{P_{ro}^2}{B_o}\right)\frac{(\bar{V})_{R_r}}{H_o R_r^{1/2} P_{ro}} - \frac{2R_r^{3/2} P_{ro}}{B_o H_o} C_5 \left(\frac{\partial \bar{V}}{\partial r}\right)_{R_r} = 0 \tag{22b}$$

in which the constants K_6 and C_6 have been replaced by means of the identity

$$K_6 = \frac{2P_{ro}^2}{A_o} K_5 \tag{23a}$$

$$C_6 = \frac{2R_r P_{ro}^2}{B_o} C_5 \tag{23b}$$

Finally, the boundary condition (17) transforms to

$$\bar{V}(P_{at}^2, s) = 0 \tag{24a}$$

$$\bar{V}(R, s) = 0 \tag{24b}$$

Furthermore, by integrating equation (16) between the appropriate limits, we can replace the integral in equation (21). For this purpose we have:

$$\int_{P_{ro}^2}^{P_{at}^2} \frac{\bar{V}}{z^{1/2}}\, dz = \frac{2}{3}\left(P_{ro}^3 - P_{at}^3\right)\frac{\bar{h}}{H_o} - \frac{1}{\epsilon s}\left[\left(\frac{\partial \bar{V}}{\partial z}\right)_{P_{ro}^2} - \left(\frac{\partial \bar{V}}{\partial z}\right)_{P_{at}^2}\right] \tag{25a}$$

$$\int_{R_r}^{R} \frac{\bar{V} r^{1/2}}{B_o}\, dr = -\frac{2}{3}\frac{\bar{h}}{H_o} - \frac{1}{\eta^3}\left[R_r^{1/2}\left(\frac{\partial \bar{V}}{\partial r}\right)_{R_r} - R^{1/2}\left(\frac{\partial \bar{V}}{\partial r}\right)_{R} - \frac{(\bar{V})_{R_r}}{2R_r}\right] \tag{25b}$$

in which

$$\ldots = \int_{R_t}^{R} P_o(r)r\,dr \tag{26b}$$

The foregoing integral is expressed explicitly in Appendix I.

A particular solution of equation (19) is:

$$\begin{cases} \bar{V}_p = -\dfrac{\bar{I}_t}{H_o}\, z & (27a) \\ \bar{V}_p = -\dfrac{\bar{I}_t}{H_o}\, r^{1/2} I_o^2(r) & (27b) \end{cases}$$

If v_I and v_{II} are independent solutions which satisfy the homogeneous part of the differential equation (19), then

$$\begin{cases} Q(z, \zeta) = \dfrac{v_I(z)v_{II}(\zeta) - v_I(\zeta)v_{II}(z)}{v_I(P_{ro}^2)v_{II}(P_{at}^2) - v_I(P_{at}^2)v_{II}(P_{ro}^2)} & (28a) \\ Q(r, \rho) = \dfrac{v_I(r)v_{II}(\rho) - v_I(\rho)v_{II}(r)}{v_I(R_r)v_{II}(R) - v_I(R)v_{II}(R_r)} & (28b) \end{cases}$$

are also solutions having the following properties:

$$\left.\begin{aligned} &Q(z, z) = 0 \\ &Q(P_{ro}^2, P_{at}^2) = -Q(P_{at}^2, P_{ro}^2) = 1 \\ &\frac{\partial Q}{\partial \zeta}(z, z) = \text{constant}^{\dagger} \end{aligned}\right\} \tag{29a}$$

† The Wronskian of second-order differential equations having the form $y'' + f(x)y = 0$ is a constant.

$$\left.\begin{aligned} &Q(r, r) = 0 \\ &Q(P_r, R) = -Q(R, R_r) = 1 \\ &\frac{\partial Q}{\partial r}(r, r) = \text{constant}^{\dagger} \end{aligned}\right\} \qquad (29b)$$

The solution of (19) which satisfies the boundary condition (24) can then be written as:

$$\begin{cases} \bar{V} = \bar{V}Q(P_{at}^2, z) + \dfrac{\bar{h}}{H_o}\left[P_{at}^2 Q(P_{ro}^2, z) - z\right] & (30a) \\ \bar{V} = \bar{V}Q(R, r) + \dfrac{\bar{h}}{H_o}\left[R^{1/2}P_{at}^2 Q(R_r, r) - r^{1/2}P_o^2(r)\right] & (30b) \end{cases}$$

in which $\bar{V}(s)$ and $\bar{h}(s)$ are determined from equations (21) and (22).

Using equation (25) and substituting into equation (22) the boundary values of $\bar{V}$ and its derivatives obtained by differentiation of equation (30), we have at our disposal two simultaneous equations for the determination of $\bar{A}(s)$ and $\bar{h}(s)$. The "two by two" determinant formed from the coefficients of the homogeneous parts of these equations is then set equal to zero. The location in the complex plane of roots of this characteristic equation determines the stability of the bearing. The characteristic determinant referred to in the foregoing is as follows:

---

† The Wronskian of second-order differential equations having the form $y'' + f(x)y = 0$ is a constant.

$$\left[\begin{array}{c|c} \dfrac{\ln H_0 r}{2V_r}\left(\dfrac{P_{ro}^2 - P_{at}^2}{\dfrac{V}{V_r} - 1}\right) s^3 + \dfrac{2}{3}(P_{ro}^3 - P_{at}^3) r s & \\ \left[P_{ro}^2 \dfrac{\partial Q}{\partial z}(P_{at}^2, P_{ro}^2) + P_{at}^2 \dfrac{\partial Q}{\partial z}(P_{ro}^2, P_{at}^2)\right. & \\ \left. - (P_{ro}^2 + P_{at}^2)\dfrac{\partial Q}{\partial z}(z, z)\right] & \\ - P_{ro}\left(\dfrac{P_{ro}^2 - P_{at}^2}{\dfrac{V}{V_r} - 1}\right) r s & \\ + \left[P_{ro}^2 \dfrac{\partial Q}{\partial z}(P_{at}^2, P_{ro}^2) - P_{ro}^2 \dfrac{\partial Q}{\partial z}(z, z)\right] & \end{array}\right] \quad \cdot Q \text{ (31a)}$$

$$\left[\begin{array}{c|c} K - \dfrac{H_0}{2K_5}\dfrac{V_r}{P_{ro}} s - \dfrac{V_r}{2K_5}\dfrac{H_0}{P_{ro}} & \dfrac{K_2}{2K_5} s - \dfrac{K_3}{2K_5} + \left[P_{ro}^2 \dfrac{\partial Q}{\partial z}(P_{at}^2, P_{ro}^2)\right] \\ + \left[P_{ro}^2 \dfrac{\partial Q}{\partial z}(P_{at}^2, P_{ro}^2) - P_{at}^2 \dfrac{\partial Q}{\partial z}(z, z) + 1\right] & \end{array}\right]$$

$$0 \qquad (31b)$$

$$\begin{vmatrix} \dfrac{nH_o\eta}{2\pi P_{at}^2}\, s^3 + \dfrac{\eta\beta s}{P_{at}^2} - \left\{ R\,\dfrac{\partial Q}{\partial \rho}(R_r, R) \right. & \dfrac{P_{ro}R_r^2\eta}{2P_{at}^2}\, s + \left\{ \left(\dfrac{P_{ro}}{P_{at}}\right)^2 R_r \dfrac{\partial Q}{\partial \rho}(r, R_r) \right. \\ \left. + \left(\dfrac{P_{ro}}{P_{at}}\right)^2 R_r \dfrac{\partial Q}{\partial \rho}(R, R_r) - \left[\left(\dfrac{P_{ro}}{P_{at}}\right)^2 + 1 \right] \times \right. & \left. - \left(\dfrac{P_{ro}}{P_{at}}\right)^2 R_r^{1/2} R^{1/2} \dfrac{\partial Q}{\partial \rho}(r, r) + \dfrac{1}{2}\left(\dfrac{P_{ro}}{P_{at}}\right)^2 \right\} \\ \times R_r^{1/2} R^{1/2} \dfrac{\partial Q}{\partial \rho}(r, r) + \dfrac{1}{2}\left[\left(\dfrac{P_{ro}}{P_{at}}\right)^2 - 1 \right] \Bigg] \Bigg\} \Bigg\} & \end{vmatrix}$$

$$\begin{vmatrix} -\dfrac{C_1}{2C_5}\left(\dfrac{H_o}{P_{ro}}\right)\dfrac{\left(\dfrac{P_{ro}}{P_{at}}\right)^2 - 1}{\log_e \dfrac{R}{R_r}}\, s + \dfrac{C_4}{2C_5}\left(\dfrac{H_o}{P_{ro}}\right)\dfrac{\left(\dfrac{P_{ro}}{P_{at}}\right)^2 - 1}{\log_e \dfrac{R}{R_r}} & \dfrac{C_2}{2C_5}\,\dfrac{\left(\dfrac{P_{ro}}{P_{at}}\right)^2 - 1}{\log_e \dfrac{R}{R_r}}\, s - \dfrac{C_3}{2C_5}\,\dfrac{\left(\dfrac{P_{ro}}{P_{at}}\right)^2 - 1}{\log_e \dfrac{R}{R_r}} \\ + \left(\dfrac{P_{ro}}{P_{at}}\right)^2 R_r \dfrac{\partial Q}{\partial \rho}(R, R_r) - R_r^{1/2} R^{1/2} \dfrac{\partial Q}{\partial \rho}(r, r) & - \left\{ \left(\dfrac{P_{ro}}{P_{at}}\right)^2 R_r \dfrac{\partial Q}{\partial \rho}(R, R_r) + \dfrac{1}{2}\left(\dfrac{P_{ro}}{P_{at}}\right)^2 \right\} \\ + \dfrac{1}{2}\left(\dfrac{P_{ro}}{P_{at}}\right)^2 - \dfrac{\left(\dfrac{P_{ro}}{P_{at}}\right)^2 - 1}{\log_e \dfrac{R}{R_r}} & \end{vmatrix}$$

2. Stability Boundary – Evaluation of Critical (Limiting) Parameters

Further progress in the analysis depends on a knowledge of the solutions y_I and y_{II} of the homogeneous part of equation (19). Even with such knowledge the location of roots in the complex plane of the characteristic equation (31) might be very involved. Our interest lies in assessing the limiting values of certain bearing parameters which influence stability. Within limits of applicability of our equations, we are guided in the expected range of their magnitudes by results obtained by means of a simplified analysis summarized in Appendix 2. Different approaches were employed for the rectangular and circular geometries and these are described in what follows:

a. Rectangular Geometry

The homogeneous part of equation (19a) is

$$y'' - \epsilon s z^{-1/2} y = 0 \tag{32a}$$

Two independent solutions, which involve Bessel functions of order $\pm 2/3$, can be written as [60]:

$$\begin{matrix} y_I \\ y_{II} \end{matrix} = z^{1/2} J_{\pm 2/3}\left(\frac{4}{3} z^{3/4} i \sqrt{\epsilon s}\right) \tag{33a}$$

in which $+2/3$ corresponds to I and $-2/3$ to II.

The series representing the Bessel function $J_\nu(2i\lambda^{1/2})$ can be written as [61]:

$$J_\nu(2i\lambda^{1/2}) = (i\lambda^{1/2})^\nu \left[\frac{1}{\Gamma(\nu+1)} + \frac{\lambda}{1!\Gamma(\nu+2)} + \frac{\lambda^2}{2!\Gamma(\nu+3)} + \cdots\right] \tag{34a}$$

With

$$\nu = \pm\frac{2}{3} \quad \text{and} \quad 2i\lambda^{1/2} = \frac{4}{3} z^{3/4} i\epsilon s$$

the first four terms of the expansion of y_I and y_{II} become

$$y_I = \frac{\left(\frac{2}{3} i\sqrt{\epsilon s}\right)^{2/3}}{\Gamma\left(\frac{2}{3}\right)} \left[\frac{3}{2} z + \frac{2}{5} z^{5/2} (\epsilon s) + \frac{1}{30} z^{8/2} (\epsilon s)^2 + \frac{2}{1485} z^{11/2} (\epsilon s)^3\right] \tag{35a}$$

$$y_{II} = \frac{\left(\frac{2}{3} i\sqrt{\epsilon s}\right)^{-2/3}}{\Gamma\left(\frac{1}{3}\right)} \left[1 + \frac{4}{3} z^{3/2} (\epsilon s) + \frac{2}{9} z^{6/2} (\epsilon s)^2 + \frac{8}{567} z^{9/2} (\epsilon s)^3\right] \qquad (36a)$$

These truncated series are then substituted into the expression $Q(z, \zeta)$ in (25a) and the derivative $(\partial Q / \partial \zeta)(z, \zeta)$ evaluated with z and ζ assuming the boundary values of P_{ro}^2 and P_{at}^2 in the order indicated in the components of the characteristic determinant (31a). In forming the expressions for the afore-mentioned derivatives of Q, we retain in all subsequent multiplications, divisions, and so on, terms of order s^3 and lower. The assumption that higher powers in s may be neglected, because of the relative smallness of their coefficients, is justified post factum by noting the magnitude of the complex-conjugate roots $s = \pm i\omega$ of the characteristic cubic to which the determinant (31a) is thus reduced. The coefficients of this cubic and those obtained by means of the simplified analysis in Appendix 2 are not the same. The effect of this difference in results is best illustrated by reference to Figure 3 and the table of results in Appendix 3.

b. Circular Geometry

In this case computer solutions for a set of discrete values of $s = i\omega$ on the axis of the imaginaries were obtained. The probable range of values for ω was dictated by predictions based on the simplified analysis and by taking into account the difference in the results of the two analyses for the rectangular case. Stability was assessed from physical conditions rather than proved. If we write

$$\xi = \frac{r}{R_r} \; ; \; \alpha = \frac{12\mu\omega R_r^2}{P_{ro} H_o^2} \; ; \; s = i\omega; \; \beta = \frac{1 - \left(\frac{P_{at}}{P_{ro}}\right)^2}{\log_e \frac{R}{R_r}}$$

then the homogeneous part of equation (19b) becomes

$$y'' + \left[\frac{1}{4\xi^2} - \frac{i\alpha}{(1 - \beta \log_e \xi)^{1/2}}\right] y = 0 \qquad (37b)$$

the solutions of which

$$y_{II} = \phi_{II}(\xi) + i\psi_{II}(\xi) \qquad (38b)$$

satisfy the conveniently chosen boundary conditions

$$\begin{aligned} y_I(1) &= 1; \quad y_I'(1) = 0 \\ y_{II}(1) &= 0; \quad y_{II}'(1) = 1 \end{aligned} \tag{39b}$$

The set of equations corresponding to (34b) is

$$\boxed{\begin{aligned} \phi'' + \frac{\phi}{4\xi^2} &= -\frac{a\psi}{(1-\beta \log_e \xi)^{1/4}} \\ \psi'' + \frac{\psi}{4\xi^2} &= \frac{a\phi}{(1-\beta \log_e \xi)^{1/4}} \end{aligned}} \tag{40b}$$

and the boundary conditions (36b) become

$$\begin{aligned} \phi_I(1) &= 1; \quad \phi_I'(1) = 0 \\ \phi_{II}(1) &= 0; \quad \phi_{II}'(1) = 1 \\ \psi_I(1) &= \psi_I'(1) = \psi_{II}(1) = \psi_{II}'(1) = 0 \end{aligned} \tag{41b}$$

Solutions of equations (40b), satisfying the conditions (41b) were obtained for a fixed value of β and a set of values of the parameter

$$a = \frac{12\mu\omega R_r^2}{P_{ro} H_o^2}$$

by means of a digital computer. Those shown in Figure A7–1, Appendix 7, correspond to the value of $a = 0.093$.

We are now in a position to express numerically the derivatives

$$\frac{\partial Q}{\partial \rho}(P, P_s) \quad \frac{\phi_I'(R) + i\,\psi_I'(R)}{\phi_{II}(R) + i\,\psi_{II}(R)}$$

$$\frac{\partial Q}{\partial \rho}(P_s, R) \quad \frac{\phi_{II}'(R) + i\,\psi_{II}'(R)}{\phi_{II}(R) + i\,\psi_{II}(R)} \tag{42b}$$

$$\frac{\partial Q}{\partial \rho}(r, r) = \frac{1}{\phi_{II}(R) + i\,\psi_{II}(R)}$$

for discrete values of α.

Let the bearing geometry and the parameters P_{at}, P_{ro}, T_o, γ and μ be specified. One may then express the gap width, H_o, as a function of the supply pressure, P_s, by simply equating the outflow through the slot and the inflow through the nozzle. When explicit expressions are substituted for quantities which constitute the elements of the determinant (31b) and α is also specified, the only undetermined parameters are the recess depth, δ, and the supply pressure, P_s. In fact, the characteristic equation assumes the form:

$$a\left[\delta - g_1(\alpha, P_s)\right] + ib\left[\delta - g_2(\alpha, P_s)\right] = 0 \tag{43b}$$

in which a and b are numerical constants. Therefore, given α, the problem reduces to finding specific values of P_s and δ, for which the real and imaginary parts of equation (43b) vanish simultaneously, e.g.,

$$\delta = g_1 = g_2 = \delta_c \tag{44b}$$

The experimental counterpart of this procedure is to maintain the ambient and recess pressures constant and to set the gap width by varying the supply pressure. The recess depth is then increased and its value on the threshold of instability is noted.

Critical recess depths and supply pressures were determined by trial and error. The method is illustrated by a numerical example in Appendix 4, and in the corresponding Figure A4–1 for $\alpha = 0.050$. A plot of g_1 and g_2 against P_s, showing the points of intersection of loci corresponding to $\alpha = 0.093$, is contained in Figure 4. Results of the present analysis are shown and compared with those based on the lumped-parameter theory in Figure 5.

3. Discussion of Theoretical Results

The effect of varying several important parameters on the stability of the rectangular bearing is shown in Figure 4. Here one finds reasonably good agreement between results based on the distributed and lumped-parameter theories, although the latter theory tends to predict values of allowable recess depth which are either conservative, when the pressure drop across the supply nozzle is relatively small, or slightly optimistic, when the pressure difference is increased. The agreement is particularly good for values of the pressure parameter between 0.10 and 0.11, that is for pressure differences in the order of 10 psi.

It is seen that when the recess width is decreased by a factor of two, the allowable recess depth is nearly doubled, so that the critical recess volume remains essentially unaffected by a 12.5 percent increase in slot length. (Compare curves I and II in Figure 3.) Values of allowable recess depth, approximately twice as large again, may be obtained if the supply nozzle diameter is increased by a factor of 3. (Compare curves II and III in Figure 3.)

It is appropriate to remark at this point that decreasing the recess area and increasing the supply nozzle diameter have the adverse effect of lowering the load capacity and static stiffness of the bearing, so that stabilization is often a compromise in the selection of important design parameters.

A number of calculations were also carried out in order to investigate the effect of bearing mass on stability. An increase in mass by a factor of ten showed that the effect was generally negligible for those rectangular geometries which were considered. The effect was greatest for the configuration corresponding to curve II, Figure 3, characterized by a relatively long slot and and a relatively small supply nozzle. At a value of the pressure parameter of approximately 0.04, for example, the tenfold increase in mass had the effect of reducing the allowable recess depth by 7.5 percent.

On the other hand, as shown in Figure 5, the mass had a significant effect in the case of the circular bearing geometry, particularly at low values of the pressure parameter. At $P_s - P_{ro}/P_a = 0.14$, for example, an increase in mass by a factor of 3.25 resulted in a 50 percent decrease in the allowable recess depth. The deviation of results based on lumped-parameter theories was also significantly greater than for the rectangular configurations, particularly in the case of the smallest mass. (Curve III, Figure 5.)

The smaller the mass of a bearing, the greater the rate of response of the bearing tends to become. When the response rate is large, the gas in each volume element becomes partly trapped, rather than squeezed out. This, in turn, causes the film "stiffness" in the annulus to increase and permit greater "compliance" of a deeper recess without compromising stability. When the mass is large, the changes in pressure and density approach the quasi-static condition, and the foregoing effects are reversed. These two cases are illustrated diagrammatically and in an exaggerated manner in Figure 6 for the conditions of a heavily (a) and lightly (b) loaded bearing. In the circular bearing the surface corresponding to the annulus constitutes 96 percent of the total area and the slot converges toward the recess periphery where the gas density is highest. If the boundary motion is relatively rapid, the gas cannot readily escape from the slot.

Because of this, the influence of mass on stability is more pronounced in the case of the circular bearing, as may be observed by comparing curves I, II and III in Figure 5. It can also be seen, that, as the mass increases, the agreement between results of the present and simplified analyses improves, because changes occur nearly quasi-statically.

The simplified, lumped-parameter analysis is based on assumptions of quasi-static changes in pressure and gas content and does not properly account for the effect of mass on the rate of response. In fact, when the inequality (A2–7), Appendix 2, is written explicitly:

$$\left(\frac{A_3 + A_5}{A_2}\right)\left(\frac{A_e}{m}\,\frac{A_1}{A_2}\right) > \left(\frac{A_e}{m}\,\frac{A_4}{A_2}\right) \tag{45}$$

the quantity m cancels in the denominators on both sides, so that the stability condition is independent of mass. Though perhaps deficient from the quantitative point of view, the simplified, lumped-parameter analysis, Appendix 2, does give explicit relationships between other parameters which affect the stability of the bearing. The inequality (A2–8), for example, can be written as follows:

$$\frac{\left(\dfrac{\partial M}{\partial H}\right)_o}{\left(\dfrac{\partial M}{\partial P_r}\right)_o} > \frac{\dfrac{\partial}{\partial H}(W_2 - W_1)_o}{\dfrac{\partial}{\partial P_r}(W_2 - W_1)_o} \tag{46}$$

in which, for a given bearing geometry,

$$M = f(T_o, P_{at}, P_{so}, P_r, H)$$

$$(W_2 - W_1) = g(T_o, P_{at}, P_s, P_r, H)$$

It follows that the stability condition also can be written as:

$$-\left[\frac{dP_r}{dH}\right]_{M = M_o} > -\left[\frac{dP_r}{dH}\right]_{W_1 = W_2 = W_o} \tag{47}$$

The physical interpretation of the above inequality is as follows. The left side of inequality (47) represents a change in recess pressure caused by a rapid change in gap width, each volume element acting as a closed cylinder in which the gas becomes trapped. The right side of inequality (47) represents a change in pressure caused by slow, quasi-static displacement, the annulus having a flow-regulating, rather than gas-trapping effect.

4. Conclusions and Summary of Theoretical Results

An analysis was made which treats the flow within the gas film in a thrust bearing on a continuous, rather than "lumped," basis. It shows that within the limitation imposed upon the design of the bearing by other considerations, the following parameters or combinations of parameters should be minimized or maximized in order to insure stability:

Minimized	Maximized
1. Depth of pockets	1. Supply nozzle diameter
2. Difference between supply and recess pressures	2. Length of annulus
3. Effective mass of bearing	3. Area ratio of annular and pocket regions

One may also conclude from results of earlier, simplified analyses [2,3,6] which show good qualitative agreement with the present one, that high gas temperatures and adiabatic conditions favor stability.

The most important factor influencing the stability of externally pressurized bearings is the interaction of the total gas-storage capacitance with the flow-regulating characteristics of a fixed supply restrictor and a variable annulus. The ratio $L_o mg$, where L_o is the load and m the mass of the bearing, also influences stability. Deeper pockets can generally only be tolerated for very moderate pressure drops across the supply nozzle. The limiting depth increases with the nozzle diameter and decreases with the length of the annulus, as well as with the mass of the bearing. These trends are illustrated in Figures 3 and 5.

The advantage of recessed pockets, channels, or grooves is that they increase the load capacity and static stiffness. The recess perimeter also provides a larger area of admission into the annulus, thereby reducing entrance velocities and preventing shocks [19, 20, 21]. These advantages, however, must be weighed very carefully in the light of requirements dictated by stability conditions.

IV. EXPERIMENTAL APPARATUS

1. Design of Experimental Apparatus

The main objectives in designing the test apparatus were the following:

a) To insure maximum flatness and rigidity, as well as hardness and material stability, of the thrust plates.

b) To provide means of precise alignment, and of accurate methods for checking the parallelism and separation of the bearing surfaces.

c) To restrain the mode of oscillation to an axial direction with minimum friction.

d) To place a variable-volume, leak-proof cavity in the center of one thrust plate and to provide for precise adjustment and accurate measurement of its depth.

e) To furnish means of applying a constant, non-massive, axial load which would not be affected by the motion of the bearing.

f) To design for maximum rigidity of the entire structure for the purpose of minimizing deflections and preventing a condition of system resonance within the anticipated frequency band width of oscillations.

g) To provide sensitive vibration pick-ups and associated electronic and photographic apparatus in order to detect, measure and record amplitudes of oscillations which are small in comparison with the magnitude of the equilibrium gap width.

h) To furnish a well regulated, relatively clean and moisture-free air supply. To select reliable pressure gauges, manometers and flow meters for the purpose of determining the static characteristics of the test bearing.

The test bearing assembly drawing is shown in Figure 7. The sturdy construction of individual parts and of the entire structure, particularly the plates and columns, is self-evident. All members were machined to extremely close tolerances, subjected to proper heat treatment and inspected prior to, during, and after manufacture and assembly. The upper thrust plate (1)† could be aligned by means of three 2 inch diameter, 48 threads per inch, adjusting screws (2) and securely bolted to the supporting columns (27). The lower thrust plate was integral with the shaft (21). Both thrust faces were chrome plated and lapped flat to the order of 1-1/2 interference bands of monochromatic sodium light (approx. 20×10^{-6} inches). Final inspection, using 6 inch diameter optical flats, showed that the convexity of a symmetric and almost circular central region of both plates did not exceed one interference band up to approximately four-fifths of the 5 inch diameters. Three sensing probes (4) of an air gauging system, incorporated in the upper plate, were equally spaced along the perimeter of the thrust face and lapped together with it.

An exact determination of deflections and modes of vibrations of the thrust plates was not attempted because of the complexity of the problem involving thick plates, supported in a manner described in the foregoing. A conservative estimate, based on approximate plate theory, the

† Circled numbers correspond to those in Figure 7 and refer to assembly components.

assumption of simple support along the column pitch circle diameter and a concentrated 500 lb. load, showed that the deflection of the center of the upper thrust plate with reference to its edge was of the order of 20×10^{-6} inches. The deflection of the lower thrust plate, in which the cantilever-like, overhung ring was subjected to pressures which in that region approached the ambient value, would be negligible in comparison. It was concluded, therefore, that load deflections would have a compensating effect on the slight, initial convexity of the surfaces.

The theory of vibrations of plates, even in approximate form, is very complex. On the assumption that the frequency, p_1, of the lowest mode of vibration could be estimated from the equation [58]:

$$p_1 = a \left(\frac{Et^2}{12r^4\rho(1-\nu^2)} \right)^{1/2}$$

in which a is a constant depending on the method of support, E is Young's modulus, ν the Poisson's ratio, ρ the mass density of the metal, r and t the radius and thickness of the plate, the lowest frequency was estimated to be of the order of 10^4 radians per second and, consequently, considerably higher than the anticipated frequency band width of auto-oscillations.

The upper thrust plate, Figure 8, contained a centrally located recess, the depth of which could be varied by means of a sliding piston (12) and a micrometer adjusting screw (15). Three tightly fitted "O" rings served as sliding seals. The piston could be locked in any desired position and the recess depth varied from zero to 3/4 inches. Air was supplied through a 7/16 inch diameter, drilled duct in the upper plate. It flowed into a chamber, the volume of which comprised the sealed space between the "O" rings and the bore of the piston, Figure 9. The flow restriction between this chamber and the recess was effected by means of a short, bell-mouthed, 0.055 inch diameter nozzle (8).† The drawing of an auxiliary piston (Figure 10), shows a 0.0135 inch diameter pressure tap, located at 11/32 inches from the piston center. The tap was connected through an internal duct to a gauge which indicated the static, equilibrium recess pressure at that point. In the modified version of the piston, the pressure tap was centrally located and plugged when dynamic tests were conducted.

The drawing of the shaft and lower thrust plate is shown in Figure 13. The shaft was hollow so that the movable mass corresponded to 25 lbs. The weight could be increased to 45 lbs. by attaching a 20 lb. bronze disc to the shaft, midway between the 3 inch journals. (It will be recalled that theoretical stability loci were constructed for masses corresponding to weights of approximately 13.9, 25 and 45 lbs.) Rigidity requirements, however, precluded a less massive shaft and thrust plate construction and no provision could be made to obtain experimental data for the smallest mass. The 4 inch diameter, circular area of the plugged end of the shaft provided a surface exposed to the pressure of the loading cylinder (20) by means of which the thrust load was applied to the bearing. Impulse turbine buckets were milled in the shaft to provide a high speed drive for experimental work involving rotation which could be conducted upon the completion of

† In a modified, later version of the piston, provision was made for replacement of a single restrictor by 4, 9 or 12 nozzles. In each case the total cross-sectional throat area was adjusted to give approximately calculated rates of mass flow (see Figures 11 and 12).

this phase of the investigation.† The size, concentricity, roundness and taper of the shaft journals were found to fall within tolerances specified on drawings. The total indicated runout of the thrust face with respect to the journals, measured on a 5-3/4 inch diameter, was less than 0.0002 inches. The shaft was balanced to within 0.00095 inch-pounds.

The problem of restraining the motion of the shaft and the lower thrust plate to a purely axial direction by means of low-friction guides was solved by mounting the shaft in externally-pressurized, air-lubricated journal bearings (24). There were two main reasons for this type of lateral support. First, the analysis was predicted on the assumption that external damping was negligible, since primary interest was only in those stability parameters associated with the bearing itself. Secondly, from the practical point of view, it was desirable to simulate as far as possible a method of gas bearing support which could be eventually applied to small turbines, compressors or centrifugers. A drawing of one bearing is shown in Figure 14. The main feature of the design was a set of six, grooved, centrally located and equally spaced pressure pads. Air was supplied to the pads through 0.016 inch diameter, brass orifices (25), shown in Figure 15. The purpose of the grooves was to minimize the storage volume of each pad, while extending a relatively uniform, high-pressure area to the outer perimeter of the grooves. The net effect of this arrangement contributed to the stability of the bearing and produced a steep load-deflection characteristic, Figure 16. Needle valves (26) were placed in drilled air ducts leading to alternate pads. The function of the needle valves was to allow for controlled misalignment of the shaft within the limits of the bearing clearance. By varying the pressure in individual pads, an excellent method of making final adjustments in the alignment of the thrust surfaces was provided, furnishing improvement on the high degree of parallelism already achieved by means of the levelling screws.

The bearings were dowel-pinned and bolted to the supporting members prior to final machining, during which both bearings and the sleeve of the loading cylinder were ground to size in one operation. The grinding operation was performed with a degree of precision which made it possible to span all three bores with a single plug gage, barely 0.0003 inches smaller than the minimum, measured bearing diameter. All grooves were indicated on the grinding machine and their depth varied from 0.006 to 0.007 inches. The orifice inserts, needle valves and the annular supply chamber of the bearing were sealed by means of "O" rings.

It was estimated that a bearing stiffness†† of 3×10^5 to 5×10^5 pounds per inch could be realized when the journal was in the concentric position. With the bearings and the shaft assumed as a spring-mass system having two degrees of freedom, approximate values of frequencies of natural modes of vibrations were calculated by considering a pivoting motion of the shaft about one bearing, the other journal being acted upon by a spring. The "constrained" frequencies, p, are given by [58]:

$$p = \ell \left(\frac{k}{I_c + m \ell_c} \right)^{1/2}$$

† The grooves at the end of the shaft were intended to suppress instability (whirl) due to hydrodynamic forces in the clearance of the loading cylinder sleeve.

†† The slope of the load-deflection curve is sometimes referred to as the "stiffness" or more correctly, as the "static stiffness". (See Figure 16.)

in which ℓ is the span length between the bearings, ℓ_c the distance from the chosen pivot to the center of gravity of the shaft, k the stiffness, m and I_c the mass and moment of inertia of the shaft about its center of gravity. The estimated natural frequencies for the 25 lb. shaft were in the order of 3×10^3 and 5×10^3 radians per second and could be substantially reduced, or raised, by varying the supply pressure. It was concluded, therefore, that in the absence of appreciable unbalance and moments in planes passing through the shaft axis, the pitching motion of the shaft within the clearance of the journal bearings would be relatively small in comparison with the amplitude of axial oscillations.

The loading cylinder (20) consisted of a 7-1 2 x 3-1 2 inches pressure vessel attached to the lower end of the supporting structure. A constant pressure maintained in the cylinder and acting on the end of the shaft protruding through the cylinder sleeve, balanced the load applied to the lower thrust plate. For small amplitudes of oscillation, volume changes would be negligible in comparison with 155 cubic inches of cylinder displacement. The cylinder pressure would remain essentially constant, unless the frequency of oscillation produced a condition of accoustic resonance in the chamber, in which case the shape and volume of the cylinder could be altered accordingly. The cylinder provided a simple method of applying a constant, non-massive load to the test bearing.

The lower ends of the 3 inch diameter columns of the test rig were press-fitted into a 2 inch thick plate (19) in which one of the journal bearings was mounted. The upper ends of the columns were located and bolted to a similar plate (18) containing the second journal bearing. The column locating holes in both plates were jig bored in tandem. The concentricity and squareness of the columns and locating bosses were indicated from the head of the precision grinder after the bearings and the bore of the loading cylinder sleeve were finished to size. The assembly was bolted to a massive and very rigid tripod. The weight of the entire structure was approximately 750 lbs. In the course of experimentation it was found necessary to interpose relatively soft isolators between the tripod and the foundation floor. The reason for doing so is discussed later in the text. A schematic of the air supply system is shown in Figure 17. Photographs of test apparatus, its components and of auxiliary equipment are shown in Figures 18 through 25.

2 Instruments -- Performance and Calibration

a. Air Gauges

The threefold function of the air gauging system, briefly referred to in the foregoing section, was (a) to facilitate the alignment of thrust surfaces to a high degree of parallelism, (b) to measure the equilibrium gap width, Ho, and (c) to provide a quick and direct method of calibrating the primary capacitance probe without disturbing its initial position and offset.

The well-known method of measuring small displacements in terms of pressure utilizes two flow restrictors, placed in series, a constant, well regulated pressure source and suitable pressure indicators. Air enters an intermediate chamber from a constant pressure reservoir through a small, fixed (or adjustable) orifice. The variation of pressure in the intermediate chamber is recorded and depends on the supply pressure level, as well as the relative size and flow characteristics of

the restrictors. The cross-sectional flow area of the second restrictor is made to vary with displacement, so that resistance to flow is furnished by a slot, or a circumferential orifice formed between the surfaces of the sensing probe and the displaced member.

For a fixed orifice size, the slope and curvature of the pressure-displacement characteristic depend mainly on the size and geometry of the probe restriction, and on the magnitude of the supply pressure. The characteristics are nonlinear and exhibit an inflection point, in the neighborhood of which the amplification is highest. The probe can be made in the shape of a short, thin-walled or a thick-walled cylinder. In the latter case, the restriction between the probe and the displaced member has the form of a narrow, radial slot rather than that of a circumferential orifice. All other factors being equal, a slot-like, variable restrictor produces a higher sensitivity, though at some expense of linearity in the pressure-displacement characteristic†. This effect is due to a more pronounced influence of viscous forces in a narrow slot than in a circumferential orifice.

It was considered that a low supply-pressure gauging system, integral with the upper thrust plate, would have an advantage over commercially available units, which generally operate at supply-pressure levels of thirty to sixty pounds per square inch. The system was to be conveniently incorporated into the test rig assembly, with probes rigidly mounted in the upper thrust plate and lapped in the plane of the thrust surface. A well regulated supply head of two to three feet of water could be obtained by means of a standpipe, with the excess air in the supply reservoir being allowed to escape at the end of an immersed tube. It was also desirable to avoid the impingement of relatively high velocity probe jets associated with high pressure systems, since this impingement might have involved difficulties in calibration, and possibly minor effects of heat transfer on the accuracy of measurement.

An estimate of orifice and probe dimensions for a low pressure system was made by assuming the flow to be incompressible. Dimensionless pressure-displacement characteristics were obtained by equating the inflow through the orifice to the outflow through the probe restriction, considered either as (a) a narrow, radial slot or (b) a circumferential orifice. Results of these preliminary calculations used in establishing approximate values of design parameters are shown in Figure 26, which also contains a schematic of the supply head regulator.

The effect of changes in barometric pressure on gap width measurement could be estimated from equations (8) and (9), the flow through the orifice and probe restrictions being essentially analogous to that through the bearing supply nozzle and gap. The gap width is then given by:

$$H_o^3 = K \frac{(P_s P_{ro} - P_{ro}^2)^{1/2}}{P_{ro}^2 - P_{at}^2}$$

† The principles involved in determining the pressure-displacement characteristic of an air gauge are essentially similar to those which apply to load-displacement characteristics of externally pressurized, gas bearings.

in which P_s, P_{ro} and P_{at} now refer to the absolute values of the supply, intermediate and ambient pressures of the air gauge system, and K is a constant. Taking the logarithm of both sides of this expression and differentiating, the approximate error in gap width measurement, associated with a change of $\pm \Delta P$ in barometric pressure, is:

$$\frac{\Delta H_o}{H_o} \simeq \mp \frac{\Delta P}{6P}$$

where in the last step it was assumed that the pressure differences are sensibly small, e.g., $P_s - P_{ro} = P_{ro} - P_{at} = P$. A change of $\pm$ 0.5 psi, relative to the barometer reading during calibration, would cause an error of $\mp$ 0.5 percent in the measured gap width, less than 10 microinches.

Initial difficulties were encountered because of supply pressure fluctuations caused by the surface tension of bubbles, forming and breaking away from drillings at the immersed end of the tube. This was circumvented by inserting a damper in the form of a series of perforated discs in the reservoir end of the tube and reduced the ripple to an almost imperceptible magnitude, relative to the smallest manometer scale reading of 1/10 of an inch. The supply pressure was adjusted to a steady, constant value of 39.2 inches of water by varying the setting of a fine, long taper needle valve.

The calibration was performed in steps of 0.0001 inches over a gap width range from 0.001 to 0.002 inches. Square, 1 x 1 inch precision gauge blocks, having a dimensional and parallelism tolerance of 1 microinches, were used for this purpose. Two blocks, differing in thickness by a multiple of 0.0001 inches, and a third block were wrung together. The step gauge created in this fashion was then wrung together with the lapped thrust surface, the step extending from the edge of and overlapping the entire probe diameter. (See schematic in Figure 27.) The probes were spaced 120 degrees apart and located 1/4 of an inch from the edge of the upper thrust surface. The blocks were taken apart and reassembled after each round of manometer readings for all three probes. This was repeated 21 times for each probe and everyone of the 11 calibration points, a total of 691 manometer readings. The gauge blocks were assembled at random and it was reasonable to assume the arithmetic average of 21 manometer readings as the most probable value corresponding to the nominal dimension of the step gauge offset. The curvature of the pressure-displacement characteristic in the experimental range of gap width was very slight. The maximum difference in any set of 21 manometer readings was less than 0.4 inches of water. At a representative sensitivity of 0.15 inches of water per 10^{-5} inches displacement, this was equivalent to 2.7 x 10^{-5} inches, or less than 3 percent of minimum gap width in the experimental range. Least-square-fitted calibration curves, Figures 27, 28 and 29, were considered adequate for gap width measurement to an accuracy of 1 to 2 percent.

b. Vibrometers – Primary and Auxiliary Probe Systems

The validity of assumptions made in the analysis increases if the amplitude of motion is small in comparison with the equilibrium gap width. For reasons arising from practical considerations, such as the impossibility of insuring perfect isolation, ideal rigidity of structural members, absolute steadiness of air flow etc., the feasibility of detection and, above all, meaningful interpretation of amplitudes in the order of 10^{-6} inches was doubtful. It was arbitrarily decided, therefore, to regard amplitudes of oscillation in the order of one percent of the gap width, or to 10^{-5} inches, as small.

The function of each probe system was to interpret dynamic capacitance changes in terms of relative displacement of the probe and the vibrating member. The primary probe system, shown on the left side of Figure 30 was used in measuring the axial oscillations with which the experiments were mainly concerned. The auxiliary probe system, shown on the right side of Figure 30 employed another capacitive pick-up, which could be mounted in several positions along and around the shaft. The function of the auxiliary probe was to detect transverse modes of vibrations which might be coupled with the axial motion of the shaft. It was, of course, desirable to reduce this motion to a bare minimum and the air-lubricated guide bearings were designed with this objective in mind. The auxiliary system thus played a monitoring role in the course of experimentation. The operation of the two vibrometers was based on different principles. Their operation, performance and calibration are described in the next paragraphs.

In the schematic diagram of the primary instrument, a Fielden Proximity Meter, model TM-051-1, the variable probe capacitance, C_x, is shown shunted across one of the two balancing capacitors of an impedance bridge. The source of bridge excitation was a 450 K.C. oscillator (modulated by 2.4 Kc when using the meter for static measurements). The out-of-balance voltage of the bridge was applied in succession to the tube grids of a two-stage, radio frequency amplifier. The gain of the amplifier could be controlled by a variable, tapped resistor in the cathode circuit of the amplifier tubes by means of a six-position switch, providing an attenuation up to 50 decibels in steps of 10 decibels. The bridge could be balanced by means of capacitors, C_1 and C_2, and the potentiometer R, which balanced the probe lead resistance. The frequency response of the instrument, Figure 31, was flat from D.C. to approximately 1200 cps and, therefore, more than adequate for measurements in the anticipated 100–400 cps band width. The output of the instrument was applied to one pair of terminals of a dual trace preamplifier of the cathode ray oscilloscope. The preamplifier had nine sensitivity settings in the range of 0.05v cm to 20v cm and a variable vernier adjustment. The frequency response of the oscilloscope-preamplifier unit (Tektronix, type 532, and dual-trace preamplifier, type 53-540) was 3 decibels down at 5 megacycles, considerably higher than that of the Fielden instrument.

The primary probe, Figure 24, consisted of a 1.4 inch diameter, brass disc, mounted in an insulating sleeve. The disc and sleeve assembly were pressed into the opening of a 4.5 inch long probe holder and lapped square with its axis. The body of the holder, which also served as part of the outer shield was rigidly clamped to the upper thrust plate by means of a split collet and a gripping clamp. With the radio frequency amplifier control adjusted to approximately one third of maximum gain and a preamplifier vertical deflection setting of 1 volt per centimeter, the sensitivity corresponded to 1 centimeter vertical deflection per 10^{-5} inches displacement when viewed on the display of the cathode ray oscilloscope. The observed noise level and drift at this setting, after an initial, one hour warm up period, was of the order of 0.05 to 0.1 volts. The maximum room temperature variation during one test run was between 74 and 77 degrees Fahrenheit. The probe lead was double-shielded to prevent stray voltage pick-up and relatively short, in order to minimize capacitance in parallel with that of the probe. For this reason also, the thickness of the insulating sleeve (dielectric constant of 2) was 0.1 inches and, therefore, large in comparison with the initial, 0.003 inch offset between the 1.4 inch probe disc and the lower thrust plate.

Following the parallel alignment of thrust surfaces, the probe was calibrated in three positions, corresponding to approximately the end and midpoints of the experimental range of gap widths. The calibration was performed, using the air gauges as a standard. The calibration

curves are shown in Figures 32, 33 and 34. From their appearance it could be concluded that for amplitudes of displacement in the order of 1×10^{-5} inches to 2×10^{-5} inches, no large error would be incurred by assuming linearity in the neighborhood of null points and a sensitivity value of 1 volt per 10^{-5} inches displacement.

The schematic diagram of the auxiliary instrument (Decker, Delta Unit 902–1 and Monitor Unit 902–2) shows the capacitance probe, C_y, mounted in one of the positions along the shaft. The main component of the system was a tube, containing gas† under reduced pressure. The tube was placed between two external electrodes connected to a 250 KC oscillator. The excitation caused a luminous discharge, so that the two internal electrodes of the tube were in contact with the plasma of the ionized gas. Any asymmetry in the spacing of the electrodes would give rise to a potential difference between the internal electrodes, caused by the migration of electrons. The effect of asymmetry could also be produced by changing the magnitude of the sensing capacitor, C_y, relative to the balancing capacitor, C_o, both shown in the schematic diagram of Figure 30. The operation of the auxiliary probe, C_y, was based on this principle [62]. A cathode follower provided a low impedance output at the preamplifier terminals and a filtering network eliminated the fraction of oscillator excitation passed through the transducer.

The transducer, probe, mounting bracket and calibration fixture are shown in Figure 25. The probe and the balancing capacitor were mounted very close to the internal tube electrodes and all leads were relatively short in order to minimize the adverse effect of stray capacitances on the output and frequency response of the instrument.

The frequency response of the auxiliary instrument, Figure 31, was flat to approximately 300 cps. 0.5 decibels down at 400 cps and, therefore, quite adequate in the anticipated, experimental frequency band width. The calibration curve of the auxiliary probe, Figure 35, displayed a greater degree of nonlinearity than that of the primary probe. The sensitivity in the neighborhood of the null point was approximately 0.02 volts per 10^{-5} inches displacement, representing 0.4 centimeters vertical deflection per 10^{-5} inches displacement on the cathode ray oscilloscope display at the highest preamplifier sensitivity setting of 0.05 volts per centimeter. The observed noise level was of the order of 0.005 volts. Since the maximum sensitivity of the auxiliary probe was considerably less than that of the primary, the monitoring test of lateral shaft vibrations was conducted under conditions of axial oscillations approximately twenty times as large as the amplitudes of self-excited vibrations observed in the course of stability experiments.

c. Pressure Gauges, Flowmeters and Regulators

The supply, recess and loading piston pressures were measured by means of two, dual bourdon tube construction, eight and one half inch dial diameter, absolute pressure gauges. The gauges were graduated in steps of 0.1 and 0.2 psi over their respective ranges of 0–100 and 0–150 psia. The ranges were covered in approximately two revolutions of the pointer. The scale had a mirrored surface reflecting the fine blade of the pointer to eliminate parallax errors and the size of the smallest subdivisions permitted reliable estimates of readings to the nearest 0.05 psi. The pressure gauges were calibrated by the manufacturer by means of a precision mercury barometer, a duplicate of a Bureau of Standards reference barometer. The accuracy of the pressure

† Argon or Neon

gauges was guaranteed to be within 0.02 percent of full scale deflection, representing 0.027 percent for the 0–100 psia gauge and 0.041 percent for the 0–150 psia gauge at 73.5 psia, the lowest test reading. The scales were hand drawn to match over one hundred calibration points. The hysteresis, backlash and "dead zone" of the pressure gauges were entirely negligible. As an additional check, the gauges were connected to the same pressure source and their readings compared. No perceptible difference was observed in the range from 50 to 100 psia.

Flow measurements were performed, using a variable area flow meter (rotameter) in conjunction with a calibrated, bellows type gas meter. The position of the rotameter float was registered along the scale of the 600 millimeter long tube for any particular setting of the bearing supply pressure. With this setting maintained and the position of the rotameter float unchanged, the flow was diverted to the gas meter. The rotameter thus merely served as a secondary, indicating instrument, while the actual flow rates were determined from the gas meter readings. The calibration of the gas meter was performed by means of an inverted-tank-type gasometer of 20 cubic feet capacity, used as a standard by a local utility company. Five calibration runs were made after the final adjustment of the counter mechanism, at a representative bearing flow rate of 5 cubic feet per minute, referred to standard atmospheric conditions. (Approximately four revolutions of the five cubic foot counter.) The differences in the final positions of the gasometer displacement indicator were negligible in comparison with the total displacement. The speed of the counter, approximately one revolution per minute, was slow enough to enable the observer to estimate the position of the pointer, relative to a predetermined mark, at least within 6 degrees, or better. This, in terms of displacement, corresponded to approximately 0.5 percent of the gasometer volume.

The rotameter float was operated in the upper half of the tube. Slight oscillations of the float were observed in the uppermost tube region, but their peak-to-peak amplitudes were of the order of 2 to 3 millimeters, as compared with the 600 millimeter length of the tube. An estimate of 2 to 3 percent accuracy in flow measurements obtained in the course of experimentation appears to be reasonable.

The pressure regulators used in the test-bearing air-supply system, Figure 17, were capable of maintaining a downstream pressure setting within 0.1 psi over their entire flow range and a 25 psi variation in delivery pressure. One regulator was installed ahead of a 7.5 cubic foot air storage tank and individual regulators were placed in the supply lines leading from the tank to the loading cylinder, test and guide bearings. The performance of the regulators was most satisfactory and pressure settings could be accurately maintained, with drift not exceeding 0.05 psi over periods of two to three hours.

V. EXPERIMENTS

1. Static Tests

The calibrated air gauges furnished the means of bearing surface alignment. The 25 lb. shaft was inserted in the pressurized guide bearings and the upper thrust plate placed on the column locating bosses. The concentricity of the bearing members was checked and adjusted to within 0.001 inches. The thrust surfaces were then brought into parallel alignment by means of levelling screws and air gauges and secured to the columns. As an additional check of parallelism, the following test was performed. A spacer, approximately 1.53×10^{-3} inches thick, was lightly clamped between the thrust surfaces at twelve, thirty degree, angular intervals along their perimeter. Manometer readings of one air gauge only were registered for each position. This was repeated nine times and the arithmetic average of readings for each position was taken. An identical test was carried out with a spacer 1.53×10^{-3} inches thick. In the process of sampling, the spacer was clamped by a constant, 0.5 lb. force, at an average distance of two inches from the bearing center. The guide bearings, spaced nine inches apart, were pressurized and their stiffness in the concentric position, Figure 16, was in the order of 4×10^{5} pounds per inch per bearing. Any misalignment, therefore, caused by the moment of the small clamping force, would be negligible. The results of this alignment test are shown in Figure 36, in which the circularly spaced numbers refer to average manometer readings of gauge No. 1 obtained with both spacers. By reference to the calibration curve in Figure 27, it can be seen that the thrust surfaces were parallel within one percent of the equilibrium gap widths, that is inclined under an angle in the order of 4×10^{-5} degrees.[+] The shaft thrust face had a total indicated runout of 0.0002" relative to the journals and the alignment would have been destroyed, had the shaft been allowed to drift in its guide bearings. An additional constraint, therefore, was furnished by means of two opposed and narrowly spaced air jets, impinging on a light, thin fin, attached to the shaft collar. (See Figure 18, for example.) The alignment was thus performed with the shaft maintained in the same position relative to the guide bearings.

The analysis was based on the existence of an essentially uniform pressure plateau, extending over the entire, one inch diameter recess area. It was postulated that, as long as the recess depth remained relatively large, the effects of the air jet issuing from the 0.055 inch diameter nozzle and the acceleration of the fluid as it entered the slot would be local and detract little from the recess volume conceived as a constant pressure reservoir. Since the stability boundary for critical recess depths in the order of 10^{-2} inches was of particular interest, the question which presented itself was the degree to which the foregoing suppositions were valid for recess depths smaller than the nozzle throat diameter and comparable in magnitude with the bearing gap width. The following possibilities deserved careful consideration. (a) For $\delta < 1.1\, d_N$, the cylindrical area bounded by the nozzle diameter and its projection on the lower thrust surface would replace the nozzle throat as the controlling restriction. (b) The recess, its depth sufficiently reduced, could act as an inward extension of the bearing gap and virtually cease to function as an intermediate reservoir. (c) The accelerating flow in the diverging region adjoining the nozzle exit could attain supersonic velocities, followed by shock and a transition to the predominantly viscous flow

[+] Interference fringes observed under optical flats indicated that each 1.5×10^{-5} to 2.0×10^{-5} plate convexity was quite concentric and approaching in shape that of a surface of revolution. Parallelism, or inclination, therefore could be thought of in reference to the circular bases of those elevations.

region in the slot [19, 20, 21], a situation analogous to that which might exist in a duct fed by a converging diverging nozzle.

A test was conducted, in which the load and a representative value of the supply pressure, 87.5 psia, were held constant, while the recess was being reduced in depth from a relatively large value. The objective of the test was to determine the "cut-off" point, at which any further reduction in depth would cause a change in the steady-state flow pattern and pressure distribution. This would then be accompanied by an easily detectable and measurable departure from the initial gap setting. Results of this test, presented in Figure 37, indicate that, for a centrally located, 0.055 inch diameter nozzle, the "cut-off" point corresponded to a recess depth of 0.030 inches.

Since the critical stability parameter was the volume, rather than the depth of the recess, the first, successful attempt at a remedy was to surround the nozzle exit by a 0.375 inch diameter, 0.020 inches deep cavity (see Figure 22). This, however, did not provide an entirely satisfactory solution, since for small critical volumes the delineation between slot and recess became rather vague. For this reason, and because the linear dimensions of the restrictor influence the degree of unsteadiness in pulsating flow, the pistons were replaced by modified version, Figure 11, which incorporated interchangeable, multiple-nozzle plates. The plates, Figures 12 and 23, contained either four, nine, or twelve short, bell-mouthed nozzles, equally spaced on a 0.5 inch diameter circle. Their total, cross-sectional throat area was later adjusted to give a flow characteristic approximating the calculated one within the experimental range of pressure ratios. The twelve nozzle plate was used in the concluding experiments. At a supply pressure of 87.5 psia, a reduction in recess depth from 0.008 inches, the "cut-off" point, to 0.006 inches, was accompanied by a decrease in gap width in the order of two percent only. The effects of decrease in recess depth on gap width for single and multiple nozzle feeders are compared in Figure 37.

The calculated load capacity of the test bearing, based on the assumptions of laminar flow in the clearance, negligible entrance effects and uniformity of pressure in the recess, varies almost linearly with the recess pressure and is independent of the supply pressure.

In order to ascertain the extent to which the actual conditions conformed to this idealization, a test was conducted in which (a) the load was adjusted to maintain a constant recess pressure† of 73.5 psia, or (b) the load was held constant at the calculated value of 472 lb, and the recess pressure† was allowed to seek its own level. The result of this test is presented in Figure 38 and indicates that in case (a) an increase of the supply pressure to 95 psia was accompanied by a 5 percent reduction in load capacity, whereas in case (b) a 4 percent increase in recess pressure was observed. Throughout this test the recess depth was larger than the "cut-off" value, referred to in the preceding paragraphs.

The desirability of minimizing differences between the actual and theoretical, steady-state bearing characteristics was of considerable importance with reference to the mass flow rate, since its partial derivatives with respect to gap width and pressure, equation (16b), played an important role in establishing the limiting values of stability parameters. Preliminary flow measurements indicated, that in the experimental range of pressure ratios the overall discharge coefficient, using a

† Pressure tap centrally located. Recess depth $\delta > 0.010$ inches.

twelve nozzle supply restrictor, varied approximately from 0.78 to 87. The total throat area was then increased by approximately 10 percent, in order to narrow down the gap between actual and calculated flow rates.†

Mass flow rates were determined in the range of supply pressures from 75 to 95 psia, with a constant load of 172 lb., the calculated value corresponding to a recess pressure of 73.5 psia and ambient pressure of 14.7 psia., applied to the test bearing. (A condition analogous to that under which stability tests were conducted.) The recess depth was always maintained at a value greater than the "cut-off" point referred to in the preceding paragraphs. The mass flow rates in the experimental range of supply pressures, obtained with slightly increased nozzle diameters, were 94 to 97 percent of the calculated values, while deviations of measured gap widths were in the order of one percent. Results of mass flow and gap width measurements are presented in Figures 39 and 40. The corresponding bearing Reynolds number, $N_R = \left(\frac{\rho_r \bar{V}_r H}{\mu}\right)_o = \frac{\dot{W}_o}{2\pi\mu R_r}$ is given in Figure 41. Mass flow rates and the Reynolds number obtained in a supply pressure range for a bearing load of 222 lb. and corresponding to a stability test conducted under reduced load conditions are given in Figures 42 and 43.

It seemed highly unlikely that absolute, one-to-one correspondence between calculated and actual, steady-state values of pressure, gap widths, and flow rates could be simultaneously achieved throughout the experimental range. It was considered that, in the presence of slight deviations in several static bearing characteristics from those of the idealized model, a fair compromise was reached, and that the deviations were of a reasonable order of magnitude.

2. Preliminary Stability Tests

Initial differences in calculated and actual, static bearing characteristics and subsequent modifications incorporated in the apparatus, were described in the preceding section. These modifications, and a number of additional changes, were also dictated by evidence gathered from results of simultaneously conducted, preliminary, dynamic tests. The preliminary tests furnished considerable information, of interest in itself, and useful from a practical point of view. In this section an account of preliminary experiments is presented in a logical sequence, not necessarily in the order in which they were conducted.

The dynamic experiments, with the exception of one comparison test, in which the recess pressure†† was held constant, were conducted under constant load conditions. Results of static tests indicated that, in the latter case, deviations from predicted, static bearing characteristics appeared to be less pronounced. Moreover, since the recess pressure could be measured at one point only, there remained the uncertainty of its distribution in the entire recess region, whereas, on the other hand, the load could easily be set and held constant at a predetermined value.

---

† Flow rates were calculated for a 0.055 inch diameter nozzle. The single nozzle was replaced by twelve nozzles and their diameters were enlarged from 0.0159 inches to approximately 0.0167 inches.

†† As measured by a tap located at a radial distance of 0.344 inches from the center.

The tests were conducted in the following manner. The load was applied to the bearing and the supply pressure adjusted to a predetermined value. The recess depth was then increased (or decreased) quasi-statically, until the bearing became unstable. In certain cases the procedure was reversed, e.g., the recess depth was set and the supply pressure increased (or decreased) very gradually, up to the threshold of instability. The axial oscillations were detected by the primary capacitance probe and viewed on the display of the cathode ray oscilloscope. Photographic records of amplitudes and frequencies of oscillations were made for a number of runs. Simultaneous check on previously determined, steady-state flow rates and gap widths was made for points in the immediate vicinity of the stability boundary. The boundary was always approached by proceeding from the stable region to the unstable region.

The first, preliminary stability test was performed, using a plane-faced piston and a centrally located, 0.055 inch diameter supply nozzle. Results of this test are shown in Figure 44. The abrupt transition in the lower branch of the curve, associated with the reduction in recess depth below the "cut-off" point (see Figure 37), provided yet another, valid reason for modifying the piston and, finally, replacing the single nozzle feed by a multiple-nozzle supply system. The upper branch of the stability boundary constituted a marked departure from the predicted, single-valued locus. This trend persisted in all subsequent experiments.

Figure 45 contains results obtained with a modified piston, having a secondary, fixed volume recess, but retaining a single, centrally-located supply nozzle. It can be seen that the unstable region enclosed by the curve corresponding to a constant load test was smaller than the unstable region bounded by the locus obtained when the load was adjusted in order to maintain a constant pressure at a tap, located 0.334 inches from the recess center. Both curves were marked by the absence of the sharp transition observed in the case of the plane-faced piston†, but continued to display two characteristic branches, separated by the point at which their slopes became infinite.

A secondary stability experiment was conducted in parallel with previously described, static tests, the objective of the latter being to assess the reliability of the assumption that the recess pressure at constant load was not appreciably affected by changes in the supply pressure. It was observed that when the 0.0135 inch diameter, 0.125 inch long pressure tap (see Figure 10) connected the recess to a relatively large, closed volume††, an effect, which could justly be described as hysteresis, was present. With the bearing stable at a particular setting of supply pressure and recess depth, the supply pressure was increased very slowly and gradually, particular care being exercised to avoid disturbances which might be large in comparison with those randomly produced and always present in the fluid, foundation and the surrounding atmosphere. Under those conditions it was possible to increase the supply pressure considerably above the value at which the bearing became unstable if jarred, displaced or subjected to a sharp impact in the axial direction. The supply pressure was then reduced to a level at which the bearing became stable again. At this level, instability could no longer be induced, regardless of rate and magnitude of the disturbance. Identical effects could be observed with the supply pressure held constant and the recess depth being varied quasi-statically. With an isolated recess volume (e.g., closed pressure tap), no hysteresis effects were present, at least no measurable difference in limiting values

† It is to be noted that in Figure 45 the ordinate is the mean recess depth, that is the recess volume divided by the recess area.

†† Consisting of drilled passages, bores and the volume of the bourdon tube.

of stability parameters was noted. Results of observations of this rather remarkable phenomenon are shown in Figure 16. It is realized, of course, that the region "unstable if disturbed" is quite arbitrary, since, ideally, the system could remain in the state of unstable equilibrium if all disturbances, no matter how small, were absent.

In the conduct of stability experiments described up to this point, the emphasis was rather on qualitative than on quantitative aspects of bearing performance. Subsequent tests, although their description and results are presented in this section under the heading of "preliminary", were carried out with considerably greater accuracy and precision of measurements. Closer spacing of intervals at which data was recorded and the determination of frequencies of auto-oscillations on the threshold of instability contributed to the detection and elimination of certain troublesome characteristics, inherent in the test rig, and subsequent improvements in correlation between predicted and experimental results.

Stability boundaries, obtained with a twelve supply nozzle, plane-faced piston, under constant load (472 lb) conditions, for 0.477×10^{-2} and 11.68×10^{-2} lb-sec^2 in effective bearing masses†, are presented and compared with predicted results in Figure 17. Measured and predicted critical frequencies, e.g., frequencies, ω_c', associated with limiting values of parameters on the observed threshold of instability and their calculated equivalents, for which the roots of the characteristic equation (31b) on the axis of imaginaries were $s = \pm i\omega_c$, are presented and compared in Figure 18. Results of a test conducted under reduced load (222 lb) conditions are shown in Figure 19. The stability boundaries referred to in the foregoing, although similar in shape to those shown in Figures 14, 15 and 16, exhibited yet another, unusual property in regions denoted by "resonance" on all pertinent figures. This peculiar trend in all loci, characterized by inflection points and abrupt changes in curvature, occurred invariably at frequencies between 220 and 225 cps, indicating the existence of resonance in the system.

The phenomenon described in the foregoing, a major source of discrepancy between theory and experiment, might well have gone unnoticed if critical values of supply pressure and recess depth had been recorded at more widely spaced intervals. Fortunately also, frequencies were recorded in parallel with other measurements, so that the local, abrupt transitions in several stability loci could be identified with the same value of approximately 220-225 cps. The preliminary dynamic tests indicated a considerable area of qualitative and quantitative disagreement between predicted and experimental results, far in excess of what might have been anticipated on the basis of comparison between calculated and actual, static bearing characteristics. There remained the problem of detecting possible sources of trouble inherent in the experimental apparatus and the application of corrective measures before final experiments could be conducted. Specifically, it was essential to (a) identify parts and components of the test rig assembly responsible for the existence of resonance at particular frequencies of auto-oscillation and (b) to investigate possible interactions between the motion of the bearing and fluid flow in regulators, supply lines, guide bearings, the loading cylinder, etc.

† The effective bearing mass consisted of the shaft and lower thrust plate assembly (Approximately 25 lb and 45 lb weights respectively).

3. Concluding Stability Experiments

This section begins with the description of preparatory test and adjustments of the experimental apparatus which preceded and were of considerable importance in the final, experimental phase of the investigation. It contains a detailed report of concluding experiments and ends with an account of tests, performed as a check of repeatability of results.

It was suspected that the singular trends associated with frequencies of auto-oscillations in the neighborhood of 220-225 cps, were caused by resonance of the entire test stand, considered as one large mass (Approximately 750 lb weight), supported on an elastic foundation. The possibility of exciting one of the graver modes of vibrations in the massive, upper thrust plate was highly unlikely, but could not be entirely excluded, because of the rather indefinite manner of support on three columns and levelling screws.

The test bearing assembly was bolted to a very sturdy and rigid tripod (see Figure 18 for example), mounted directly on an asphalt tile covered, concrete floor. Crude deflection measurements, using a dial indicator and the experimenter's weight for the applied load, yielded a natural frequency for the test rig mass and the elastic foundation of approximately 230 cps, a value extremely close to the narrow band in which the inflection points and rapid changes in curvature of stability loci were observed. In order to measure the natural frequency of this mass-spring system, as well as the response of the upper thrust plate, it was decided to utilize the floated shaft as the seismic mass and the primary capacitance probe as the sensing element of a vibrometer, providently, though inadvertently incorporated in the test rig design. A very minute, axial load was applied to the shaft thrust plate and balanced by the pressure in the loading cylinder. This furnished the desired "soft" suspension for the floated mass and a correspondingly low, damped natural frequency of less than 1 cps. The base of the tripod was then pressed down and released, while the response was recorded by means of the oscilloscope camera. As anticipated, the recorded frequency was approximately 220-225 cps.

The cause of the observed singularities thus firmly established, the entire test rig structure was mounted on isolators. A record, obtained in a manner similar to that described in the foregoing, showed that the natural frequency of the isolated structure was 15 cps. A further, effective reduction of isolator stiffness, such as could be obtained with a Julius suspension, for example, or a sizeable increase in mass, was not feasible because of physical limitations. The response of the upper thrust plate was also of interest. The plate was struck a light blow near the center with a small hammer, and the resulting trace of the cathode ray oscilloscope was recorded. The frequency observed in this case was approximately 730 cps, twice as large as the maximum frequency of auto-oscillations encountered in all experiments, but considerably lower than the magnitude estimated in the design phase of the apparatus.

Photographic records of frequency measurements described in the preceding paragraphs are presented in Figure 50, in which I represents the trace obtained with the structure resting on the floor, II is the response of the upper thrust plate and III refers to the structure when mounted on isolators. The uneven appearance of traces is due to the fact that, with relatively low pressures applied at the extremities, the shaft drifted continuously in the axial direction, producing corresponding changes in the electrostatic field between the probe and the "seismic mass". The oscillations of the upper thrust plate, II, are superimposed on the trace produced by vibrations of the test rig structure on isolators, both induced by the light hammer blow.

The concluding stability experiments were conducted with an isolated structure, so that the ratios of lowest, observed frequencies of auto-oscillations and the natural frequency of the assembly-isolator system were in the order of 14:1. Bearing alignment, leakproofness, zero settings of instruments, etc., were thoroughly checked. A low frequency generator, the triangular wave output of which was used as a marker on photographic records, was checked against an electronic, digital counter and found to be stable and accurate within 1 percent in the 500-1200 cps band width. Particularly great care was taken in recording readings of the recess depth dial indicator and of pressure gauges. Pressure readings were corrected to the nearest 0.05 psi for head loss between the measuring station and the point of admission to the reservoir, upstream of the supply nozzles. In approaching the critical recess depth on the threshold of instability, the piston was locked, so that final setting was made by very gradually overcoming a substantial amount of frictional resistance, in addition to that offered by four sliding "O"-ring seals. Whenever the stability boundary was approached by varying the supply pressure, the regulator control was adjusted in a manner, for which, perhaps, the most apt description would be "creeping".

It will be recalled that the stability analysis was based on linearized equations of motion, implying among other things that, in the course of the experimental part of this investigation, sensibly small amplitudes of oscillation, possibly one percent of the equilibrium gap width, would have to be observed. Now, since sources of small, random disturbances were always present in the fluid, the foundation and the surrounding atmosphere, more or less well defined wave forms of bearing response, with amplitudes of the order of microinches, were always observed on the display of the cathode ray oscilloscope. On the other hand, no significant growth of oscillation was noted for very large changes of supply pressure and recess depth, until their values approached a region in which relatively minute changes in either parameter were accompanied by an abrupt increase in the amplitude of vibration. In this narrow transition region the amplitude of vibration was modulated by irregular "beats". Their incidence, relative to the frequency of oscillation, occurred at more frequent intervals for large values of the critical recess depth.

Perfect discontinuities and ideally defined boundaries seldom occur in physical problems. Nevertheless, the transition in amplitudes from the microinch noise level to magnitudes approaching the gap width itself was invariably accomplished by changes smaller than one percent of the stability parameters designated as "critical". Thus the stability boundary is sharply defined. Otherwise stated, the amplitude of oscillation remained very small and its rate of change (slope) with recess depth, at constant supply pressure†, remained essentially zero, until both changed almost discontinuously in a narrow region, the amplitude becoming very large and its rate of change (slope) almost infinite.

The concepts and ideas contained in the preceding paragraphs can be summarized by stating (a) that amplitudes of the order of 10^{-5} inches were small compared with the bearing gap width, (b) that the available instruments were capable of detecting and measuring amplitudes of the order of 10^{-5} inches and, moreover, (c) that the order of magnitude of 10^{-5} inches was not very critical as a criterion of limiting values of recess depth and supply pressure at the inception of instability.

† Alternatively, with supply pressure at constant recess depth.

The foregoing statement may be substantiated by referring to Figures 51, 52, 53 and 54, which illustrate the abrupt transition in amplitude growth when approaching the boundary from the stable side, by increasing (or decreasing) the recess depth at several settings of the supply pressure. The lower photographs in each row were obtained with long exposure times and slow sweep rates. They show quite clearly the irregular "beats" referred to previously. The upper photographs display wave forms of auto-oscillations in random time intervals and were obtained with sweep rates one thousand times faster than corresponding photographs in the lower rows.

Stability boundaries and frequency determinations, obtained in the course of concluding experiments, are presented and compared with predicted results in Figures 55 and 56. A corresponding tabulation of experimental data is included in Appendix 4, following the table of calculated results. Photographic records of frequency measurements are shown in Figures 57 and 58. In the presence of irregular beats, described in the foregoing paragraphs, particularly at large values of the recess depth, the determination of critical parameters was unavoidably subject to judgment of the observer. The procedure followed by the experimenter was to observe the trace on the cathode ray oscilloscope while adjusting the recess depth, without noting the reading of the recess depth dial indicator. The objective was to avoid bias in selecting the point at which auto-oscillations were estimated to have reached amplitudes between approximately 1×10^{-5} to 2×10^{-5} inches. An alternative method was used for a number of points in which the stability boundary was approached by varying the supply pressure instead of the recess depth, thus providing a check on the reliability of measurement. In the latter case, the attention of the experimenter was focussed on the oscilloscope display without observing the pressure gauge reading until the critical point was determined. No significant differences were noted, regardless of the manner in which the stability boundary was approached, indicating that results were reproducible according to the selected criterion, as well as rather insensitive to inexactitudes involved in its selection. Several measurements were repeated on days when the barometric pressure and the ambient (room) temperature were slightly higher, or lower, but the observed differences were negligibly small. It is estimated that the repeatability of limiting stability parameter measurements is at least within 1 to 2 percent of readings obtained in the course of concluding experimental runs.

The very substantial and, in this case, rather encouraging qualitative and quantitative differences in results of preliminary and concluding experiments were attributed entirely to the isolation of the test rig structure, since no other changes were effected in the system. On the other hand, all static and dynamic experiments seemed to indicate that, under certain conditions, even minute changes in system characteristics had an enormous effect on the stability of the bearing. In the final phase of this investigation, therefore, the experiments were accompanied by numerous and important secondary tests, with the objective of (a) discovering possible causes of remaining divergencies between theory and experiment, particularly such as might have accounted for the double-valuedness of stability loci, or, failing this, (b) adding weight to the reliability of the concluding experiments. Those secondary tests will now be described.

All experimentally determined stability boundaries were double-valued and exhibited an "upper" branch, not predicted by the present analysis. Although no calculated data were available for recess pressures other than 73.5 psia (corresponding to a 472 lb load), a stability test under reduced load conditions (222 lb), cursorily mentioned in a preceding section, was carried out. The reduced load test was dictated by the desirability of ascertaining whether the double-valuedness would also persist in the case of a locus corresponding to lower Reynolds number,

$N_R = \frac{\rho_{ro} \bar{\Gamma}_{ro} H_o}{\mu} = \frac{W_o}{2\pi\mu R_r}$. A qualitative comparison of results in Figures 55 and 49, or Figures 57 and 49, in conjunction with reference to Figures 41 and 42, clearly indicates the presence of "upper branches" in loci corresponding to different ranges of the Reynolds number.

Next, effects of misalignment were investigated. This investigation was accomplished by two methods. First, the constraint of the shaft† was removed and the relative positions of the upper and lower thrust plates were altered. These changes produced a very substantial misalignment due to the run-out of the lower thrust surface with respect to the shaft journals. Although very minor differences in limiting values of supply pressure and recess depth were noted, no significant change in the trend of the stability boundary was observed. Similar conditions prevailed when the rotor was brought up to a speed of approximately 2500 rpm, by means of an air jet directed onto the turbine wheel buckets. In the second misalignment test, the supply pressures of the upper and lower guide bearings was varied in such a manner as to tilt the shaft within the limits of bearing clearance. In addition to misalignment, the accompanying effect, in this case, was to change the "stiffness" of the guide bearings. No substantial changes in stability characteristics were observed, except when the supply pressure to one, or both bearings, was reduced to the extent which precluded their functioning as effective guides and constraints, or when metal to metal contact between the bearings and journals was established.

Experiments conducted with the isolated and non-isolated test rig assembly not only demonstrated local effects in the vicinity of the resonance point, but also the presence of large differences in limiting values of stability parameters for frequencies of incipient auto-oscillation considerably removed from the critical. The existence of other critical frequencies in the experimental band width and the effect of small vibrations of the upper thrust plate could not be entirely dismissed. It was, therefore, decided to stiffen the upper thrust plate. This was accomplished by placing three 3/4 inch diameter jacking screws between the column-bracing, upper plate and the upper thrust plate (see assembly drawing Figure 7) in positions straddling the columns and as close to the bearing center as was feasible. The jacking resulted in a slight amount of controlled, symmetrical deflection. It was known, however, at that time that no significant changes in stability characteristics were associated with a moderate degree of misalignment. The rigidity of the upper thrust plate thus considerably increased, measurements for a series of previously established, experimental points were repeated, without, however, any appreciable departures having been noted.

In the analysis, as well as in the course of experimental investigations, it was postulated that the load, and consequently the pressure in the loading cylinder, remained constant at all times. Accordingly, the design of the loading cell was such that fractional pressure changes, associated with motion of the shaft, were to be of the order of smallness of the displaced to total cylinder volume ratio. (Approximately 10^{-6}.) There existed the possibility, however, that the lower end of the shaft, acting as a piston type source, gave rise to a complex, stationary wave system. If wave reinforcement occurred, the effect could be appreciable and divergence between the theoretical and the physical models significant. The possible presence and influence of resonance in the chamber could be confirmed or disproved by altering the shape and volume of the

† Two jets impinging on each side a guide fin, mounted on the shaft collar. See Figure 18

loading cylinder. A simple method was employed, in which the cylinder was fitted with a small standpipe and then filled with water in increments up to three quarters of the total volume. No significant departures, however, from previously established results were observed.

The validity of the concept of a "constant pressure supply reservoir" depends on the absence of pulsations transmitted through the supply restrictors. The likelihood of propagation and, possibly, of reflection of pressure disturbances upstream of the supply nozzles could not be dismissed. Unfortunately, installation of suitably small and sensitive pressure pick-ups, or hot wire anemometers, in the immediate vicinity of the nozzle entrance and exit would have involved substantial changes in design and elaborate, additional instrumentation. Subsequent investigations, therefore, were limited to a series of simple checks, intended to locate possible sources of trouble in the air supply system. It is, perhaps, both useful and instructive to render a brief account of some of the steps taken in this "trouble-shooting" procedure:

a. A two cubic foot plenum chamber was installed at a point where the supply line was joined to the upper thrust plate (see assembly drawing in Figure 7), in order to increase the capacity of the supply reservoir and provide a zone in which small fluid disturbances could be dissipated.

b. The compressor was shut off so that no unsteadiness of flow originated at the source of the supply while readings were recorded. The capacity of the main supply reservoir was sufficiently large to maintain a constant supply pressure in a time interval required to perform one or several measurements.

c. Since instabilities of pressure regulators and pneumatic valves are similar in nature to those occurring in externally pressurized bearings, the regulators and valves, placed in series in lines supplying the bearings and loading cylinder (see Figure 17), were adjusted for varying degrees of pressure drop across the metering elements. Various settings were made and a number of measurements were repeated.

None of the foregoing measures had any effect on the stability characteristics of the test bearing.

Valid comparisons between theoretical and experimental results could only be made if the pitching (cocking) motion of the shaft remained small in comparison with the amplitudes of axial oscillation. In the course of several preliminary and final runs, the auxiliary capacitance probe was mounted in different positions around and along the shaft, but no measurable amplitudes were detected. Preliminary calculations carried out in the design phase of the apparatus, in which the shaft and the guide bearings were considered as a simple, two-degree-of-freedom system, indicated that the slower mode of vibration could possibly be excited by frequencies of auto-oscillation in the upper range of the experimental band width. Figure 39 shows results of a vibration test, in which the auxiliary capacitance probe was located in a position† where large amplitudes at frequencies close to that of the first mode could most readily be observed. Since the sensitivity of the auxiliary probe was considerably less than that of the primary probe, the gain of the primary instrument was decreased and the amplitude of auto-oscillations increased to 4×10^{-4} inches, approximately twenty times the value observed on the threshold of instability. This permitted a better comparison to be drawn between amplitudes of axial and pitching motions. The frequency of

† Along the six inch outer diameter of the shaft thrust plate.

auto-oscillations was then varied from 375 to 200 cps. The second photograph in Figure 59 shows the largest amplitude of pitching which was observed, occurring at 2091 rad/sec (333 cps). It is realized, of course, that more than two modes could be excited, that the shaft could perform small, conical gyrations within the limits of bearing clearance, or momentarily rotate and translate, but not necessarily in the plane of the capacitance probe. Nevertheless, Figure 59 shows the largest amplitude ratio (approximately 100:1) ever observed in any of the monitoring positions and there is little doubt, therefore, that the overall effect of pitching was a very minor one.

VI. DISCUSSION

1. Comparison of Analytical and Experimental Results

Comparison of theoretical and experimental loci in Figure 55 indicates reasonably good agreement in the region in which pressure differences between the supply reservoir and the recess are highest. In this region, values of δ_c obtained from the present analysis represent a considerable improvement when compared with those predicted from the simplified, lumped-parameter theory. (See Figure 5.) As the pressure drop across the supply nozzle decreases, the deviation of empirical curves from their calculated equivalents becomes more pronounced. Beyond the points of vertical tangency neither quantitative nor qualitative agreement exists.

With regard to slope, empirical curves and those predicted from the present analysis agree fairly well over a considerable part of the experimental range. The same situation does not hold true for the single locus obtained from the simplified theory. The present analysis predicts conservative (safe) limits, whereas the simplified analysis gives values which are either too high or too low. A most significant improvement, introduced through the refinements incorporated in the distributed-parameter approach, is that the present analysis gives a correct interpretation with regard to influence of the effective bearing mass on stability, a fact to which experimental results lend unequivocal support. As emphasized previously, one of the major shortcomings of lumped-parameter analyses is that they predict the same locus, regardless of magnitude of the bearing mass.†

It is interesting to note that for the larger mass, agreement between predicted and experimental loci in Figure 55 is better and extends over a larger portion of the curves. It is more than probable that, as the bearing mass increases, agreement between predicted and actual results progressively improves. This inference may be particularly important, because in actual applications one might expect considerably higher than 0.05 or 0.10 weight to thrust ratios. Since from a practical point of view one is usually interested in "safe depths" of the order of 5×10^{-3} to 20×10^{-3} inches, it is encouraging to note that is is precisely in this region where predictions based on the present analysis constitute a considerable improvement in comparison with those based on simplified analyses.

2. Remarks on the Relative Validity of Theoretical Assumptions and Their Applicability To the Experimental Model

The present investigation included both, analysis and experiment. To the investigators, who have engaged in this dual undertaking, falls the difficult task of accounting for disagreement in areas where it exists. In what follows, an attempt is made to inquire into likely causes of divergence between analytical predictions and empirical results, particularly the single and double valuedness of the respective stability loci.

It will be recalled that the dynamic lubrication equation (4) was required to satisfy a boundary condition at the outer bearing perimeter and to be compatible with the continuity equation

† The bearing mass becomes a significant stability parameter in presence of appreciable, external damping only. (See Appendix 2.)

(7) of the variable-volume recess. Now, in implementing the continuity equation, it was assumed that the inflow through the supply nozzle, W_1, and the outflow along the perimeter of the recess, W_2, could be considered as quasi-steady. The quasi-steady approach has been successfully employed in problems of charging and discharging of fixed and variable-volume cylinders, in research related to the dynamics of pneumatic controls. A typical example is the work of Shearer [9]. The flow through orifices, under pulsating downstream conditions, has been investigated, both theoretically and experimentally, by Schultz-Grunow [46]. He suggested that limits of applicability of steady-flow equations could be assessed from the reciprocal of the Strouhal number:

$$\phi = \frac{\bar{V} t_o}{d}$$

$\bar{V}$ = Average velocity

t_o = Period of pulsation

d = Orifice diameter

which is directly related in unidimensional, inviscid flow to the ratio of local and convective acceleration components, $V\frac{\partial V}{\partial x}\Big/\frac{\partial V}{\partial t}$. He concluded, that for values of $\phi > 510$, the steady-flow equations yielded accurate values of instantaneous flow rates and the effect of local acceleration was negligible. Although maximum values of ϕ obtained in the course of present experiments were larger that 10^3, magnitudes of ϕ were generally comparable with 510, and thus not sufficiently far removed from a transition zone, in which the effect of pulsation may have been significant. Other investigators discovered that pulsations propagated upstream of the restrictor and gave rise to complex wave patterns and phase relations on either side of the orifice [47, 48, 49, 50].

The adequacy of considering inflow as quasi-steady is questionable, at least in certain ranges of pressure, frequency and geometrical parameters of the system. In the limiting case, when P_r/P_s approaches the critical ratio, no pressure pulses propagate upstream of the nozzle throat. If one assumes, however, that acoustic equations hold when P_r/P_s approaches unity, the theory of horns, to which the supply nozzles bear a geometrical resemblance, predicts definite phase relations between pressure and flow at the throat [64]. The existence, therefore, of appreciable time lags cannot be entirely dismissed, in which case the assumed, "in phase" perturbation relation, $w_1 = C_3 p_r$, may be deficient.

If we now direct our attention to the outflow from the recess, the entrance effects, associated with fluid acceleration and flow separation at the sharp edge of the perimeter, are further complicated by the variation of the area of admission to the slot under dynamic conditions. Here, the use of equation (9), e.g., $W_2 = -(\text{constant})\, H^3 P_r \frac{\partial P_r}{\partial r}$, implied not only that the outflow could be considered as quasi-steady, but also that there was no transition and that a viscous, laminar flow regime became at once established as the fluid entered the slot. Consider also the case of small-amplitude vibration when the recess volume is large and the pressure deviation, p_r, tends to become very small in comparison with p, the perturbation pressure in the slot. In fact, due to

the trapping and squeezing of the fluid in the slot, maxima of p are likely to occur close to the recess perimeter, where the gas is relatively dense. This may cause large variations of the perturbation pressure gradient in the immediate vicinity of the recess and, in turn, induce appreciable fluid acceleration and deceleration in the region of admission to the slot.† Moreover, the linearized expression, $w_2 = C_4 h - C_5 p_r - C_6 \frac{\partial p_r}{\partial r}$, corresponding to equation (9), does not take into account the complex phenomena occurring in the neighborhood of the recess boundary. Divergence of results may be due partly to the cumulative discrepancy involved in expressing the difference, $w_1 - w_2$, corresponding to the righthand side of the continuity equation (7).

Next, we consider possible shortcomings of assumptions involved in the mass accumulation term on the left-hand side of the continuity equation (7). If we now focus our attention on the internal processes occurring within the variable-volume recess, such as the impingement, deflection or disintegration of jets and the interaction between decelerating and accelerating fluid streams, the physical picture defies mathematical description. Unavoidably, therefore, it becomes necessary to resort to simplifying assumptions, or postulates, such as uniformity of "average" properties at any instant of time. The latter assumption was made in the qualitative analysis of periodic charging and discharging of the variable-volume recess in Appendix 5.

Granted the high degree of oversimplification in formulating the problem, the analysis of Appendix 5 nevertheless suggests that the pressure and density in the recess are not in phase. Numerical examples, indicate, however, that no appreciable time lag is involved, so that the process may be reasonably well described by P/ρ^n = constant. Although the exponent, n, is then not a function of time, it does depend on a dimensionless "charge" parameter, $Q = W_0/\rho_{r0} V_{r0} \omega$, in which W_0, ρ_{r0}, V_{r0} refer to the equilibrium flow rate, the recess gas density and the recess volume, and ω is the circular frequency of oscillation.

Considerable numerical discrepancies between theoretical and experimental results could be accounted for by selecting more "representative" values of the exponent n, but it is by no means implied here that an explanation for the double valuedness of experimental loci could be supplied by assigning to n values other than $\gamma = 1.4$ in the recess, or other than $n = 1$ in the slot.††

It is considered that the inertia forces in the slot were small in comparison with viscous forces. Although no definite, analytical proof has been furnished to support this contention, the derivation in Appendix 6 suggests that the neglect of inertia contributions in the bearing clearance may not be very serious, and that the dynamic lubrication equation (4) is a good first approximation. Moreover, in the presence of significant inertia effects, one would expect the disagreement between predicted and empirical results to be greatest at relatively high frequencies of oscillation, whereas exactly the opposite is indicated by the experimental evidence.

In the foregoing discussion, an attempt was made to probe into the more questionable aspects of the present analysis. A critical inquiry would be incomplete and one-sided, if one omitted to

---

† An analogous condition is illustrated diagrammatically in Figure 6a., showing a greatly exaggerated, instantaneous pressure profile and positive gradient at the edge of the recess.

†† In the slot, P/ρ = constant was the assumed relation. See also Appendix 6.

examine both the experimental model and the procedure for possible deficiencies and inherent sources of error. Here, however, the numerous, secondary tests, described in the section "Concluding Stability Experiments," have narrowed down considerably the area of search for a feasible explanation.

The turning point in the experimental phase of this investigation was the discovery of resonance in the system. Isolation of the test structure not only altered the shape of stability loci near resonance points, but also displaced the curves in a direction effecting considerably better agreement between theory and experiment. The extent to which actual motion departs from the idealization of an undamped, single-degree-of-freedom system, for which perfect isolation, absolute rigidity and absence of external damping are assumed, is most difficult to assess. One need only to consider that 10^{-6} inches represented 10 percent of what could be meaningfully interpreted in the presence of mechanical and electronic noise. Since the capacitance gauge measured the relative, instantaneous displacement of bearing surfaces at a point, it provided no information with regard to the absolute motion of boundaries. Nor was it feasible to furnish a perfectly isolated and immobile reference frame for external mounting of capacitance probes.

The extension of the simplified analysis of Appendix 2 to a damped, two-degrees-of-freedom system suggested the possibility of a double valued stability locus. An approximate value of the critical damping ratio was estimated from Figure 50.† Calculations then indicated limiting values of critical recess depth on the upper branch of the locus to be of the order 10^{-3} inches, as compared with 10^{-1} inches in the case of experimental results. The simplified analysis, of course, lends only qualitative support to the existence of double valued loci and may predict grossly exaggerated results in this region. It is not unlikely that the extension of the present analysis, to include the effects of elastic and damping properties of the entire system, would result in more realistic predictions of critical values of stability parameters. At any rate, one cannot rule out that the divergence between theory and experiment was due to appreciable motion and accompanying dissipative effects in the supporting structure, whereas perfect rigidity and motion of one boundary only were hypothesized for the idealized model. The importance of considering the motion of the supporting structure was stressed in a recent paper by Tsai [52], who investigated the stability of a simple, pneumatic pressure reducer. The author states in the introduction: "Also, the vibrations of the mounting structure may induce undesirable oscillations in the reducer. To obtain a complete picture of performance, one must therefore investigate the dynamic characteristics of the reducer and analyze the problems of its natural frequency and stability."

Of equal importance is the extent to which the actual, steady-flow pattern conforms to that which has been postulated for the idealized model. The flow of compressible fluids in narrow channels and in externally-pressurized bearings, in various Reynolds and Mach number regimes, has been the subject of numerous, theoretical and experimental investigations.†† The Reynolds and Mach numbers corresponding to the flow range of the present investigation were sensibly small, and deviations from calculated, steady-flow bearing characteristics were of the order 2-5 percent. Nevertheless, once oscillation is started, the influence of entrance phenomena on stability may become of far greater significance than their effect under static conditions would seem to indicate.

---

† Trace of decaying vibration of structure when mounted on isolators.

†† Representative examples may be found in references [19, 20, 21, 22, 23, 24, 27, 31]. A rather complete list is contained in a bibliography by E.B. Sciulli, reference [44].

The flow pattern in the entrance region does not conform to the idealization of the experimental model. In the flow around the sharp edge of the recess, separation takes place and the fluid stream converges, a toroidal vortex forming beneath the contraction. McGinn [35] obtained streamline patterns for water flow in a sharp-edged, radial diffuser by means of dye filaments, and recorded photographically the vortices formed downstream of the separation point. Shires [22], using air, demonstrated the existence of pressure depressions in the entrance zones of several, parallel-surface slot configurations, for various pressure ratios and slot width. The Reynolds number in the experiments of McGinn and Shires was of the same order as that in the present investigation.

Consider a vibration, induced by a random disturbance and accompanied by changes of pressure gradient in the entrance region. The vibration will cause alternate elongation and contraction of the transverse dimension of the vortex ring. The outflow from the recess depends on the width of the effective flow area, rather than on the width of the slot. The amplitude and phase relation between the slot width and the width of the contraction may not be a simple one, and the representation of the instantaneous flow rate by the expression $W_2 = -(\text{constant})\, H^3 P_r \frac{\partial P_r}{\partial r}$ may lead to considerable error. Conceivably also, an appreciable amount of energy, which would otherwise be dissipated in the process of a self-sustained oscillation, may instead be absorbed through entrance losses. The entrance zone may thus provide an internal damping mechanism and have a stabilizing effect on the bearing. Since entrance phenomena were not considered in the analysis, theory may predict instability in regions which experiments show to be stable.

On the other hand, as one follows the lower branch of a locus and departure from predicted results increases, the Reynolds and Mach numbers decrease, so that entrance effects become less pronounced, up to the point of vertical tangency of the locus. Thereafter, the opposite holds true. It would be inconsistent, therefore, to attribute the main cause of divergence of results solely to entrance effects, except to say that their disregard may have been a contributing factor.

In this section, a number of likely, but by no means proven sources of divergence between theoretical and empirical results, have been discussed. Although the observed discrepancies may have been a cumulative result of several deficiencies involved in the theoretical assumptions, as well as those inherent in the experimental apparatus, the most probable sources of error which suggest themselves are:

a. The possible insufficiency of the quasi-steady flow representation and of the postulation of uniform properties in expressing the continuity equation of the variable-volume recess.

b. An appreciable effect of the relative motion of the bearing surfaces, associated with imperfect isolation and/or the distributed, elastic and damping properties of the supporting structure.

c. The presence of entrance effects in the experimental model, contrasted with their neglect in the present analysis.

VII. CONCLUDING REMARKS AND RECOMMENDATIONS

It has been demonstrated that the dynamic lubrication equation (4) gives a more faithful representation of the effects of partial trapping and squeezing of a compressible fluid film in a long and narrow slot than could hitherto be obtained with the aid of arbitrary assumptions with regard to the flow and the pressure profile. Moreover, the present results have been obtained without resort to any correction factors. It has also been shown, both theoretically and experimentally, that the bearing mass has a significant effect on stability and that stability is enhanced by decreasing the bearing mass. As a consequence of results of the analytical and the experimental phases of this investigation, it has been possible to assess more critically the earlier, lumped-parameter theories.

Experience gained in the course of experiments has shown that the response of the bearing can be greatly influenced by the method of mounting, indicating that in actual applications the mass and the elastic and damping properties of the supporting structure may have to be considered in formulating the stability problem. Stability has been shown to be extremely sensitive to minute changes of certain bearing parameters and system characteristics and quite insensitive to others. The experimental results have also substantiated the validity of the linearized approach in the analysis, since, in regions designated as "stable," no self-sustaining oscillation could be induced, regardless of the magnitude of disturbance.

There are several areas in which the present research could be further extended and supplemented. The stability of bearings cannot be treated as a separate problem and must be dealt with in conjunction with other aspects of an entire system. It is not always possible, however, to formulate the equations of motion of a complex system in terms of distributed parameters. Even after such formulation, it may be impossible to obtain solutions. Moreover, system considerations aside, the distributed parameter approach may also become too involved for more complex bearing configurations. Clearly, therefore, a more rational method of lumped-parameter representation would be desirable.

The present analysis suggests an approach in this direction which merits further consideration. It will be recalled in the case of the rectangular geometry, that in the series representation of the functions y_I and y_{II} we capitalized on the smallness of the parameter ϵ and neglected higher order terms of the expansion. When the first four terms of the expansion are retained, the characteristic equation reduces to a polynomial of the third degree in s. On the other hand, it has been shown in Appendix 2 that the characteristic equation obtained from the simple analysis is also a polynomial of the third degree in s. Since both equations are of the same form and differ only in their coefficients, the results of the present analysis may be helpful in devising a rational method of correlation of the distributed and the lumped-parameter representations. In the case of the circular bearing, it may also be feasible to obtain solutions y_I and y_{II} in the form of series expansions in terms of the small parameter, $12\mu R_r^2 / P_{ro} H_o^2$, which characterizes the equation (19b). It may then be possible to represent the characteristic equation (31b) by a polynomial in s, following a procedure analogous to that which was employed in the case of the rectangular bearing configuration. A correlation of coefficients in the characteristic polynomials of the present and simplified analyses would give the advantage of formulating the problem in terms of ordinary, rather than partial differential equations.

It would be desirable to retain the quasi-steady flow representation in future stability analyses, because of the inherent simplicity of such an approach. Nevertheless, in certain flow regimes, it may become necessary to modify the mathematical description of the charging – discharging process of variable volumes in bearings. Perhaps, as a first step in this direction, it would be useful to determine the throat impedances of simple nozzles and of orifices, in a manner analogous to derivations in the acoustic theory. A better understanding of the phenomena occurring in the entrance region of a slot is also needed, before an attempt is made at an improved mathematical description of fluid flow in the entrance zone. A study of streamline patterns and of vortex formations should be made, when the boundary oscillates in the transverse direction of the slot. A complete time record of flow patterns could be obtained with the aid of a high speed camera, using otherwise methods similar to those employed by McGinn [35] for the case of steady flow of water in the entrance zone of a sharp-edged, radial slot.

Advance consideration has been given to future experiments which would involve rotation of the shaft. For this purpose, an air-driven turbine has been incorporated in the design of the shaft, details of which are shown in Figure 14. The effect on stability and other rotor characteristics due to pressure gradients, induced through centrifugal forces and surface misalignment, is of considerable interest and should be studied. Also, additional stability loci could be obtained for various pressure ratios and types of supply restrictor, under rotating, or non-rotating conditions. The experimental apparatus is capable of providing a great deal of valuable information and can also be modified for the purpose of experimentation with other bearing configurations. It is recommended, therefore, that full use be made in the future of the existing facility.

Finally, since the simple geometries considered in this investigation are frequently encountered in practical applications, and since good agreement exists in regions which are of interest, the programming of a digital computer, to provide design data for a representative range of parameters, is warranted and recommended.

VIII. BIBLIOGRAPHY

1. "Effect of Compressibility at Low Reynolds Number," G. I. Taylor and P. G. Saffman, Journal of the Aeronautical Sciences, Vol. 21, No. 8, 1857, pp. 553–562.

2. "A Dynamic Analysis of Externally Pressurized Air Bearings," H. H. Richardson, MIT, Department of Mechanical Engineering, MS thesis, Cambridge, Mass., January 1955.

3. "Static and Dynamic Characteristics of Compensated Gas Bearings," H. H. Richardson, Trans. ASME, Vol. 80, No. 7, 1958, pp. 1503–1509.

4. "Instability of an Externally Pressurized Non-Rotating Air Thrust Bearing," L. Licht, General Electric Laboratories, Technical Information Series Report No. R55GL341, September 1955.

5. "Self-Excited Vibrations of an Air-Lubricated Thrust Bearing," L. Licht, D. D. Fuller and B. Sternlicht, Trans ASME, Vol. 80, No. 2, 1958, pp. 411–414.

6. "An Analysis of the Effect of Several Parameters on the Stability of an Air-Lubricated Hydrostatic Thrust Bearing," W. H. Roudebush, NACA TN4095, September 1957.

7. "Air-Hammer Instability in Pressurized Journal Gas Bearings," L. Licht, Paper No. 60-WA-10, presented at the ASME Winter Annual Meeting, New York, December 1960. (To be published Trans. ASME.)

8. "Transient Flow Forces and Valve Instability," S. Y. Lee and J. F. Blackburn, Trans. ASME, Vol. 74, 1952, pp. 1005–1011.

9. "Study of Pneumatic Processes in Continuous Control of Motion with Compressed Air," J. L. Shearer, Trans. ASME, Vol. 78, No. 2, 1956, pp. 233–249.

10. "Nonlinear Analog Study of a High-Pressure Pneumatic Servomechanism," J. L. Shearer, Trans. ASME, Vol. 79, No. 3, 1957, pp. 465–472.

11. "Axial Relative Motion of a Circular Step Bearing," L. Licht, Trans. ASME, Journal of Basic Engineering, Vol. 81, No. 2, 1959, pp. 109–117.

12. "The Effect of Conduit Dynamics on Control-Valve Stability," F. D. Ezekiel, Trans. ASME, Vol. 80, No. 4, 1958, pp. 904–908.

13. "The Design and Analysis of a Servovalve with Flow Feedback," E. Bahniuk and S. Y. Lee, Trans. ASME, Journal of Basic Engineering, Vol. 82, No. 1, 1960, pp. 73–80.

14. "Analysis and Design of a Servomotor Operating on High-Pressure Compressed Air," G. Beethof, Trans. ASME, Vol 79, No. 4, 1957, pp. 875–885.

15. "The Effect of Oil-Column Acoustic Resonance on Hydraulic Valve 'Squeal'," F. W. Ainsworth, Trans. ASME, Vol. 78, No. 4, 1956, pp. 773-778.

16. "Static and Dynamic Control Characteristics of Flapper-Nozzle Valves," Tsun-Ying Feng, Trans. ASME, Journal of Basic Engineering, Vol. 81, No. 3, 1959, pp. 275–284.

17. "Die Vorgänge in Federbelasteten Einspritzdüsen von Kompressorlosen Ölmaschinen," O. Lutz, Ing. Archiven, 1953, p. 155.

18. "Dynamical Instability in Flow Systems," T. Baron, Proceedings of the First Midwestern Conference on Fluid Dynamics, Ann Arbor, Mich., 1951, pp. 216–225.

19. "Ecoulement d'un Fluide entre Deux Plans Paralleles. Contribution a l'Etude des Butees d'Air," R. Comolet, Publications Scientifiques et Techniques de l'Air, No. 334, September 1957.

20. "Ecoulement Radial d'un Palier à Air," E. A. Deuker and A. Wojtech, Revue Générale l'Hydraulique, Vol. 17, 1951, pp. 228–234, 285–294.

21. "A Theoretical Investigation of Pressure Depression in Externally Pressurized Gas-Lubricated Circular Thrust Bearings," H. Mori, MIT Lubrication Laboratory, Contract NOrd 17366 (53–138), Cambridge, Mass., 1959.

22. "The Viscid Flow of Air in a Narrow Slot," G. L. Shires, National Gas Turbine Establishment, Memo No. 46, December 1948, Farnborough, England.

23. "Flow of a Compressible Fluid in a Thin Passage," S. K. Grinnell, Trans. ASME, Vol. 78, No. 4, 1956, pp. 765–771.

24. "Heat Transfer Effects in Hydrostatic Thrust Bearing Lubrication," W. F. Hughes and J. F. Osterle, Trans. ASME, Vol. 79, No. 6, 1957, pp. 1225–1228.

25. "Air Bearing Studies at Normal and Elevated Temperatures," J. D. Pigott and E. F. Macks, NACA paper presented at the ASLE Meeting, Boston, Mass., April 1953.

26. U. S. Patent No. 2, 683, 635, Air Bearing, R. M. Wilcox, 1954.

27. "A Preliminary Investigation of an Air-Lubricated Hydrostatic Thrust Bearing," L. Licht and D. D. Fuller, ASME paper No. 54-Lub-18, presented at the First, Joint ASME/ASLE Conference, October 1954.

28. "Air Bearings for Guidance Components of Ballistic Missiles and their Production Aspects," H. C. Rothe, Paper presented at the First International Symposuim on Gas-Lubricated Bearings, Washington, D. C., October 1959.

29. "The Static Strength of Pressure Fed Gas Journal Bearings," C. H. Robinson and F. Sterry, Atomic Energy Research Establishment, Report No. AERE-ED/R 1672, 1958, Harwell, England.

30. "Air Bearings – Research and Applications at the National Engineering Laboratory, Scotland," M. Granoek and J. Kerr, paper presented at the First International Symposuim on Gas-Lubricated Bearings, Washington, D. C., October 1959.

31. "The Analysis and Design of Hydrodynamic Gas Bearings," R. R. Weber, North American Aviation Co., Aerophysics Laboratory Report, No. AL-699, September 1949.

32. "The Performance of Externally Pressurized Bearings Using Simple Orifice Restrictors," D. S. Allen, P. J. Stokes and S. Whitley, ASLE paper No. 60LC-17, presented at the Joint ASME-ASLE Lubrication Conference, Boston, Mass., October 1960.

33. "Air Driven Spinners," L. W. Wightman, Machine Design, Vol. 20, No. 5, 1948, pp. 121–125.

34. "Air-Lubricated Bearings," P. M. Mueller, Product Engineering, Vol. 22, No. 8, 1951, pp. 112–115.

35. "Observations of Radial Flow of Water Between Fixed Parallel Plates," J. R. McGinn, Applied Scientific Research, Sec. A, Vol. 5, 1955, pp. 255–264.

36. "Varieties of Shaft Disturbances Due to Fluid Films in Journal Bearings," B. L. Newkirk, Trans. ASME, Vol. 78, 1956, pp. 985–989.

37. "Whirling of a Journal in a Sleeve Bearing," D. Robertson, Philosophical Magazine, Series 7, Vol. 15, January 1933, pp. 113–120.

38. "Contribution to the Theory of Oil Whip," H. Poritsky, Trans. ASME, Vol. 75, 1953, pp. 1153–1161.

39. "Investigation of Translatory Fluid Whirl in Vertical Machines," G. F. Boeker and B. Sternlicht, Trans. ASME, Vol. 78, 1956, pp. 13–19.

40. "Experimental Investigation of Oil Whip," O. Pinkus, Trans. ASME, Vol. 78, 1956, pp. 975–983.

41. "Gleitlager mit Luftschmierung," H. Drescher, Zeit. V.D.I., Vol. 95, No. 35, 1953, pp. 1182–1190.

42. "A Theory of Oil Whip," Y. Hori, Trans. ASME, Journal of Applied Mechanics, Vol. 26, No. 2, 1959, pp. 189–198.

43. "Some Instabilities and Operating Characteristics of High Speed Gas-Lubricated Journal Bearings," G. K. Fischer, J. L. Cherubim and D. D. Fuller, ASME paper No. 58-A-231.

44. "Bibliography of Gas-Lubricated Bearings," E. B. Scuilli, The Franklin Institute Laboratories, Philadelphia, Pa., Interim Report No. I-A2049-6, Contract Nonr 2342 (00), Task NR 097 317, September 1959.†

† A comprehensive bibliography, containing 290 references directly related to gas lubrication.

45. "Refinement of the Theory of the Infinitely-Long, Self-Acting, Gas Lubricated Journal Bearing," H. G. Elrod, Jr. and A. Burgdorfer, presented at the First International Symposium on Gas-Lubricated Bearings, Washington, D. C., October 1959

46. "Durchflussmessverfaarungen für Pulsierende Strömungen," F. Schultz Grunow, Forschung auf dem Gebiete des Ingenieurewesens, Vol 12, 1941, pp. 117–126.

47. "Basic Difficulties in Pulsating Flow Metering," A. R. Deschere, Trans. ASME., Vol 74, 1952, pp. 919–924.

48 "Pulsation and its Effects in Flowmeters," E. J. Lindahl, Trans. ASME, Vol. 68, 1946, pp. 883–894.

49. "The Dynamics of Pulsative Flow Through Sharp-Edged Restrictions," R. C. Baird and I. C. Bechtold, Trans. ASME, Vol. 74, 1952, pp. 1381–1388.

50. "Pulsating-Flow Measurement – A Literature Survey," A. K. Oppenheim and E. G. Chilton, Trans. ASME, Vol. 77, No. 2, 1955, pp. 231–248.

51. "Nonsteady Discharge of Subcritical Flow," G. Rudinger, ASME paper No. 60-WA-152, presented at the ASME Winter Annual Meeting, New York, December 1960. (To be published Trans. ASME.)

52. "Dynamic Behavior of a Simple Pneumatic Pressure Reducer," D. H. Tsai and E. C. Cassidy, ASME paper No. 60-WA-186, presented at the ASME Winter Annual Meeting, New York, December 1960. (To be published Trans. ASME.)

53. "Rapid Discharge of Gas from a Vessel into the Atmosphere," E. Giffen, Engineering, Vol. 150, 1940, p. 134.

54. "Discharge of Exhaust Gases in Two-stroke Engines," J. H. Weaving, Proc. Inst. Mech. Eng., Vol. 161, 1949, pp. 98–120. (See also discussion by J. Kestin.)

55. "The Rapid Discharge of Gases from Vessels," J. Kestin and J. S. Glass, Aircraft Engineering, Vol. 23, 1951, pp. 300–304.

56. "A Study of the Processes of Charging and Discharging Constant Volume Tanks with Air," C. K. Skinner and F. D. Wagner, MIT, Department of Mechanical Engineering, SB thesis, Cambridge, Mass., 1954.

57. "Mechanical Vibrations," J. P. Den Hartog, McGraw-Hill Book Co., 3rd. ed., New York, 1947, pp. 356–358, 363–366, 382–386, 386–392.

58. "Vibration Problems in Engineering," S. Timoshenko, Van Nostrand Co., 3rd. ed., New York, 1955, pp. 199–204, 450–452.

59. "Mathematics of Engineering Systems," D. F. Lawden, Wiley and Sons, 1st. ed., New York, 1951, pp. 119–128.

60. "Differentialgleichungen Lösungsmethoden und Lösungen," E. Kamke, Chelsea Publishing Co., 3d ed., New York 1948, p. 440, equation 2.162(10).

61. "Tables of Functions," E. Jahnke and F. Emde., Dover Publications, 4th. ed., New York, 1945, p. 128.

62. "Mechanic-Electric Transducer," K. S. Lion, Review of Scientific Instruments, Vol. 27, No. 4, 1956, pp. 222–225.

63. "Laplace Transformation – Theory and Engineering Applications," W. T. Thomson, Prentice-Hall Inc., 1st. ed., New York, 1950.

64. "Elements of Acoustical Engineering," H. F. Olson, Van Nostrand Co., 1st. ed., New York, 1940, pp. 86–94.

65. "Viscous Flow Theory," Vol. I, S. I. Pai, Van Nostrand Co., 1st. ed., New York, 1956, p. 134.

IX. NOMENCLATURE

SYMBOL	MEANING	FIRST APPEARANCE	UNITS
A_e	"Equivalent" area, defined in Appendix 2	A2–1	in^2
A_r	Recess area	A2–1, A5–6	in^2
A_c	Coefficient, $-(P_{ro}^2 - P_{at}^2)/X - X_r$	16a	lb^2/in^5
$\bar{A}(s)$	Complex function, defined in text	30	
B_o	Coefficient, $-(P_{ro}^2 - P_{at}^2)/\log_e \frac{R}{R_r}$	16b	lb^2/in^4
$C_{1\cdots6}$	Coefficients, defined in text	15b	
c	Damping constant associated with mass M, Appendix 2	A2–10	lb–sec/in
c_p	Specific heat at constant pressure	A6–4	in^2/sec^2–°F
c_v	Specific heat at constant volume		in^2/sec^2–°F
d_N	Nozzle throat diameter	8	in
e	Base of natural logarithms, 2.718...	16b	No dim.
F(t)	Forcing function	6	lb
f(t)	Perturbation forcing function in text, $(H(t)/H^*$ in Appendix 6)	14, (A6–7)	lb, (No dim.)
$g_{1,2}$	Functions of α and P_s, defined in text	43b	in
H	Bearing gap width	1	in
H_o	Bearing gap width, equilibrium	12	in
h	Bearing gap width perturbation, $H - H_o$	12	in
h_o	Bearing gap width perturbation amplitude	A5–6	in
$\mathfrak{I}$	Pressure integral, defined in text and expressed in Appendix I	26b, A1–3	lb
i	$\sqrt{-1}$	33a	No dim.
J	Bessel function	33a	No dim.
K	Spring constant associated with mass M, Appendix 2	A2–10	lb/in
$K_{1\ldots6}$	Coefficients, defined in text	15a	
k	Thermal conductivity	A6–4	lb/sec–°F

SYMBOL	MEANING	FIRST APPEARANCE	UNITS
F	Bearing load in text and in Appendix 1 ($R - R_r$ in Appendix 6)	A1–9, (A6–7)	lb, (in)
M	Mass of bearing support	A2–10	lb-sec^2/in
M_t	Total gas mass-content of bearing	A2–4	lb-sec^2/in
M_r	Gas mass-content of recess volume	7	lb-sec^2/in
m	Bearing mass (Movable, or effective bearing mass)	6	lb-sec^2/in
P	Pressure in bearing gap, abs.	1	lb/in^2
P_{at}	Atmospheric (ambient) pressure, abs.	5	lb/in^2
P_o	Pressure in bearing gap, abs., equilibrium	6	lb/in^2
P_r	Pressure in bearing recess, abs.	6	lb/in^2
P_{ro}	Pressure in bearing recess, abs., equilibrium	6	lb/in^2
P_s	Supply pressure, abs.	8	lb/in^2
p	Perturbation pressure in bearing gap†, $P - P_o$	12	lb/in^2
p_r	Perturbation pressure in bearing recess, $P_r - P_{ro}$	12	lb/in^2
Q	Complex function, defined in test	28	No dim.
Q	"Charge" parameter, defined in Appendix 5	A5–11	No dim.
R	Radius of bearing	5b	in
R_r	Radius of recess	6b	in
R	Gas constant, $c_p - c_v$	8	in^2/sec^2-°F
r	Radial coordinate	1b	in
s	Complex variable	19	1/sec
T	Temperature, abs.	A6–4	°F
T_o	Temperature in bearing gap, abs., equilibrium ($T_o = T_{at} = T_{ro} = T_s$)	8	°F
t	Time	3	sec
$\bar{F}_i$	Velocity functions, defined in Appendix 6	A6–15	No dim.
u	Component of velocity in the x and/or r-direction	A6–1	in/sec
$\bar{u}$	Mean velocity in the x and/or r-direction	1	in/sec

† Except in Appendix 5, in which $p = P_r - P_{ro}$ is the perturbation pressure in the recess and the subscript "r" is omitted.

SYMBOL	MEANING	FIRST APPEARANCE	UNITS
$\bar{V}_1$	Velocity functions, defined in Appendix 6	A6–15	No dim.
V_r	Recess volume	A5–6	in^3
V_{ro}	Recess volume, equilibrium	A5–6	in^3
v	Pressure variable, defined in text	18	
W_o	Bearing gas mass flow rate, equilibrium	16	lb-sec/in
W_1	Rate of gas inflow through supply nozzle	7	lb-sec/in
W_2	Rate of gas outflow from recess (or outflow from bearing)	7, A5–1, (A2–4)	lb-sec/in
$\bar{W}_1$	Velocity functions, defined in Apnendix 6	A6–15	No. dim.
w	Velocity component in the z-direction	A6–1	in/sec
w_1	Perturbation inflow, $W_1 - W_o$	A5–6	lb-sec/in
w_2	Perturbation outflow, $W_2 - W_o$	A5–6	lb-sec/in
X	Half width of rectangular bearing	5a	in
X_r	Half width of rectangular recess	6a	in
x	Coordinate	1a	in
$x_{I,II}$	Solutions of homogeneous part of equation 19	28, 32a, 37b	
z	Substitution variable, $z = P_o^2$	18	lb^2/in^4
z	Coordinate	A6–1	in
a	Compressibility parameter, $12\mu\omega R_r^2/P_{ro}H_o^2$	37b	No dim.
β	Parameter, $1 - \left(\frac{P_{at}}{P_{ro}}\right)^2 \Big/ \log_e \frac{R}{Rr}$	37b, A1–3	No dim.
β	Parameter, $H_o/\delta + H_o$	A5–6	No dim.
γ	Ratio of specific heats, c_p/c_v	8	No dim.
δ	Recess depth	10	in
δ_c	Critical recess depth	44b	in
ϵ	Coefficient, $\frac{12\mu}{H_o^2}\left(\frac{X - X_r}{P_o^2 - P_{at}^2}\right)^2$, text	20a	in^6-sec/lb^3
ϵ	Dimensionless amplitude, h_o/H_o, Appendix 5	A5–6	No dim.

SYMBOL	MEANING	FIRST APPEARANCE	UNITS
λ	Parameter, $H^*/R - R_r$. Appendix 6	A6–7	No dim.
ξ'	Dummy variable, corresponding to $z = P_o^2$	28a	lb^2/in^4
η	Coefficient, $12\mu/H_o^2$	20b	lb-sec/in^4
$\overline{\theta}_i$	Temperature functions, defined in Appendix 6	A6–15	No dim.
$\lambda_{1\ldots5}$	Coefficients, defined in Appendix 2	A2–4	
$\Lambda_{0\ldots5}$	Coefficients, defined in Appendix 2	A2–17	
μ	Dynamic viscosity	1	lb-sec/in^2
ν	Kinematic viscosity	A6–7	in^2/sec
ξ	Dimensionless radius, r/R_r	37b	No dim.
π	Constant, 3.141...	8	No dim.
$\overline{\pi}_i$	Pressure functions, defined in Appendix 6	A6–15	No dim.
ρ	Dummy variable, corresponding to r	28b	in
ρ	Gas density in bearing gap	2	lb-sec^2/in^4
ρ_r	Gas density in bearing recess	10	lb-sec^2/in^4
ρ_{ro}	Gas density in bearing recess, equilibrium	11	lb-sec^2/in^4
σ	Perturbation density, $\rho_r - \rho_{ro}$	A5–6	lb-sec^2/in^4
$\phi_{I,II}$	Real component of $y_{I,II}$	38b	
$\psi_{I,II}$	Imaginary component of $y_{I,II}$	38b	
ϕ	Phase angle, defined in Appendix 5	A5–36	No dim.
ψ	Phase angle, defined in Appendix 5	A5–37	No dim.
ω	Circular frequency of forced vibration	A5–6	1/sec
ω	Imaginary part of s, $\omega = \alpha P_{ro} H_o^2/12\mu R_r^2$	37b	1/sec
ω_c	Critical frequency, corresponding to $s = \pm i\omega_c$ and $\delta = \delta_c$	37b	1/sec

SUBSCRIPTS AND OPERATORS

SYMBOL	REFERS TO	FIRST APPEARANCE
at	Atmospheric, or ambient	5
c	Critical	44b
e	Equivalent	A2–1
i	Ordering subscript (also isentropic, Appendix 5)	16, (A5–20)
m	Amplitude of perturbation, Appendix 5	A5–36
r	Recess and recess boundary	6
s	Supply	8
o	Equilibrium condition	6
D	Differentiation with respect to $\bar{t} = \omega t$, Appendix 5	A5–12
$(\cdot)$	Differentiation with respect to $\bar{t} = \frac{\nu^*}{L^2} t$, Appendix 6	A6–8
$(\cdot)$	Differentiation with respect to t, Appendix 2	A2–1
$(')$	Differentiation with respect to $z = P_o^2$, text	19a, 32a
$(')$	Differentiation with respect to x and r (or $\xi = r/R_p$), text	13, (37b)
$(-)$	Laplace transform of variable	19
$(-)$	Dimensionless quantity	A5–6, A6–7
$(-)$	Mean value of quantity	1
$(*)$	Reference quantity	A6–7

APPENDIX I

THE PRESSURE INTEGRAL, $\mathcal{J}$.†

In the integral

$$\mathcal{J} = \int_{R_r}^{R} P_o(r)\, r\, dr \tag{A1-1}$$

in which

$$P_o(r) = P_{ro}\left(1 - \beta \log_e \frac{r}{R_r}\right)^{\frac{1}{2}} \tag{A1-2}$$

and

$$\beta = \frac{1 - \left(\frac{P_{at}}{P_{ro}}\right)^2}{\log \frac{R}{R_r}} \tag{A1-3}$$

let

$$2P_o^2 = P_{ro}^2\, \beta\, \theta^2 \tag{A1-4}$$

The quantities under the integral sign then become:

$$\left.\begin{aligned} P_o &= P_{ro}\left(\frac{\beta}{2}\right)^{\frac{1}{2}} \theta \\ r &= R_r \left(\frac{R}{R_r}\right)^{\frac{P_{ro}^2}{P_{ro}^2 - P_{at}^2}} e^{-\frac{\theta^2}{2}} \\ dr &= -R_r \left(\frac{R}{R_r}\right)^{\frac{P_{ro}^2}{P_{ro}^2 - P_{at}^2}} e^{-\frac{\theta^2}{2}}\, \theta\, d\theta \end{aligned}\right\} \tag{A1-5}$$

†The method employed here in deriving an expression for $\mathcal{J}$ is identical with that contained in reference [27]

so that:

$$\mathcal{S} = -P_{ro} P_r^2 \left(\frac{R}{R_r}\right)^{\frac{2P_{ro}^2}{P_{ro}^2 - P_{at}^2}} \left(\frac{\beta}{2}\right)^{\frac{1}{2}} \int_{\theta(R_r)}^{\theta(R)} \theta^2 e^{-\theta^2} d\theta \qquad \text{(A1-6)}$$

Integration by parts and introduction of the factor $\frac{2}{\sqrt{\pi}}$ then yields:

$$\mathcal{S} = P_{ro} R_r^2 \left(\frac{\pi\beta}{32}\right)^{\frac{1}{2}} \left(\frac{R}{R_r}\right)^{\frac{2P_{ro}^2}{P_{ro}^2 - P_{at}^2}} \left[\frac{2}{\sqrt{\pi}} \theta e^{-\theta^2} - \frac{2}{\sqrt{\pi}} \int e^{-\theta^2} d\theta\right]_{\theta(R_r)}^{\theta(R)} \qquad \text{(A1-7)}$$

in which

$$\left.\begin{aligned} \theta(R_r) &= \left(\frac{2}{\beta}\right)^{\frac{1}{2}} \\ \theta(R) &= \frac{P_{at}}{P_{ro}} \left(\frac{2}{\beta}\right)^{\frac{1}{2}}. \end{aligned}\right\} \qquad \text{(A1-8)}$$

Values of the probability integral

$$\frac{2}{\sqrt{\pi}} \int_0^{\theta} e^{-x^2} dx$$

are listed in standard mathematical tables. The integral can be used in determining the load capacity, L, of the bearing:

$$L = \pi R_r^2 P_{ro} \left(\frac{2\mathcal{S}}{P_{ro} R_r^2} - \frac{P_{at}}{P_{ro}} \frac{R^2}{R_r^2} + 1\right) \qquad \text{(A1-9)}$$

A good approximation of the load capacity, L, may also be obtained in assuming a linear pressure profile. (See Figure A1-1.)

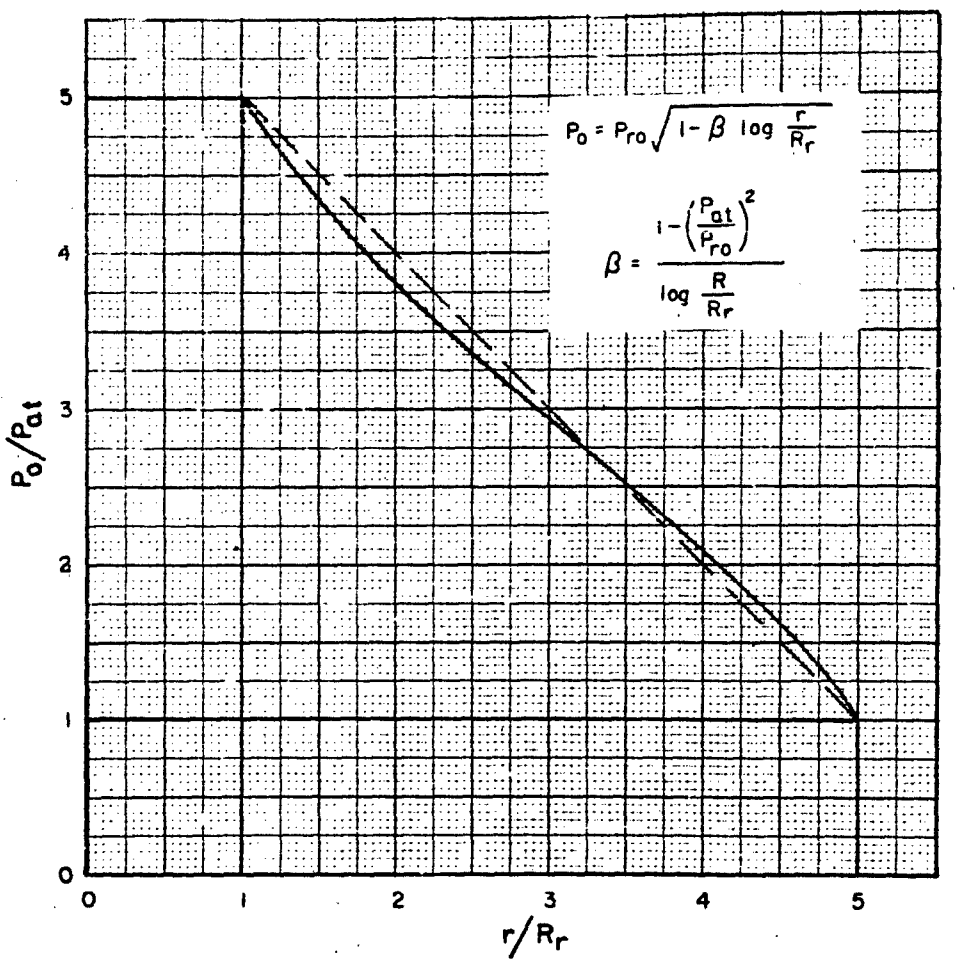


FIG. AI-1. PRESSURE PROFILE AND COMPARISON WITH STRAIGHT LINE APPROXIMATION

APPENDIX 2

SIMPLIFIED STABILITY ANALYSES

In the simplified approach to the stability problem [2,5], the flow in the slot is treated in terms of lumped parameters, and the dynamic pressure profile at any instant of time is assumed to be similar to that which would result if the bearing load were applied statically. In the case of the circular bearing, a further simplification is made by assuming a linear profile along the annulus. (See Figure A1–1, Appendix 1.)

The perturbation equations, which correspond to equations (13), (14) and (15) of the more exact analysis, can then be expressed in terms of the variation of gap width, h, and of the recess pressure, p_r, as follows.

$$m\ddot{h} = A_r p_r + \iint_{A_a} p\,dA + f(t) = A_e p_r + f(t) \qquad \text{(A2–1)}$$

$$\lambda_1 \dot{h} + \lambda_2 \dot{p}_r = -\lambda_4 h - (\lambda_3 + \lambda_5) p_r \qquad \text{(A2–2)}$$

in which

A_r = area over recess

A_a = area over annulus

A_e = "equivalent" area

the explicit expression for A_e being:

$$A_e = 2Y_r \left[1 + \frac{2}{3}\left(\frac{X}{X_r} - 1\right) \frac{\left(\frac{P_{ro}}{P_{at}}\right)^2 + 2\left(\frac{P_{at}}{P_{ro}}\right) - 3}{\left(\frac{P_{ro}}{P_{at}} - \frac{P_{at}}{P_{ro}}\right)^2} \right] \qquad \text{(A2–3a)}$$

$$A_e = \frac{1}{3}\pi R_r^2 \left[\left(\frac{R}{R_r}\right)^2 + \left(\frac{R}{R_r}\right) + 1 \right] \qquad \text{(A2–3b)}$$

while the coefficients λ_i are as follows:

$$\left.\begin{aligned}
\lambda_1 &= \left(\frac{\partial M}{\partial H}\right)_o = \frac{2X_r P_{ro}}{RT_o}\left\{1 + \frac{2}{3}\left(\frac{X}{X_r} - 1\right)\left(\frac{P_{ro}}{P_{at}}\right)\frac{1 - \left(\frac{P_{at}}{P_{ro}}\right)^3}{\left(\frac{P_{ro}}{P_{at}} - \frac{P_{at}}{P_{ro}}\right)}\right\} \\
\lambda_2 &= \left(\frac{\partial M}{\partial P_r}\right)_o = \frac{2X_r H_o}{\gamma R T_o}\left\{\left(\frac{\delta}{H_o} + 1\right) + \frac{2}{3}\gamma\left(\frac{X}{X_r} - 1\right)\frac{\left(\frac{P_{ro}}{P_{at}}\right)^2 + 2\left(\frac{P_{at}}{P_{ro}}\right) - 3}{\left(\frac{P_{ro}}{P_{at}} - \frac{P_{at}}{P_{ro}}\right)^2}\right\} \\
\lambda_3 &= -\left(\frac{\partial W_1}{\partial P_r}\right)_o = \frac{W_o}{2\bar{P}_{ro}}\left(\frac{2 - \frac{P_a}{P_{ro}}}{\frac{P_a}{P_{ro}} - 1}\right) \\
\lambda_4 &= \left(\frac{\partial W_2}{\partial H}\right)_o = \frac{3W_o}{H_o} \\
\lambda_5 &= \left(\frac{\partial W_2}{\partial P_r}\right)_o = \frac{2\Gamma_{ro} W_o}{P_{ro}^2 - P_{at}^2}
\end{aligned}\right\} \quad \text{(A2-4a)}$$

$$\left.\begin{aligned}
\Lambda_1 &= \left(\frac{\partial M}{\partial H}\right)_o = \frac{\pi R_r^2 P_{ro}}{RT_o}\left\{1 + \left(\frac{P_{at}}{P_{ro}}\right)\left[\left(\frac{R}{R_r}\right)^2 - 1\right] + \left(1 - \frac{P_{at}}{P_{ro}}\right)\frac{\left(\frac{R}{R_r}\right)^3 - 3\left(\frac{R}{R_r}\right) + 2}{3\left(\frac{R}{R_r} - 1\right)}\right\} \\
\Lambda_2 &= \left(\frac{\partial M}{\partial P_r}\right)_o = \frac{\pi R_r^2 H_o}{RT_o}\left\{\left(\frac{\delta}{H_o} + 1\right) + \gamma\,\frac{\left(\frac{R}{R_r}\right)^2 - 3\left(\frac{R}{R_r}\right) + 2}{3\left(\frac{R}{R_r} - 1\right)}\right\} \\
\Lambda_3 &= -\left(\frac{\partial W_1}{\partial P_r}\right)_o = \frac{W_o}{2P_{ro}}\left(\frac{2 - \frac{P_s}{P_{ro}}}{\frac{P_s}{P_{ro}} - 1}\right) \\
\Lambda_4 &= \left(\frac{\partial W_2}{\partial H}\right)_o = \frac{3W_o}{H_o} \\
\Lambda_5 &= \left(\frac{\partial W_2}{\partial P_r}\right)_o = \frac{2P_{ro}W_o}{P_{ro}^2 - P_{at}^2}
\end{aligned}\right\} \quad \text{(A2–4b)}$$

Combining equations (A2–1) and (A2–2) we obtain:

$$\dddot{h} + \left(\frac{\lambda_3 + \lambda_5}{\lambda_2}\right)\ddot{h} + \left(\frac{A_e}{m}\,\frac{\lambda_1}{\lambda_2}\right)\dot{h} + \left(\frac{A_e}{m}\,\frac{\lambda_4}{\lambda_2}\right)h = g(t) \qquad \text{(A2–5)}$$

The characteristic equation is a cubic:

$$s^3 + \Lambda_2 s^2 + \Lambda_1 s + \Lambda_o = 0 \qquad \text{(A2–6)}$$

and stability conditions require that all coefficients A_i be positive, and that

$$A_1 A_2 > A_o \tag{A2–7}$$

The inequality (A2–7) reduces to:

$$\frac{\lambda_1}{\lambda_2} > \frac{\lambda_4}{\lambda_3 + \lambda_5} \tag{A2–8}$$

from which the critical recess depth, indicated with broken lines in Figures 3 and 5, was calculated.

It is to be noted that the coefficients λ_i are analogous to the coefficients C_i and K_i in the more exact analysis, but that there is no one-to-one correspondence, except for λ_3 and λ_4.

Next, consider the relative, axial motion of two thrust plates, one integral with a rotor, of of mass m, the other with a support, of mass M, resting on isolators, or on an elastic foundation. We associate with the supporting structure the lumped elastic and damping constants, K and c. For the resulting, two-degrees-of-freedom system, shown diagrammatically in the appended Figure A2–1, the following set of equations may be written:

$$m\ddot{x}_2 - A_e p_r = f(t) \tag{A2–9}$$

$$M\ddot{x}_1 + c\dot{x}_1 + Kx_1 + A_e p_r = 0 \tag{A2–10}$$

$$\lambda_1 \dot{h} + \lambda_2 \dot{p}_r + \lambda_4 h + (\lambda_3 + \lambda_5) p_r = 0 \tag{A2–11}$$

in which

$$h = x_2 - x_1 \tag{A2–12}$$

Applying the Laplace transformation to equations (A2–9) through (A2–11), and assuming initial rest conditions, we obtain:

$$s^2 \bar{x}_2 - \frac{A_e}{m} \bar{p}_r = \frac{\bar{f}(s)}{m} \tag{A2–13}$$

$$\left(s^2 + s\frac{c}{M} + \frac{K}{M}\right)\bar{x}_1 + \frac{A_e}{m}\bar{p}_r = 0 \qquad \text{(A2–14)}$$

$$\bar{x}_2 - \bar{x}_1 - \left(\frac{s\lambda_2 + \lambda_3 + \lambda_5}{s\lambda_1 + \lambda_4}\right)\bar{p}_r = 0 \qquad \text{(A2–15)}$$

The characteristic determinant, corresponding to the set of equations (A2–13) through A2–15) is:

$$\begin{vmatrix} 0 & s^2 & -b \\ s^2 + Cs + \Omega^2 & 0 & B \\ -1 & 1 & \dfrac{s\lambda_2 + \lambda_3 + \lambda_5}{s\lambda_1 + \lambda_4} \end{vmatrix} = 0 \qquad \text{(A2–16)}$$

in which

$$\Omega^2 = \frac{K}{M}\ ; \qquad B = \frac{A_e}{M}\ ;$$

$$C = \frac{c}{M}\ ; \qquad b = \frac{A_e}{m}\ ;$$

The characteristic equation is a quintic:

$$A_5 s^5 + A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_o = 0 \qquad \text{(A2–17)}$$

in which the coefficients, A_i, are as follows:

$$\left.\begin{aligned} A_5 &= \lambda_2 \\ A_4 &= C\lambda_2 + (\lambda_3 + \lambda_5) \\ A_3 &= \Omega^2\lambda_2 + C(\lambda_3 + \lambda_5) + (B + b)\lambda_1 \\ A_2 &= \Omega^2(\lambda_3 + \lambda_5) + Cb\lambda_1 + (B + b)\lambda_4 \\ A_1 &= b(\Omega^2\lambda_1 + C\lambda_4) \\ A_o &= b\Omega^2\lambda_4 \end{aligned}\right\} \qquad \text{(A2–18)}$$

Without loss of generality, one may assume that $A_5 > 0$. The necessary and sufficient conditions that all roots of the quintic have negative real parts and the system be stable can be stated as follows [59]:

$$\Delta_1 = A_4 > 0 \tag{A2–19}$$

$$\Delta_2 = \begin{vmatrix} A_4 & A_5 \\ A_2 & A_3 \end{vmatrix} = (A_4 A_3 - A_5 A_2) > 0 \tag{A2–20}$$

$$\Delta_3 = \begin{vmatrix} A_4 & A_5 & 0 \\ A_2 & A_3 & A_4 \\ A_0 & A_1 & A_2 \end{vmatrix} = A_4(A_5 A_0 - A_4 A_1) + A_2 \Delta_2 > 0 \tag{A2–21}$$

$$\Delta_4 = \begin{vmatrix} A_4 & A_5 & 0 & 0 \\ A_2 & A_3 & A_4 & A_5 \\ A_0 & A_1 & A_2 & A_3 \\ 0 & 0 & A_0 & A_1 \end{vmatrix} = -\left[(A_5 A_0 - A_4 A_1)^2 + (A_3 A_0 - A_2 A_1)\Delta_2\right] > 0 \tag{A2–22}$$

$$\Delta_5 = A_0 \Delta_4 > 0 \tag{A2–23}$$

Since A_5, A_4 and A_0 are generally positive, it is sufficient to examine the signs of Δ_2, Δ_3 and Δ_4.

Two points in the latter part of this analysis are of particular interest:

a. If the damping coefficient, c, is zero, the stability conditions of this system are identical with those given in the inequality (A2–8) for the case of a rigidly supported bearing.

b. Since $A_5 = \lambda_2$ is the only coefficient containing the recess depth, δ, as a parameter, the determinants Δ_3 and Δ_4 yield a quadratic and a cubic in δ. Consequently, unlike in the case of the undamped, rigidly supported bearing, more than one value of δ_c may correspond to the same value of the pressure parameter, $P_s - P_{ro}/P_{ro}$.

In the foregoing analysis, it was assumed that the external damping for the rotor, of mass m, was negligible. This, indeed, would be the case if gas bearings were employed as a means of lateral support, but no difficulty, other than an algebraic one, is involved in adding a damping term, $-c_2\dot{x}_2$, to the left hand side of equation (A2–9).

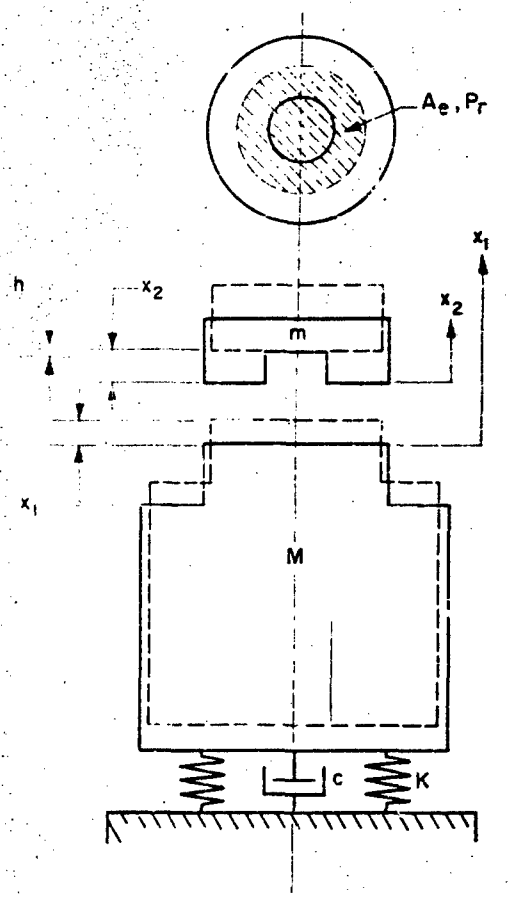


FIG. A2-1. SCHEMATIC DIAGRAM OF BEARING AND FLEXIBLE SUPPORT

APPENDIX 3

RECTANGULAR GEOMETRY – TABULATION OF CALCULATED RESULTS

$2X = 1$ in
$P_{at} = 14.7$ psia
$P_s = 88.2$ psia
$m = 1.65 \times 10^{-3}$ lb-sec^2/in./unit length.
$T_o = 530°$ F abs
$\rho = 2.65 \times 10^{-9}$ lb-sec/in^2
$\gamma = c_p/c_v = 1.4$

I. $2X_r = 0.2$ in; $a_N = 7.54 \times 10^{-4}$ in^2
II. $2X_r = 0.1$ in; $a_N = 7.54 \times 10^{-4}$ in^2
III. $2X_r = 0.1$ in; $a_N = 6.86 \times 10^{-3}$ in^2

Note: δ_c' refers to results of the simplified analysis.

P_{ro}	$P_s = P_{ro}/P_{ro}$	α_c	δ_c	δ_c'
psia	–	$\frac{\text{rad}}{\text{sec}} \times 10^{-3}$	in $\times 10^3$	in $\times 10^3$
		I		
72.5	0.217	4.93	1.90	2.21
75.0	0.176	4.82	2.65	2.93
80.0	0.103	4.35	5.75	5.64
83.2	0.060	3.75	11.8	9.70
85.0	0.0376	3.19	22.6	15.0
		II		
72.5	0.217	4.72	3.57	4.10
75.0	0.176	4.61	5.17	5.63
80.0	0.103	4.16	12.0	11.0
83.2	0.060	3.58	26.2	19.1
85.0	0.0376	3.02	54.1	29.7
		III		
72.5	0.217	3.26	7.45	8.55
75.0	0.176	3.19	10.8	11.8
80.0	0.103	2.89	24.8	22.0
83.2	0.060	2.51	52.5	39.8
85.0	0.0376	2.16	104.0	62.0

APPENDIX 4.

CIRCULAR GEOMETRY – DETERMINATION OF CRITICAL RECESS DEPTHS. TABULATION OF CALCULATED AND EXPERIMENTAL RESULTS.

When explicit expression of the coefficients C_{ij}, in equation (16b), are substituted into the characteristic determinant (31b) and the complex variable, **s**, is taken as equal to $i\omega$, the latter can be written as:

$$\|\Delta\| = \left\| \begin{array}{c|c} T_{11} & T_{12} \\ \hline T_{21} & T_{21} \end{array} \right\| = \left\| \begin{array}{c|c} F_{11} + G_{11} & F_{12} + G_{12} \\ \hline F_{21} + G_{21} & F_{22} + G_{22} \end{array} \right\| = 0 \qquad \text{(A4–1)}$$

in which

$$\left.\begin{aligned}
F_{11} &= i\left[\frac{2\sigma_s a}{P_{ro}H_r^2} - \frac{mP_{ro}H_o^2}{144\pi\mu^2 R_r^6}(aH_o)^3\right] \\
F_{12} &= F_{21} = ia \\
F_{22} &= -i\left(\frac{a}{\gamma H_o}\delta + \frac{a}{\gamma}\right) - \frac{\beta}{2}\left(\frac{2 - \dfrac{P_s}{P_{ro}}}{\dfrac{P_s}{P_{ro}} - 1}\right) \\
G_{11} &= -\left\{2\frac{\partial Q}{\partial \xi}(5,1) + 2\left(\frac{P_{at}}{P_{ro}}\right)^2\left(\frac{R}{R_r}\right)\frac{\partial Q}{\partial \xi}(1,5)\right. \\
&\qquad \left. - 2\left[1 + \left(\frac{P_{at}}{P_{ro}}\right)^2\right]\left(\frac{R}{R_r}\right)^5\frac{\partial Q}{\partial \xi}(1,1) + \left[1 - \left(\frac{P_{at}}{P_{ro}}\right)^2\right]\right\} \\
G_{12} &= -\left\{2\frac{\partial Q}{\partial \xi}(5,1) - 2\left(\frac{R}{R_r}\right)^5\frac{\partial Q}{\partial \xi}(1,1) + 1\right\}
\end{aligned}\right\} \qquad \text{(A4–2)}$$

$$G_{21} = \left(\frac{\partial \theta}{\partial \xi} (5,1) - 2\left(\frac{P_{at}}{P_{ro}}\right)^2 \left(\frac{R}{R_r}\right)^{\frac{1}{2}} \frac{\partial \theta}{\partial \xi} (1,1) + (1+\beta) \right)$$

$$G_{22} = - \left(\frac{\partial \theta}{\partial \xi} (5,1) + 1 \right)$$

(A4–2) Cont

and

$$\frac{\partial \theta}{\partial \xi} (5,1) = \left[\frac{(\phi_I \phi_{II} + \psi_I \psi_{II}) + i(\phi_{II} \psi_I - \phi_I \psi_{II})}{\phi_{II}^2 + \psi_{II}^2} \right]_{\xi = 5}$$

$$\frac{\partial \theta}{\partial \xi} (1,5) = \left[\frac{(\phi_{II} \phi'_{II} + \psi_{II} \psi'_{II}) + i(\phi_{II} \psi'_{II} - \phi'_{II} \psi_{II})}{\phi_{II}^2 + \psi_{II}^2} \right]_{\zeta = 5}$$

$$\frac{\partial \theta}{\partial \xi} (1,1) = \left[\frac{\phi_{II} - i\,\psi_{II}}{\phi_{II}^2 + \psi_{II}^2} \right]_{\xi = 5}$$

(A4–3)

In the foregoing expressions:

$$\alpha = \frac{12 \mu R_r^2 \omega}{P_{ro} H_o^2}$$

$$\beta = \frac{1 - \left(\frac{P_{at}}{P_{ro}}\right)^2}{\log_e \frac{R}{R_r}}$$

$$\delta = \int_{R_r}^{R} P_o(r)\, r\, dr \quad \text{(Appendix 1.)}$$

(A4–4)

and $\phi(\xi)$, $\psi(\xi)$ are the variables in equation (40b), solutions of which, for discrete values of α and for $\beta = 0.596481$, are tabulated in Appendix 7.

Let the bearing geometry and the parameters P_{at}, P_{ro}, T_o, γ and μ be specified. The gap width, H_o, can then be expressed in terms of P_s, the supply pressure, by equating the steady-state mass-flow rates through the nozzle and through the slot, e.g.:

$$W_o = \frac{\pi P_{ro}^2}{12\mu RT_o} H_o^3 = 2\left(\frac{\pi d_N^2}{4}\right)\left[\frac{\gamma}{RT_o}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} P_{ro}^2\left(\frac{P_a}{P_{ro}}-1\right)\right]^{\frac{1}{2}} \qquad \text{(A4-5)}$$

Given α, the unknown parameters which appear explicitly in the characteristic determinant are P_s and δ, which must be selected in such a manner as to reduce simultaneously to zero the real and imaginary parts of the characteristic equation. (e.g., $g_1 = g_2 = \delta_c$ in equation 43b.)

This reduction was accomplished by trial and error, as shown in the following sample calculation.

Sample Calculation

Given:

$m = 6.477 \times 10^{-2}$ lb-sec^2/in

$R_r = 0.5$ in

$R = 2.5$ in

$P_{at} = 14.7$ psia

$P_{ro} = 73.5$ psia

$T_o = 530°$ Fabs.

$R = 2.47 \times 10^5$ in^2/sec^2-°F

$\gamma = 1.4$

$\mu = 2.65 \times 10^{-9}$ lb-sec/in^2

Let

$\alpha = 0.050$

and assume

$P_s = 79.6$ psia

Then

$\alpha = 0.3965$

$\lambda = 0.1124 \times 10^{3}$

and from Appendix 7 have:

$\phi_I(5) = 0.3733$

$\phi_I'(5) = 0.4772$

$\phi_{II}(5) = 0.3598 \times 10^{1}$; $\phi_{II}'(5) = 0.6841$

$\phi_{II}(5) = 0.9151$; $\phi_{II}'(5) = 0.8906$

From equation (A4–3) have:

$$2\frac{\partial()}{\partial\xi}(5,1) = 0.2640 + i\,0.2020$$

$$2\frac{\partial()}{\partial\xi}(1,5) = 0.4860 + i\,0.3788$$

$$2\frac{\partial()}{\partial\xi}(1,1) = 0.5312 - i\,0.1378$$

Equation (A4–5) yields:

$$W_o = 0.2069 \times 10^{-4}\left(\frac{P_s}{P_{ro}} - 1\right)^{\frac{1}{2}} = 0.6019 \times 10^{-5}$$

so that

$H_o^3 = 0.4096 \times 10^{-3}\, W_o = 0.2465 \times 10^{-8}$

$H_o^2 = 0.1825 \times 10^{-5}$

$H_o = 0.1351 \times 10^{-2}$

and

$(\alpha H_o)^3 = 0.3082 \times 10^{-12}$

The elements of the determinant (A4–1) then become:

$$\left.\begin{aligned} F_{11} &= i\,0.5577 \\ G_{11} &= -0.8620 \times 10^{-1} - i\,0.5983 \\ T_{11} &= -0.8620 \times 10^{-1} - i\,0.4060 \times 10^{-1} \end{aligned}\right\}$$

$$\left.\begin{aligned} F_{22} &= -0.3295 \times 10^{-1} - i\,0.3571 \times 10^{-1} - i\,0.2644 \times 10^{2}\,\delta \\ G_{22} &= -0.1251 \times 10^{1} - i\,0.2020 \\ T_{22} &= -0.1559 \times 10^{1} - i\,0.2377 - i\,0.2644 \times 10^{2}\,\delta \end{aligned}\right\}$$

$$\left.\begin{aligned} F_{12} &= i\,0.5000 \times 10^{-1} \\ G_{12} &= 0.7600 \times 10^{-1} - i\,0.5101 \\ T_{12} &= 0.7600 \times 10^{-1} + i\,0.5601 \end{aligned}\right\}$$

$$\left.\begin{aligned} F_{21} &= i\,0.5000 \times 10^{-1} \\ G_{21} &= 0.1812 \times 10^{1} + i\,0.2143 \\ T_{21} &= 0.1812 \times 10^{1} + i\,0.2643 \end{aligned}\right\}$$

For $\alpha = 0.050$ and $P_s = 79.6$, the characteristic determinant (A4–1) reduces to:

$$\begin{aligned} \|\Lambda\| &= T_{11}T_{22} - T_{12}T_{21} \\ &= a(\delta - g_1) + ib(\delta - g_2) \\ &= \underline{-1.073(\delta - 0.3668) + i\,2.279(\delta - 0.3639)} \end{aligned}$$

Results of similar calculations for other values of P_s are as follows:

P_s	g_1	g_2
81.0	0.2550	0.3724
80.0	0.3274	0.3675
79.8	0.3455	0.3655
79.6	0.3668	0.3639

A plot of g_1 and g_2 vs. P_s, for $\alpha = 0.050$, is shown in the appended Figure A4-1.

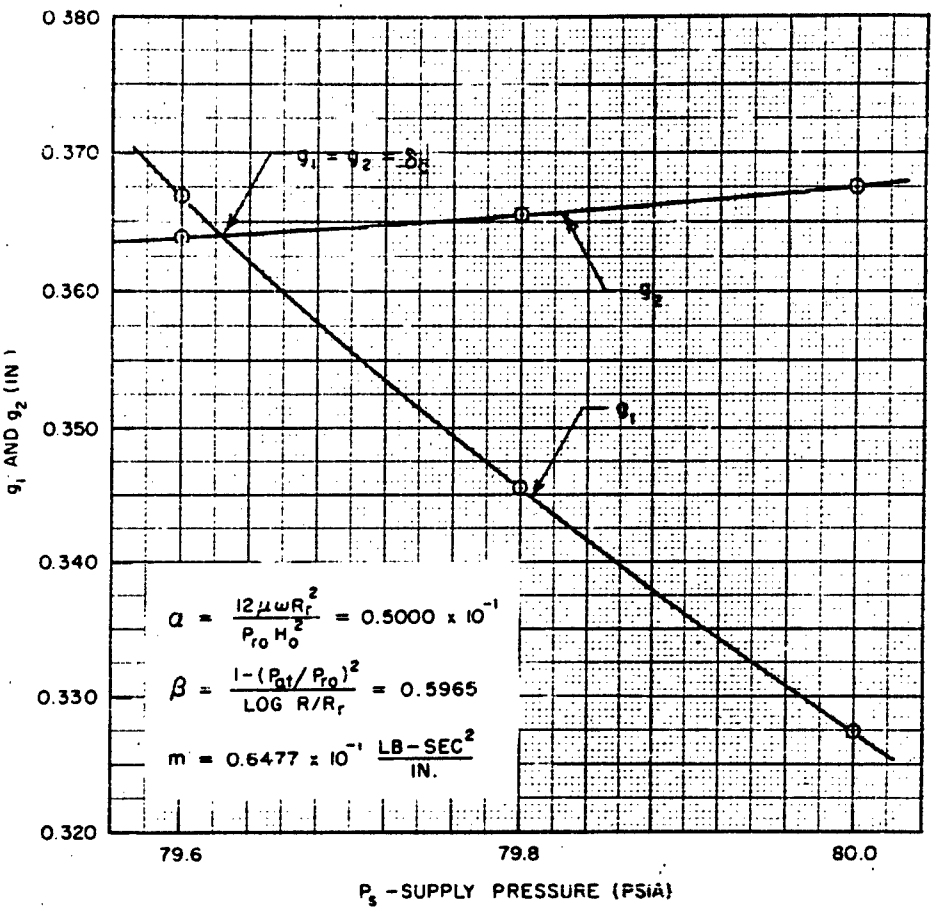


FIG. A4-1. DETERMINATION OF THE CRITICAL RECESS DEPTH FOR $\alpha = 0.050$

TABULATION OF CALCULATED RESULTS

PRESENT ANALYSIS

P_s	$P_s - P_{ro}/P_{ro}$	δ_c	ω_c
psia	–	in × 10^3	$\frac{rad}{sec} \times 10^{-3}$
I. $m = 3.592 \times 10^{-2}$ lb-sec^2/in			
82.8	0.127	143	1.55
83.0	0.129	110	1.72
83.1	0.131	95.9	1.81
83.2	0.132	90.0	1.85
83.3	0.133	83.5	1.90
83.4	0.135	78.1	1.94
84.2	0.146	49.0	2.24
II. $m = 6.477 \times 10^{-2}$ lb-sec^2/in			
79.4	0.0803	2650	0.334
79.6	0.0830	364	0.845
80.3	0.0925	117	1.22
80.6	0.0966	116	1.33
81.0	0.102	90.1	1.44
82.2	0.118	54.6	1.67
82.8	0.127	45.4	1.75
83.1	0.131	41.6	1.79
83.6	0.137	36.6	1.84
84.2	0.146	31.4	1.89
85.2	0.159	25.4	1.97
87.1	0.185	18.1	2.09
89.6	0.219	12.3	2.19
91.8	0.249	9.09	2.27
93.8	0.276	6.96	2.32
III. $m = 11.68 \times 10^{-2}$ lb-sec^2/in			
78.1	0.0626	273	0.769
81.4	0.108	50.8	1.29
82.4	0.121	39.9	1.36
88.4	0.203	13.8	1.62
91.0	0.238	9.60	1.68

$2R = 5$ in

$2R_p = 1$ in

$d_N = 0.055$ in ($a_N = 0.237 \times 10^{-2}$ in^2)

$P_{a2} = 14.7$ psia

$P_{ro} = 73.5$ psia

$T_o = 530°$ Fabs.

$\mu = 2.65 \times 10^{-9}$ lb-sec/in^2

$\gamma = \frac{c_p}{c_v} = 1.4$

SIMPLIFIED ANALYSIS

P_s	$P_s - P_{ro}/P_{ro}$	δ_c	ω_c
psia	–	in × 10^3	$\frac{rad}{sec} \times 10^{-3}$
75.0	0.0204	158	1.05
80.0	0.0834	42.0	1.76
85.0	0.157	23.3	2.07
90.0	0.225	15.2	2.25
95.0	0.293	10.5	2.37

TABULATION OF EXPERIMENTAL RESULTS

$2R_c$ = 5 in

$2R_r$ = 1 in

Twelve 0.0167 in diam. nozzles (Total throat area = 0.263×10^{-2} in^2)

I = 472 lb (Theoretical load at $P_{to} = 5\ P_{at}$ = 73.5 psia)

I. m = 6.477×10^{-2} lb-sec^2/in; P_{at} = 14.74 psia; T_o = 537° Fabs.

II. m = 11.56×10^{-2} lb-sec^2/in; P_{at} = 14.63 psia; T_o = 536° Fabs.

I

P_s	δ_c	ω_c
psia	in × 10^3	rad/sec
96.45	7.1	2242
95.45	7.7	2227
94.45	8.3	2214
93.45	9.4	2185
92.45	10.7	2153
91.45	13.0	2126
90.50	16.8	2076
89.50	23.5	1987
88.50	32.7	1884
87.50	46.0	1802
86.50	59.0	1727
86.05	69.0	1689
85.75	89.5	1610
85.75	122	1487
86.05	146	1424
86.50	170	1380
87.50	200	1330
89.50	244	1307
91.45	274	1277
93.45	293	1232
95.45	302	1334
97.40	308	1346

II

P_s	δ_c	ω_c
psia	in × 10^3	rad/sec
96.45	6.1	1713
95.45	6.9	1706
94.45	7.5	1699
93.45	8.3	1684
92.45	9.2	1676
91.45	10.6	1647
90.50	12.5	1621
89.50	14.7	1590
88.50	17.7	1552
87.50	22.2	1515
86.50	29.3	1471
85.55	38.2	1421
84.55	49.1	1362
83.55	62.5	1313
82.60	84.5	1239
82.10	107	1179
81.60	147	1094
81.60	206	1055
82.10	290	929
82.60	335	910
83.55	405	873
84.55	451	864
85.55	487	857
87.50	545	866
89.50	590	873
91.45	620	903
93.45	635	908
95.45	649	937
97.40	630	970

APPENDIX 5

THE PERIODIC CHARGING AND DISCHARGING OF A VARIABLE-VOLUME RECESS

The purpose of this appendix is to investigate, at least qualitatively, the adequacy of the assumption that the process of alternate charging and discharging of the variable-volume recess could be represented by the isentropic pressure-density relation $P\ \rho^{\gamma} = \text{constant}$ to a reasonable degree of approximation.

In the course of experimentation, small amplitude auto-oscillations and sine waves have been viewed simultaneously and compared on the display of the cathode ray oscilloscope. Observed differences in wave form were slight. We shall consider, therefore, the case of prescribed, sinusoidal motion of one bearing surface and, making a series of simplifying assumptions, similar to those in Appendix 2, utilize the energy equation, instead of resorting to an a priori assumption of the pressure-density relation in the recess region.

In what follows, pertinent equations will be developed and linearized. Resulting perturbation amplitudes of pressure and density, as well as phase angles, will be related to those obtained when $P\ \rho^{\gamma} = \text{constant}$. Numerical examples will be given for parameter values corresponding to representative points on the double valued, experimental stability locus I, Figure 55.

It will be assumed that perfect mixing of the entering gas occurs, but that fluid velocities in the region V_r are negligible and properties uniform throughout at any instant of time. Adiabaticity and equality of temperatures T_s and T_r of initially steady flow will also be assumed (e.g., $T_s = T_{ro} = T_o$). The flow rates and their perturbations will be expressed identically as in Appendix 2., implying the quasi-steady flow and lumped parameter approach to the problem. The fluid will be treated as a perfect gas with constant specific heats, e.g.:

$$\frac{P}{\rho} = RT;\ R = c_p - c_v;\ \frac{c_p}{c_v} = \gamma$$

so that

$$\left.\begin{aligned} \underline{u} - c_v T &= \frac{1}{\gamma - 1}\ \frac{P}{\rho} \\ \underline{h} = c_p T &= \frac{\gamma}{\gamma - 1}\ \frac{P}{\rho} \end{aligned}\right\}$$

where $\underline{u}$ is the internal energy and $\underline{h}$ the enthalpy.

The continuity and energy equations for the system shown in Figure A5–1 are as follows:

$$W_1 - W_2 = \frac{dM_r}{dt} = \frac{d}{dt}(\rho_r V_r) \tag{A5–1}$$

$$W_1 \underline{h}_s - W_2 \underline{h}_r = \frac{d}{dt}(\rho_r V_r \underline{u}_r) + P_r \frac{dV_r}{dt} \tag{A5–2}$$

which can be written as:

$$W_1 - W_2 = \rho_r \frac{dV_r}{dt} + V_r \frac{d\rho_r}{dt} \tag{A5–3}$$

$$W_1 \frac{P_s}{\rho_s} - W_2 \frac{P_r}{\rho_r} = P_r \frac{dV_r}{dt} + \frac{1}{\gamma} V_r \frac{dP_r}{dt} \tag{A5–4}$$

The following dimensionless perturbation quantities are now introduced:

$$\left.\begin{aligned}
W_1 &= W_o + w_1 = W_o(1 + \bar{w}_1) \\
W_2 &= W_o + w_2 = W_o(1 + \bar{w}_2) \\
P_r &= P_{ro} + p = P_{ro}(1 + \bar{p}) \\
\rho_r &= \rho_{ro} + \sigma = \rho_{ro}(1 + \bar{\sigma}) \\
H &= H_o + h = H_o(1 + \bar{h}) \\
V_r &= A_r(\delta + H_o) + A_r h = V_{ro} + A_r h = V_{ro}(1 + \beta\bar{h});\ \beta = \frac{H_o}{\delta + H_o} \\
t &= \frac{1}{\omega}\bar{t}
\end{aligned}\right\} \tag{A5–5}$$

Let also h be prescribed as:

$$\left.\begin{aligned}
h &= h_o \sin \omega t = H_o \epsilon \sin \omega t;\ \epsilon = \frac{h_o}{H_o} \ll 1 \\
&\text{so that} \\
\bar{h} &= \epsilon \sin \bar{t}
\end{aligned}\right\} \tag{A5–6}$$

Also, as in Appendix 2., we take;

$$\left.\begin{aligned}
\lambda_3 &= -\left(\frac{\partial W_1}{\partial P_r}\right)_o = \frac{W_o}{P_{ro}}\,\bar{\lambda}_3 \;;\; \bar{\lambda}_3 = \frac{1}{2}\left(\frac{2-\frac{P_s}{P_{ro}}}{\frac{P_s}{P_{ro}}-1}\right)\\
\lambda_5 &= -\left(\frac{\partial W_2}{\partial P_r}\right)_o = \frac{W_o}{P_{ro}}\,\bar{\lambda}_5 \;;\; \bar{\lambda}_5 = \frac{2}{1-\left(\frac{P_{at}}{P_{ro}}\right)^2}\\
\lambda_4 &= \left(\frac{\partial W_2}{\partial H}\right)_o = \frac{W_o}{H_o}\,\bar{\lambda}_4 \;;\; \bar{\lambda}_4 = 3\\
\lambda_3 + \lambda_5 &= \frac{W_o}{P_{ro}}\,(\bar{\lambda}_3 + \bar{\lambda}_5) = \frac{W_o}{P_{ro}}\,\bar{\Lambda}
\end{aligned}\right\} \quad \text{(A5–7)}$$

so that with:

$$\bar{w}_1 - \bar{w}_2 = -\bar{\Lambda}\bar{p} - \bar{\lambda}_4\bar{h} \qquad \text{(A5–8)}$$

equations (A5–3) and (A5–4) yield:

$$\dot{\bar{\sigma}} + Q\bar{\sigma} - \frac{1}{\gamma}\,\dot{\bar{p}} - Qp = 0 \qquad \text{(A5–9)}$$

$$\dot{\bar{\sigma}} + Q\bar{\Lambda}\bar{p} = -Q\bar{\lambda}_4\,\epsilon\sin\bar{t} - \beta\epsilon\cos\bar{t} \qquad \text{(A5–10)}$$

in which

$$\left.\begin{aligned}
Q &= \frac{W_o}{\rho_{ro}\,V_{rc}\,\omega}\\
\beta &= \frac{H_o}{\delta + H_o}
\end{aligned}\right\} \qquad \text{(A5–11)}$$

and $(\cdot) = \dfrac{d}{dt} = \dfrac{1}{\omega}\dfrac{d}{dt}$ denotes differentiation with respect to dimensionless time, $\bar{t}$.

In operational form, equations (A5–9) and (A5–10) are:

$$(D + Q)\bar{\sigma} - \left(\frac{1}{\gamma} D + Q\right)\bar{p} = 0 \qquad \text{(A5–12)}$$

$$D\bar{\sigma} + Q\bar{\Lambda}\bar{p} = -\epsilon(\beta D + Q\bar{\lambda}_4)\sin\bar{t} \qquad \text{(A5–13)}$$

where

$$D = \frac{d}{d\bar{t}}$$

On the other hand, an assumed relation

$$\frac{P_r}{P_{ro}} = \left(\frac{\rho_r}{\rho_{ro}}\right)^{\gamma} \qquad \text{(A5–14)}$$

yields

$$\frac{\bar{p}}{\bar{\sigma}} = \gamma \qquad \text{(A5–15)}$$

so that equation (A5–3), with the aid of equations (A5–8) and (A5–15), reduces to:

$$\frac{1}{\gamma}\dot{\bar{p}} + Q\bar{\Lambda}\bar{p} = -Q\bar{\lambda}_4\epsilon\sin\bar{t} - \beta\epsilon\cos\bar{t} \qquad \text{(A5–16)}$$

or:

$$\left(\frac{1}{\gamma} D + Q\bar{\Lambda}\right)\bar{p} = -\epsilon(\beta D + Q\bar{\lambda}_4)\sin\bar{t} \qquad \text{(A5–17)}$$

Operation on equations (A5–12) and (A5–13) with (D) and (D + Q), followed by subtraction, and operation with (QΛ) and (1/γ D + Q), followed by addition, yields the following net of equations:

$$\bar{p} = \Im\frac{-\epsilon\left[\beta D^2 + Q(\beta + \bar{\lambda}_4)D + Q^2\bar{\lambda}_4\right]e^{i\bar{t}}}{\left[\frac{1}{\gamma} D^2 + Q(1 + \bar{\Lambda})\, D + Q^2\bar{\Lambda}\right]}$$

$$= \Im\frac{-\epsilon\left[(Q^2\bar{\lambda}_4 - \beta) + iQ(\beta + \bar{\lambda}_4)\right]}{\left(Q^2\bar{\Lambda} - \frac{1}{\gamma}\right) + iQ(1 + \bar{\Lambda})}(\cos\bar{t} + i\sin\bar{t}) \qquad \text{(A5–18)}$$

$$= \frac{-\epsilon\left[\frac{1}{\gamma}\beta^2 - Q\left(\beta + \frac{1}{\gamma}\bar{\lambda}_4\right)\beta + Q^2\bar{\lambda}_4\right]e^{i\bar{t}}}{\left[\frac{1}{\gamma}\beta^2 - Q(2+\bar{\lambda})\beta + Q^2\bar{\lambda}\right]}$$

$$+ \frac{\epsilon\left[\left(Q^2\bar{\lambda}_4 - \frac{1}{\gamma}\beta\right) - iQ\left(\beta + \frac{1}{\gamma}\bar{\lambda}_4\right)\right]}{\left(Q^2\bar{\lambda} - \frac{1}{\gamma}\right) - iQ(1+\bar{\lambda})}(\cos\bar{t} + i\sin\bar{t}) \qquad (A5\text{–}19)$$

On the other hand, from equation (A5–17) we obtain:

$$\bar{p}_i = \frac{-\epsilon(\beta^2 + Q\bar{\lambda}_4)e^{i\bar{t}}}{\left(\frac{1}{\gamma}\beta - Q\bar{\lambda}\right)} = \frac{-\epsilon(Q\bar{\lambda}_4 + i\beta)}{Q\bar{\lambda} + i\frac{1}{\gamma}}(\cos\bar{t} + i\sin\bar{t}) \qquad (A5\text{–}20)$$

In the foregoing equations, the subscript $_i$ denotes the isentropic relation assumed in obtaining equation (A5–17), and $_\beta$ refers to the imaginary part of a complex quantity.

The steady-state pressure and density responses to a sinusoidal displacement input:

$$\bar{E} = \epsilon\sin\bar{t}$$

are as follows:

$$\bar{p} = \frac{\epsilon Q\left[(Q^2\bar{\lambda}_4 - \beta)(1+\bar{\lambda}) - (\beta + \bar{\lambda}_4)\left(Q^2\bar{\lambda} - \frac{1}{\gamma}\right)\right]}{\left(Q^2\bar{\lambda} - \frac{1}{\gamma}\right)^2 + Q^2(1+\bar{\lambda})^2}\cos\bar{t}$$

$$- \frac{\epsilon\left[(Q^2\bar{\lambda}_4 - \beta)\left(Q^2\bar{\lambda} - \frac{1}{\gamma}\right) + Q^2(\beta + \bar{\lambda}_4)(1+\bar{\lambda})\right]}{\left(Q^2\bar{\lambda} - \frac{1}{\gamma}\right)^2 + Q^2(1+\bar{\lambda})^2}\sin\bar{t} \qquad (A5\text{–}21)$$

$$= \frac{\epsilon Q\left[\left(Q^2\bar{\lambda}_4 - \frac{1}{\gamma}\beta\right)(1+\bar{\Lambda}) - \left(\beta + \frac{1}{\gamma}\bar{\lambda}_4\right)\left(Q^2\bar{\Lambda} - \frac{1}{\gamma}\right)\right]}{\left(Q^2\bar{\Lambda} - \frac{1}{\gamma}\right)^2 + Q^2(1+\bar{\Lambda})^2}\cos\bar{t}$$

$$- \frac{\epsilon\left[\left(Q^2\bar{\lambda}_4 - \frac{1}{\gamma}\beta\right)\left(Q^2\bar{\Lambda} - \frac{1}{\gamma}\right) + Q^2\left(\beta + \frac{1}{\gamma}\bar{\lambda}_4\right)(1+\bar{\Lambda})\right]}{\left(Q^2\bar{\Lambda} - \frac{1}{\gamma}\right)^2 + Q^2(1+\bar{\Lambda})^2}\sin\bar{t} \qquad \text{(A5–22)}$$

$$\bar{v}_1 = \frac{\epsilon Q\left(\frac{1}{\gamma}\bar{\lambda}_4 - \bar{\Lambda}\beta\right)}{Q^2\bar{\Lambda}^2 + \frac{1}{\gamma^2}}\cos\bar{t} - \frac{\epsilon\left(Q^2\bar{\Lambda}\bar{\lambda}_4 - \frac{1}{\gamma}\beta\right)}{Q^2\bar{\Lambda}^2 + \frac{1}{\gamma^2}}\sin\bar{t} \qquad \text{(A5–23)}$$

and, on the basis of assumption [A5–14], from equation [A5–15]

$$\bar{\sigma}_1 = \frac{1}{\gamma}\bar{v}_1 \qquad \text{(A5–24)}$$

The expressions for $\bar{v}$ and $\bar{\sigma}$ contain the parameters ϵ, γ, $\bar{\lambda}_4$, $\bar{\Lambda}$, β and Q, of which ϵ, γ and $\bar{\lambda}_4$ are constants and $\bar{\Lambda}$ is fixed for any given set of pressure ratios P_s/P_{ro} and P_{at}/P_{ro}. We note, once again, the form of the parameters β and Q:

$$\beta = \frac{H_o}{\delta + H_o}$$

and

$$Q = \frac{W_o}{\rho_{ro} V_{ro} \omega}$$

Prior to solving equations (A5–21) through (A5–24) for a set of numerical values, it is of interest to consider the asymptotic behavior of $\bar{v}$ and $\bar{\sigma}$ for limiting values of the "charge" parameter Q. We shall consider the cases when:

(a) Q is a very large number

and (b) Q is a very small number

while β remains finite.

Case (a) $\bar{\Omega}$ is very large

Equations (A5–21) and (A5–22) reduce to:

$$\bar{p} = -\epsilon \frac{\bar{\lambda}_4}{\bar{\Lambda}} \sin\bar{t} \qquad \text{(A5–25)}$$

$$\bar{\sigma} = -\epsilon \frac{\bar{\lambda}_4}{\bar{\Lambda}} \sin\bar{t} \qquad \text{(A5–26)}$$

so that

$$\frac{\bar{p}}{\bar{\sigma}} = 1 \quad \text{or} \quad \frac{p}{\sigma} = \frac{P_{ro}}{\rho_{ro}} = \frac{a_o^2}{\gamma} \qquad \text{(A5–27)}$$

We note that equation (A5–23) also reduces to:

$$\bar{p}_i = -\epsilon \frac{\bar{\lambda}_4}{\bar{\Lambda}} \sin\bar{t} \qquad \text{(A5–28)}$$

but since the assumption $(P_r/P_{ro}) = (\rho_r/\rho_{ro})^\gamma$ prescribes the $\bar{p}_i$ and $\bar{\sigma}_i$ relation, we have:

$$\frac{\bar{p}_i}{\bar{\sigma}_i} = \gamma \quad \text{or} \quad \frac{p_i}{\sigma_i} = \frac{\gamma P_{ro}}{\rho_{ro}} = a_o^2 \qquad \text{(A5–29)}$$

In case (a), the following conclusions can be drawn:

1. The pressure-density relation is nearly isothermal.
2. The pressure and density are in phase and both are 180° out of phase with the displacement $\bar{h} = \epsilon \sin \bar{t}$. (e.g., increasing with decreasing gap width.) The coefficient

$$-\epsilon \frac{\bar{\lambda}_4}{\bar{\Lambda}} = -\epsilon \frac{\bar{\lambda}_4}{\bar{\lambda}_3 + \bar{\lambda}_5}$$

is a measure of the "static stiffness". The physical significance of the foregoing is that, if the motion is very slow (quasi-static), or if the mass flow rate is relatively large and the mass content of the recess region is relatively small, the changes occur nearly isothermally.

Case (b): ϵ is very small.

Equations (A5–21) and (A5–22) reduce to:

$$\bar{p} = -\epsilon\gamma\beta\sin\bar{t} \qquad \text{(A5–30)}$$

$$\bar{\sigma} = -\epsilon\beta\sin\bar{t} \qquad \text{(A5–31)}$$

so that

$$\frac{\bar{p}}{\bar{\sigma}} = \gamma \quad \text{or} \quad \frac{p}{\sigma} = \frac{\gamma P_{ro}}{\rho_{ro}} = a_o^2 \qquad \text{(A5–32)}$$

Equation (A5–23) also reduces to:

$$\bar{p}_i = -\epsilon\gamma\beta\sin\bar{t} \qquad \text{(A5–33)}$$

so that identically as in equation (A5–32):

$$\frac{\bar{p}_i}{\bar{\sigma}_i} = \gamma \quad \text{or} \quad \frac{p_i}{\sigma_i} = \frac{\gamma P_{ro}}{\rho_{ro}} = a_o^2 \qquad \text{(A5–34)}$$

which was the assumed relation.

In case (b), the following conclusions can be drawn:

1. The pressure-density relation is nearly isentropic.
2. The pressure and density are in phase and both are 180° out of phase with the displacement $\bar{h} = \epsilon\sin\bar{t}$. (e.g., increasing with decreasing gap width.) The coefficient

$$-\epsilon\gamma\beta = -\gamma\epsilon\frac{H_o}{\delta + H_o} = -\gamma\frac{v_o}{V_o}$$

is indicative of the "dynamic stiffness", such as would be the case of gas in a closed cylinder and piston system undergoing isentropic compression or expansion. The physical significance of the foregoing is that, if the motion is very fast, or if the mass flow rate is relatively small and the recess mass content is relatively large, the changes occur nearly isentropically.

To correlate the analysis with experimental results, the numerical examples were selected to correspond approximately to two points of the experimental stability locus I, Figure 55, for which the data are as follows:

		1	2
P_c	psia	91.5	
P_{ro}	psia	75.9	
P_{at}	psia	14.7	
T_o	°R abs.	537	
ρ_{ro}	lb-sec^2/in^4	0.5722×10^{-6}	
W_o	lb-sec/in	10×10^{-6}	
H_o	in	1.64×10^{-3}	
A_r	in^2	0.7854	
$\bar{\lambda}_3$	–	1.933	
$\bar{\lambda}_5$	–	2.078	
$\bar{\Lambda}$	–	4.011	
$\bar{\lambda}_4$	–	3.000	
γ	–	1.4 ($1/\gamma$ = 0.7143)	
ϵ	–	10^{-2}	
δ	in	13×10^{-3}	275×10^{-3}
ω	rad/sec	2125	1280
V_3	in^2	11.50×10^{-3}	217.3×10^{-3}
Q	–	0.7153	6.284×10^{-2}
β	–	0.1120	5.928×10^{-3}

(A5–35)

Example 3

$$\left.\begin{aligned} \bar{p} &= \epsilon\bar{p}_m \sin(\bar{t} - \phi) = \epsilon(-0.6792\sin\bar{t} + 0.1624\cos\bar{t}) \\ \tan\phi &= \frac{0.1624}{-0.6792} = -0.2391 \\ \phi &= 180° - 13.45° = 166.6° \\ \bar{p}_m &= 0.6985 \end{aligned}\right\} \quad (A5\text{–}36)$$

$$\left.\begin{aligned} \bar{\sigma} &= \epsilon\bar{\sigma}_m \sin(\bar{t} - \phi) = \epsilon(-0.5291\sin\bar{t} + 0.2232\cos\bar{t}) \\ \tan\phi &= \frac{0.2232}{-0.5291} = -0.4218 \\ \phi &= 180° - 22.87° = 157.1° \\ \bar{\sigma}_m &= 0.5743 \end{aligned}\right\} \quad (A5\text{–}37)$$

$$\phi - \phi = 9.5° \quad (A5\text{–}38)$$

$$\left.\begin{aligned} \bar{p}_i &= \epsilon\bar{p}_{mi} \sin(\bar{t} - \phi_i) = \epsilon(-0.7135\sin\bar{t} + 0.1386\cos\bar{t}) \\ \tan\phi_i &= \frac{0.1386}{-0.7135} = -0.1943 \\ \phi_i &= 180° - 11° = 169° \\ \bar{p}_{mi} &= 0.7269 \\ \bar{\sigma}_{mi} &= \frac{1}{\gamma}\,\bar{p}_{mi} = 0.5192 \end{aligned}\right\} \quad (A5\text{–}39)$$

$$\phi_i - \phi = 2.4° \quad (A5\text{–}40)$$

$$\phi_i - \psi = 11.9° \quad (A5\text{–}41)$$

Example 2.

$$\left.\begin{aligned} \bar{p} &= \epsilon\bar{p}_m \sin(\bar{t}-\phi) = \epsilon(-0.0911\sin\bar{t} + 0.2218\cos\bar{t}) \\ \tan\phi &= \frac{0.2218}{-0.0911} = -2.468 \\ \phi &= 180° - 67.93° = 112.1° \\ \bar{p}_m &= 0.2445 \end{aligned}\right\} \quad (A5\text{–}42)$$

$$\left.\begin{aligned} \bar{\sigma} &= \epsilon\bar{\sigma}_m \sin(\bar{t}-\psi) = \epsilon(-0.0611\sin\bar{t} + 0.1625\cos\bar{t}) \\ \tan\psi &= \frac{0.1625}{-0.0611} = -2.660 \\ \psi &= 180° - 69.4° = 110.6° \\ \bar{\sigma}_m &= 0.1765 \end{aligned}\right\} \quad (A5\text{–}43)$$

$$\phi - \psi = 1.5° \quad (A5\text{–}44)$$

$$\left.\begin{aligned} \bar{p}_i &= \epsilon\bar{p}_{mi} \sin(\bar{t}-\phi_i) = (-0.0923\sin\bar{t} + 0.2322\cos\bar{t}) \\ \tan\phi_i &= \frac{0.2322}{-0.0923} = -2.516 \\ \phi_i &= 180° - 68.32° = 111.7° \\ \bar{p}_{mi} &= 0.2499 \\ \bar{\sigma}_{mi} &= \frac{1}{\gamma}\bar{p}_{mi} = 0.1785 \end{aligned}\right\} \quad (A5\text{–}45)$$

$$\phi_i - \phi = -0.4° \quad (A5\text{–}46)$$

$$\phi_i - \psi = 1.1° \quad (A5\text{–}47)$$

Comparison of results of examples 1 and 2 shows that pressure-density phase angles are relatively small, approximately 7.7° and 1.5° respectively. When the assumed relation is $\bar{p} = (\rho_r \bar{\rho})^\gamma$, this is equivalent to prescribing that the density and pressure be in phase. In example 1, the angle between pressures is approximately 2.4° and that between densities 11.9°. In example 2, the corresponding angles are very small; 0.4° and 1.1° respectively.

Since all angles are relatively small, so that all densities and pressures are "almost in phase", a comparison of amplitudes is of interest as a criterion of validity of the isentropic assumption.

From example 1 we have:

$$\frac{\bar{p}_{mi}}{\bar{p}_m} = 1.01 \quad \text{and} \quad \frac{\bar{\sigma}_{mi}}{\bar{\sigma}_m} = 0.904 \tag{A5-48}$$

also:

$$\frac{\bar{p}_m}{\bar{\sigma}_m} = 1.22, \text{ as compared with } \frac{\bar{p}_{mi}}{\bar{\sigma}_{mi}} = \gamma = 1.40 \tag{A5-49}$$

In example 2 the ratios are:

$$\frac{\bar{p}_{mi}}{\bar{p}_m} = 1.02 \quad \text{and} \quad \frac{\bar{\sigma}_{mi}}{\bar{\sigma}_m} = 1.01 \tag{A5-50}$$

also:

$$\frac{\bar{p}_m}{\bar{\sigma}_m} = 1.37, \text{ as compared with } \frac{\bar{p}_{mi}}{\bar{\sigma}_{mi}} = \gamma = 1.40 \tag{A5-51}$$

In the foregoing examples, all parameters were identical, with the exception of the frequencies and the recess volumes. The ratio of the former was of the order 2; that of the latter of the order 1 20. Significantly, as might have been anticipated from physical considerations, the pressure and density amplitudes of example 2 were barely 1/3 of those in example 1.

For a sufficiently large volume, pressure and density changes in the recess, associated with small but finite motion, could become negligible. A very different condition, of course, would prevail in the bearing gap, due to the combined squeezing and trapping of the fluid between the narrowly spaced boundaries.

Results of calculations of examples 1 and 2 are illustrated in Figure A5–2.

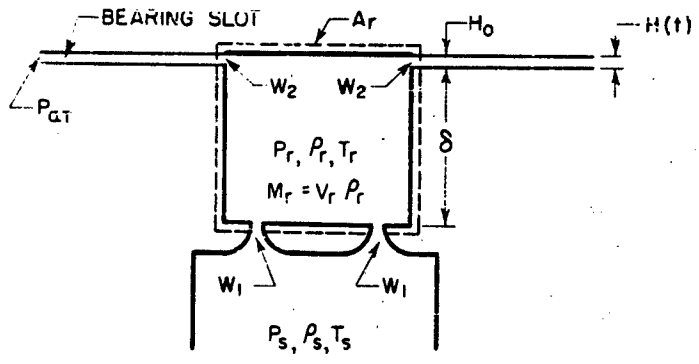


FIG. A5-1 SCHEMATIC DIAGRAM OF VARIABLE-VOLUME BEARING RECESS

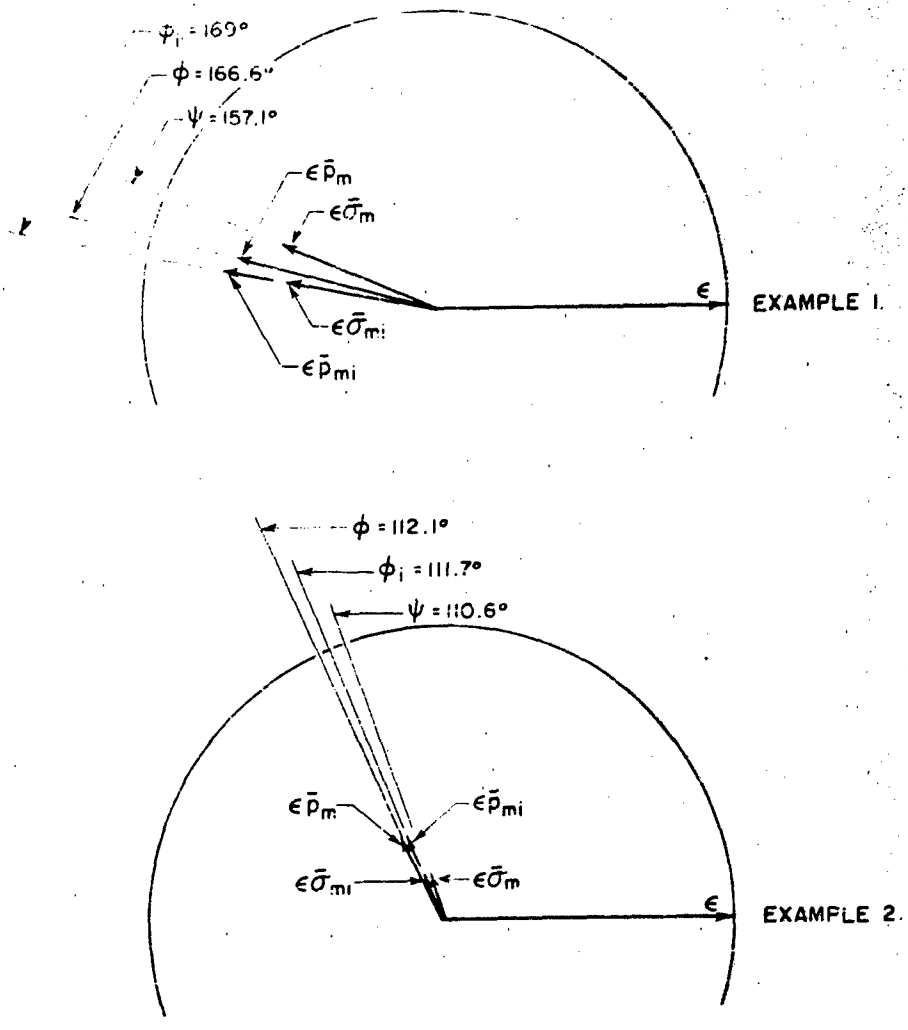


FIG. A5-2. AMPLITUDES AND PHASE RELATIONS OF PRESSURE AND DENSITY IN VARIABLE VOLUME RECESS

APPENDIX 6

DERIVATION OF THE DYNAMIC LUBRICATION EQUATION

The purpose of this appendix is to show that the differential equation (4) of the main text provides an adequate description of fluid-dynamical phenomena in the bearing slot, provided orders of magnitude of characteristic parameters fulfill certain requirements. The development follows closely the method of Elrod [15] who employed a small-parameter technique in the derivation of equations for gas lubrication.

It will be assumed that the fluid obeys the perfect gas law and that the specific heats, viscosity and thermal conductivity are constant. Continuity and constancy of temperature of the boundaries will also be assumed. In the range of pressures and temperatures considered, the foregoing assumptions are reasonable for gas flow in a narrow slot between metallic members.

The momentum, continuity and energy equations, stated in cylindrical coordinates, for axially symmetric, laminar motion, and the equation of state are [65]:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial r}+\frac{\mu}{3}\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}\right)+\mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{A6-1}$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\frac{\mu}{3}\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}\right)+\mu\left(\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right) \tag{A6-2}$$

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(r\rho u)+\frac{\partial}{\partial z}(\rho w)=0 \tag{A6-3}$$

$$c_p\rho\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)-\left(\frac{\partial P}{\partial t}+u\frac{\partial P}{\partial r}+w\frac{\partial P}{\partial z}\right)=\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+k\frac{\partial^2 T}{\partial z^2}+\mu\left[2\left(\frac{\partial u}{\partial r}\right)^2+2\left(\frac{\partial w}{\partial z}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)^2-\frac{2}{3}\left(\frac{\partial u}{\partial r}+\frac{\partial w}{\partial z}\right)^2\right] \tag{A6-4}$$

$$\frac{P}{\rho}=RT \tag{A6-5}$$

in which $\mu, k = c_p - c_v, \gamma = \frac{c_p}{c_v}$ are constant.

The boundary conditions are:

$$\left.\begin{aligned} &\text{At} \quad z = 0; && u = w = 0 \\ & && T = T^* \\ &\text{At} \quad z = H(t); && u = 0 \\ & && w = \frac{dH}{dt} \\ & && T = T^* \end{aligned}\right\} \qquad \text{(A6–6)}$$

Denoting reference quantities with $(\ )^*$ and dimensionless quantities with $(\bar{\ })$, the following set of variables is formed:

$$\left.\begin{aligned} u &= \rho^* \nu^* = \text{constant} \\ \rho &= \rho^* \bar{\rho} \\ T &= T^* \bar{T} \\ P &= \rho^* \left(\frac{\nu^*}{H^*}\right)^2 \bar{P} \\ H &= H^* f(t), \quad \text{where } f(t) \text{ is 0. (1)} \\ r &= L\bar{r}, \quad \text{where } L = R - R_r \\ z &= H^* f \bar{z} \\ t &= \frac{L^2}{\nu^*} \bar{t} \\ u &= \frac{\nu^*}{L} \bar{u} \\ w &= \frac{H^*}{L} \frac{\nu^*}{L} \bar{w} = \epsilon \frac{\nu^*}{L} \bar{w}, \quad \text{where } \epsilon = \frac{H^*}{L} \end{aligned}\right\} \qquad \text{(A6–7)}$$

also:

$$\frac{\bar{P}}{\bar{T}} = \frac{\rho^*\left(\frac{\cdot}{H^*}\right)^2}{T^*}\frac{\bar{P}}{\bar{T}} = \frac{\rho^*\left(\frac{\cdot}{H^*}\right)^2}{P^*}\frac{P^*}{T^*}\frac{\rho^*}{P^*}\frac{\bar{P}}{\bar{T}}$$

$$\bar{\rho} = \frac{\rho}{\rho^*} = \frac{1}{P^*}\frac{\bar{P}}{\bar{T}}$$

(A6-7) Cont

In the foregoing $\epsilon = \frac{H^*}{R - R_2} = \frac{H^*}{L}$ is the small parameter used in developing the dynamic lubrication equations.

In transforming the equations of fluid motion into their dimensionless equivalents, the following relations between derivatives hold:

$$\frac{\partial \bar{r}}{\partial r} = \frac{1}{L} \quad ; \quad \frac{\partial \bar{r}}{\partial z} = 0 \quad ; \quad \frac{\partial \bar{r}}{\partial t} = 0$$

$$\frac{\partial \bar{z}}{\partial r} = \frac{1}{fH^*} \quad ; \quad \frac{\partial \bar{z}}{\partial t} = -\frac{\nu^*}{L^2}\bar{z}\frac{\dot{f}}{f} \quad ; \quad \frac{\partial \bar{z}}{\partial r} = 0$$

$$\frac{\partial \bar{t}}{\partial r} = \frac{1}{L} \quad ; \quad \frac{\partial \bar{t}}{\partial z} = 0 \quad ; \quad \frac{\partial \bar{t}}{\partial t} = 0$$

(A6-8)

so that:

$$\frac{\partial}{\partial r} = \frac{1}{L}\frac{\partial}{\partial \bar{r}} \quad ; \quad \frac{\partial^2}{\partial r^2} = \frac{1}{L^2}\frac{\partial^2}{\partial \bar{r}^2}$$

$$\frac{\partial}{\partial z} = \frac{1}{fH^*}\frac{\partial}{\partial \bar{z}} \quad ; \quad \frac{\partial^2}{\partial z^2} = \frac{1}{f^2 H^{*2}}\frac{\partial^2}{\partial \bar{z}^2}$$

$$\frac{\partial}{\partial t} = \frac{\nu^*}{L^2}\left(\frac{\partial}{\partial \bar{t}} - \frac{\dot{f}}{f}\bar{z}\frac{\partial}{\partial \bar{z}}\right)$$

in which $(') = \frac{d}{d\bar{t}} = \frac{\nu^*}{L^2}\frac{d}{dt}$

The dimensionless form of equations (A6–1) through (A6–5) then is:

$$\left[\frac{\partial \bar{u}}{\partial \bar{t}} + \left(\frac{1}{f}\,\bar{w} - \frac{\dot{f}}{f}\,\bar{z}\right)\frac{\partial \bar{u}}{\partial \bar{z}} - \bar{u}\,\frac{\partial \bar{u}}{\partial \bar{r}}\right] = -\frac{1}{\epsilon^2}\,\frac{\partial \bar{P}}{\partial \bar{r}} + \frac{1}{3}\,\frac{\partial}{\partial \bar{r}}\left(\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{1}{f}\,\frac{\partial \bar{w}}{\partial \bar{z}}\right)$$

$$\left(\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\,\frac{\partial \bar{u}}{\partial \bar{r}} - \frac{\bar{u}}{\bar{r}^2} + \frac{1}{\epsilon^2}\,\frac{1}{f^2}\,\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right) \qquad \text{(A6–9)}$$

$$\left[\frac{\partial \bar{w}}{\partial \bar{t}} + \left(\frac{1}{f}\,\bar{w} - \frac{\dot{f}}{f}\,\bar{z}\right)\frac{\partial \bar{w}}{\partial \bar{z}} - \bar{u}\,\frac{\partial \bar{w}}{\partial \bar{r}}\right] = -\frac{1}{\epsilon^4}\,\frac{1}{f}\,\frac{\partial \bar{P}}{\partial \bar{z}}$$

$$+\frac{1}{3}\,\frac{1}{\bar{r}}\,\frac{\partial}{\partial \bar{z}}\left(\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{1}{f}\,\frac{\partial \bar{w}}{\partial \bar{z}}\right) + \left(\frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\,\frac{\partial \bar{w}}{\partial \bar{r}} + \frac{1}{\epsilon^2}\,\frac{1}{f}\,\frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right) \qquad \text{(A6–10)}$$

$$\frac{\partial \bar{\rho}}{\partial \bar{t}} - \frac{\dot{f}}{f}\,\bar{z}\,\frac{\partial \bar{\rho}}{\partial \bar{z}} + \frac{1}{\bar{r}}\,\frac{\partial}{\partial \bar{r}}\left(\bar{r}\,\bar{\rho}\,\bar{u}\right) + \frac{1}{f}\,\frac{\partial}{\partial \bar{z}}\left(\bar{\rho}\,\bar{w}\right) = 0 \qquad \text{(A6–11)}$$

$$\frac{c_p T^* L^2}{\nu^{*2}}\,\bar{\rho}\left[\frac{\partial \bar{T}}{\partial \bar{t}} + \left(\frac{1}{f}\,\bar{w} - \frac{\dot{f}}{f}\,\bar{z}\right)\frac{\partial \bar{T}}{\partial \bar{z}} + \bar{u}\,\frac{\partial \bar{T}}{\partial \bar{r}}\right] - \frac{1}{\epsilon^2}\left[\frac{\partial \bar{P}}{\partial \bar{t}} + \left(\frac{1}{f}\,\bar{w} - \frac{\dot{f}}{f}\,\bar{z}\right)\frac{\partial \bar{P}}{\partial \bar{z}} + \bar{u}\,\frac{\partial \bar{P}}{\partial \bar{r}}\right]$$

$$= \frac{k}{\mu c_p}\,\frac{c_p T^* L^2}{\nu^{*2}}\left[\frac{1}{\bar{r}}\,\frac{\partial}{\partial \bar{r}}\left(\bar{r}\,\frac{\partial \bar{T}}{\partial \bar{r}}\right) + \frac{1}{\epsilon^2}\,\frac{1}{f^2}\,\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right]$$

$$+\left[2\left(\frac{\partial \bar{u}}{\partial \bar{r}}\right)^2 + 2\,\frac{1}{f^2}\left(\frac{\partial \bar{w}}{\partial \bar{z}}\right)^2 + \left(\frac{1}{\epsilon}\,\frac{1}{f}\,\frac{\partial \bar{u}}{\partial \bar{z}} + \epsilon\,\frac{\partial \bar{w}}{\partial \bar{r}}\right)^2 - \frac{2}{3}\left(\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{1}{f}\,\frac{\partial \bar{w}}{\partial \bar{z}}\right)^2\right] \qquad \text{(A6–12)}$$

$$\frac{\bar{P}}{\rho} = \bar{P}^*\bar{T} \qquad \text{(A6–13)}$$

The boundary conditions are:

$$\left.\begin{array}{lll} \text{At } \bar{z} = 0 \;; & \bar{u} = \bar{w} = 0 \;, & \bar{T} = 1 \\ \text{At } \bar{z} = 1 \;; & \bar{u} = 0 & \\ & \bar{w} = \dot{f} \;, & \bar{T} = 1 \end{array}\right\} \qquad \text{(A6–14)}$$

Consider now solutions of equations (A6–9) through (A6–12) which can be expanded in terms of the small parameter ϵ as follows:

$$\left.\begin{aligned}
\bar{U} &= \bar{U}_0 + \epsilon^2 \bar{U}_1 + \epsilon^4 \bar{U}_2 + \cdots \\
\bar{W} &= \bar{W}_0 + \epsilon^2 \bar{W}_1 + \epsilon^4 \bar{W}_2 + \cdots \\
\bar{\Pi} &= \bar{\pi}_0 + \epsilon^2 \bar{\pi}_1 + \epsilon^4 \bar{\pi}_2 + \cdots \\
\bar{T} &= \bar{\theta}_0 + \epsilon^2 \bar{\theta}_1 + \epsilon^4 \bar{\theta}_2 + \cdots
\end{aligned}\right\} \quad \text{(A6–15)}\dagger$$

so that:

$$\bar{\rho} = \frac{1}{\bar{P}^*}\,\frac{\bar{\Pi}}{\bar{T}} = \frac{1}{\bar{P}^*}\,\frac{\bar{\pi}_0 + \epsilon^2 \bar{\pi}_1 + \cdots}{\bar{\theta}_0 + \epsilon^2 \bar{\theta}_1 + \cdots} = \frac{1}{\bar{P}^*}\left[\frac{\bar{\pi}_0}{\bar{\theta}_0} + \epsilon^2\left(\frac{\bar{\pi}_1}{\bar{\theta}_0} - \frac{\bar{\pi}_0 \bar{\theta}_1}{\bar{\theta}_0^{\,2}}\right) + \cdots\right]$$

The functions appearing in equation (A6–15) depend in general on $\bar{t}$, $\bar{r}$ and $\bar{z}$ and are independent of ϵ. The zeroth order functions are made to assume the boundary values (A6–14), so that approximating functions of higher order vanish at the boundaries. e.g.:

$$\left.\begin{aligned}
&\text{At } \bar{z} = 0 \quad : \quad \bar{U}_0 = \bar{U}_1 = \bar{U}_2 \cdots = 0 \\
&\qquad\qquad\quad\; \bar{W}_0 = \bar{W}_1 = \bar{W}_2 \cdots = 0 \\
&\qquad\qquad\quad\; \bar{\theta}_0 = 1 \; ; \quad \bar{\theta}_2 = \bar{\theta}_3 = 0 \\
&\text{At } \bar{z} = 1 \quad : \quad \bar{U}_0 = \bar{U}_1 = \bar{U}_2 \cdots = 0 \\
&\qquad\qquad\quad\; \bar{W}_0 = \bar{f} \; ; \quad \bar{W}_1 = \bar{W}_2 \cdots = 0 \\
&\qquad\qquad\quad\; \bar{\theta}_0 = 1 \; ; \quad \bar{\theta}_2 = \bar{\theta}_3 \cdots = 0
\end{aligned}\right\} \quad \text{(A6–16)}$$

Substituting the series (A6–15) in equations (A6–9) through (A6–12), the zeroth equations are:

$$0 = -\frac{\partial \bar{\pi}_0}{\partial \bar{r}} + \frac{1}{f}\,\frac{\partial^2 \bar{U}_0}{\partial \bar{z}^2} \qquad \text{(A6–17)}$$

† Powers of ϵ differing by two are taken since exponents of ϵ in the differential equations differ by two.

$$0 = -\frac{1}{f}\frac{\partial \bar{\pi}_o}{\partial \bar{z}} \qquad \text{(A6-18)}$$

and noting from equation (A6-18) that $\bar{\pi}_o = \bar{\pi}_o(\bar{r}, \bar{t})$ only, the continuity equation (A6-11) yields:

$$0 = \frac{\partial}{\partial \bar{t}}\left(\frac{\bar{\pi}_o}{\bar{\theta}_o}\right) + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\,\frac{\bar{\Gamma}_o}{\bar{\theta}_o}\,\bar{U}_o\right) + \frac{\bar{\pi}_o}{\bar{\theta}_o^2}\left(\frac{\dot{f}}{f}\,\bar{z} - \frac{1}{f}\,\bar{W}_o\right)\frac{\partial \theta_o}{\partial \bar{z}} + \frac{1}{f}\,\frac{\bar{\pi}_o}{\bar{\theta}_o}\,\frac{\partial \bar{w}_o}{\partial \bar{z}} \qquad \text{(A6-19)}$$

and the energy equation (A6-12):

$$-\frac{k}{c_p \rho}\,\frac{c_p T^* L^2}{\nu^{*2}}\,\frac{1}{f^2}\,\frac{\partial^2 \bar{\theta}_o}{\partial \bar{z}^2} = \frac{1}{f^2}\left(\frac{\partial \bar{U}_o}{\partial \bar{z}}\right)^2 + \bar{U}_o\,\frac{\partial \bar{\pi}_o}{\partial \bar{r}} + \frac{\partial \bar{\pi}_o}{\partial \bar{t}}$$

$$= \frac{1}{f^2}\left(\frac{\partial \bar{U}_o}{\partial \bar{z}}\right)^2 + \frac{1}{f^2}\,\bar{U}_o\,\frac{\partial^2 \bar{U}_o}{\partial \bar{z}^2} + \frac{\partial \bar{\pi}_o}{\partial \bar{t}} \qquad \text{(A6-20)}$$

where in the penultimate step, use was made of equation (A6-17).

Integration of equation (A6-20) with respect to $\bar{z}$ gives:

$$-\left(\frac{k}{c_p \mu}\,\frac{c_p T^* L^2}{\nu^{*2}}\right)\bar{\theta}_o = \frac{\bar{U}_o^2}{2} + \frac{f^2 \bar{z}^2}{2}\,\frac{\partial \bar{\pi}_o}{\partial \bar{t}} + \bar{z}F(\bar{r}, \bar{t}) + G(\bar{r}, \bar{t}) \qquad \text{(A6-21)}$$

and, applying the boundary conditions (A6-16):

$$F = \frac{f^2}{2}\,\frac{\partial \bar{\pi}_o}{\partial \bar{t}} \quad ; \quad G = -\frac{k}{c_p \mu}\,\frac{c_p T^* L^2}{\nu^{*2}} \qquad \text{(A6-22)}$$

so that equation (A6-21) becomes:

$$\bar{\theta}_o = 1 + \frac{1}{2}\left(\frac{c_p \mu}{k}\,\frac{\nu^{*2}}{c_p T^* L^2}\right)\left[\bar{z}(1-\bar{z})\,f^2\,\frac{\partial \bar{\pi}_o}{\partial \bar{t}} - \bar{U}_o^2\right] \qquad \text{(A6-23)}$$

Letting

$$U^* = \frac{\nu^*}{l^*}\,\bar{l}^*$$

be the reference velocity, corresponding to the steady-flow condition when $f = 1$, equation (A6–23) can be written in the following form:

$$\bar{\theta}_o = 1 + \frac{\gamma - 1}{2} N_{Pr} M^2 \left[\bar{r}(1 - \bar{z}) \left(\frac{f}{\bar{l}^*}\right)^2 \frac{\partial \bar{\pi}_o}{\partial \bar{r}} - \left(\frac{\bar{U}_o}{\bar{l}^*}\right)^2\right] \qquad \text{(A6–24)}$$

in which

$$N_{Pr} = \frac{c_p \mu}{k} \quad ; \quad M = \frac{U^*}{\sqrt{\gamma \mathcal{R} T^*}} \qquad \text{(A6–25)}$$

Since, generally, $\frac{\gamma - 1}{2} N_{Pr} M^2 \ll 1$, one may assume that to the first degree of approximation

$$\bar{T} \simeq \bar{\theta}_o \simeq 1 \qquad \text{(A6–26)}$$

which is equivalent to the assumption made in equation (2) of the main text, e.g., P/ρ = constant. Equation (A6–19) then reduces to:

$$\frac{\partial \bar{\pi}_o}{\partial \bar{t}} + \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} (\bar{r}\, \bar{\pi}_o \bar{U}_o) + \frac{1}{f} \bar{\pi}_o \frac{\partial \bar{W}_o}{\partial \bar{z}} = 0 \qquad \text{(A6–27)}$$

Integration of equation (A5-17) with respect to $\bar{z}$ yields:

$$\bar{U}_o = \frac{f^2 \bar{z}^2}{2} \frac{\partial \bar{\pi}_o}{\partial \bar{r}} + \bar{z} A(\bar{r}, \bar{t}) + B(\bar{r}, \bar{t}) \qquad \text{(A6–28)}$$

and application of boundary conditions (A6–16) gives:

$$A = \frac{-f^2}{2} \frac{\partial \bar{\pi}_o}{\partial \bar{r}} \quad ; \quad B = 0 \qquad \text{(A6–29)}$$

so that:

$$\bar{\Gamma}_o = \frac{1}{2}(\bar{z}^2 - \bar{z})\, f^2 \frac{\partial \bar{\pi}_o}{\partial \bar{r}} \qquad \text{(A6-30)}$$

Substitution of equation (A6-30) in equation (A6-27) gives:

$$\frac{\partial \bar{\pi}_o}{\partial \bar{t}} + \frac{1}{2}(\bar{z}^2 - \bar{z}) f^2 \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}}\left(\bar{r}\, \bar{\pi}_o \frac{\partial \bar{\pi}_o}{\partial \bar{r}}\right) - \frac{1}{f} \bar{\pi}_o \frac{\partial \bar{\pi}_o}{\partial \bar{z}} = 0 \qquad \text{(A6-31)}$$

and integration between the limits $\bar{z} = 0$ and $\bar{z} = 1$ yields:

$$\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}}\left(\bar{r}\, \bar{\pi}_o \frac{\partial \bar{\pi}_o}{\partial \bar{r}}\right) = \frac{12}{f^3} \frac{\partial}{\partial \bar{t}}(f\, \bar{\pi}_o) \qquad \text{(A6-32)}$$

which is the dimensionless equivalent of equation (4) of the main text, when it is assumed that $\bar{\pi}_o$ is a good approximation of $\bar{P}$

The error involved in neglecting higher order terms of the expansions could be estimated only if zeroth solutions were available; through substitution in the initial equations, for example. It is self-evident that the degree to which zeroth terms of ϵ-expansions approximate the actual functions depende on magnitudes of other coefficients which appear in the dimensionless equations of fluid motion. Of part.cular importance is the magnitude of $\dot{f}$ which characterizes the effect of boundary motion.

It was tacitly assumed in taking Taylor's equation [1] as the starting point of the analysis, and subsequently implied in the linearization thereof, that $\dot{f}$ was arbitrarily small, In doing so one was encouraged by the fact that equation (4) of the main text is exact in the limiting case, whenever:

$$L \to \infty, \quad \text{with} \quad \dot{f} = \frac{L^2}{\nu^*} \frac{df}{dt} \text{ finite}$$

or

$$H^* \to 0, \quad \text{with} \quad \frac{P^*}{\rho^*}\left(\frac{H^*}{\nu^*}\right)^2 \text{ finite.}$$

APPENDIX 7

TABULATION OF COMPUTER SOLUTIONS

This appendix contains a tabulation of computer solutions of the set of simultaneous equations (408) subject to the boundary condition (415). The solutions are for a fixed value of $\beta = 0.596481$ and for discrete values of the parameter α. The range of the independent variable is $1 \geq \xi \geq 5$ and the interval size is $\Delta\xi = 0.1$. The numerical method employed was that of Hoppingar and Dimsdale and is contained in "Numerical Analysis", Kaiser S. Kunz, McGraw-Hill Book Co., 1957 pp 206–208.

The following code is to be used in reading the tables. The first digit is zero if the number is positive and is a minus sign if the number is negative. The second and third digits stand for 50 + n, where n is the exponent in 10^n. The remaining digits represent a decimal, which must be multiplied by 10^n in order to read the required number.

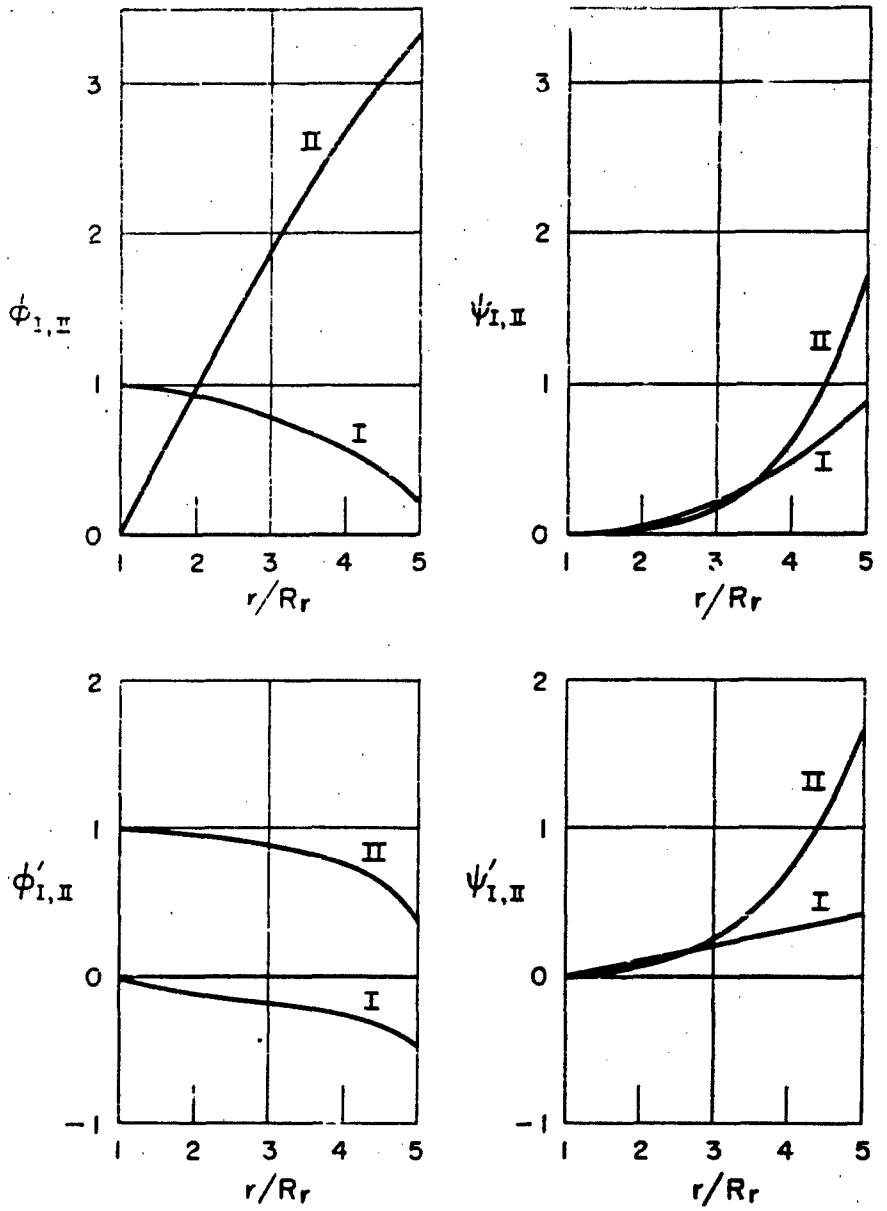


FIG. A7-1. PLOT OF COMPUTER SOLUTIONS OF EQUATIONS 40b FOR $a = 0.093$

ζ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
051100000000	049400000000	051100000000	000000000000	-50250000000	000000000000	000000000000	049999999520
051110000000		050996641948	-49227942140	-50206370756	046504433540	048101403201	049102745262
051120000000		050995581747	-49416109836	-50172446210	047203457113	048205341741	049105103931
051130000000		050990665025	-49575363250	-50146544234	0474419[illegible]	048311519348	049107175795
051140000000		050984143327	-49710963019	-50125599015	047[illegible]27949262	048419638996	049104024516
051150000000		050976445560	-49827717110	-50108508898	048130204116	048529494635	049110495982
051160000000		050967650448	-49928995152	-49945193796	048198487157	048640982734	049112223466
051170000000		050957468250	-50101753781	-49898052961	048258442404	048753946190	049113436795
051180000000		[illegible]	-50109540850	-49731390113	048539813615	048868410632	049114037404
051190000000		050936163070	-50116431014	-49648770330	048432120052	048983785154	049116157937
051200000000		050924194890	-50122550554	-49578247064	048554304110	049110080209	049117305594
051210000000		050911540480	-50126028755	-49517419963	048482252750	049121834741	049118390609
051220000000		050908187956	-50132936201	-49460116762	048780008873	049133726561	049119422770
051230000000		050884295330	-50137356486	-49419503067	048910415410	049145730721	049120404965
051240000000		050871080874	-50141345341	-49379401734	049107150242	049157086234	049121361025
051250000000		050856478436	-50144963829	-49344405820	049123829200	049169969106	049122278172
051260000000		050842049121	-50148249209	-49313561123	049141153083	049182241130	049123182890
051270000000		050827045398	-50151246635	-49286129343	049159987008	049194642909	049124054841
051280000000		050811508649	-50153981968	-49261766611	049180039593	049207073867	04912[illegible]7905
051290000000		050796241985	-50156489645	-49240095019	049201404702	049219620676	049125025498
051300000000		050780517479	-50158790349	-49220425872	049224009570	049232249297	049126719704
051310000000		050764518110	-50160908536	-49203931842	049247450951	049244963738	049127427799
051320000000		050748381272	-50162860969	-4918748[illegible]	[illegible]495	049257745862	049128571733
051330000000		050731980043	-50164606227	-49173413372	[illegible]	049270078223	049129853212
051340000000		050715412465	-50166337945	-49161012847	04932[illegible]	049283070379	049130095762
051350000000		050698669035	-50167490500	-49149473833	049364155337	049296708731	049131706858
051360000000		050681847950	-50169335297	-49139484415	049388580108	049310092815	049132918004
051370000000		050664997234	-50170483494	-49130330764	049418163152	049323304477	049134032993
051380000000		050647760148	-50171945030	-49122134073	049451130540	049336874205	049135490187
051390000000		050630402675	-50173128909	-49114815170	049488816885	049350529520	049137341506
051400000000		050613080654	-50174244080	-49108393479	0495219[illegible]1391	049384309968	049139710342
051410000000		050595507498	-50175297332	-49102409708	049589413255	049378340271	049141357439
051420000000		050577002847	-50176298783	-48976513140	049590985194	049392693296	049143862061
051430000000		050560340730	-50177253302	-48934307105	049636941310	049407147197	049145920204
051440000000		050542573460	-50178169892	-48699415776	049678429481	049422017079	049153772236
051450000000		050524709440	-50179054990	-48672719612	049721343830	049437246972	049154709962
051460000000		050506743225	-50179918179	-48854402194	049765430121	049452904796	049160120075
051470000000		050488748978	-50180707296	-48845913233	049812030375	049469334338	049167040792
051480000000		050470609846	-50181614059	-48849580673	049859799769	049486477163	049176188339
051490000000		050452465903	-50182471557	-48849494037	049909938065	04950469[illegible]	[illegible]
051500000000		050434114457	-50183361597	-48914409488	049960780535	049524441938	049207447095
051510000000		050415742509	-50184310761	-49100360472	050101933250	049546444415	049237073822

ξ	α	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
0.5110000000000	0.0100000000000	0.0000000000000	0.5110000000000	0.0000000000000	0.0000000000000	0.0000000000000	0.0000000000000
0.5111000000000	0.0099498743710	0.0099498743710	-0.4992486499204	0.0501666499949	0.0000000000000	0.0000000000000	0.0000000000000
0.5112000000000	0.0098994914405	0.0197989828420	-0.4969967796429	0.0503344967600	0.0001666833451	0.0000000000000	0.0000000000000
0.5113000000000	0.0098488578018	0.0295477406438	-0.4947447093858	0.0505033435200	0.0003333333333	0.0000000000000	0.0000000000000
0.5114000000000	0.0097979589711	0.0392957996149	-0.4924924392287	0.0506731902800	0.0005000833333	0.0000000000000	0.0000000000000
0.5115000000000	0.0097467943921	0.0490430939570	-0.4902399690716	0.0508440370400	0.0006668333333	0.0000000000000	0.0000000000000
0.5116000000000	0.0096953597431	0.0587896536991	-0.4879873989145	0.0510158838000	0.0008335833333	0.0000000000000	0.0000000000000
0.5117000000000	0.0096436507104	0.0685353134411	-0.4857347287574	0.0511887305600	0.0010003333333	0.0000000000000	0.0000000000000
0.5118000000000	0.0095916630466	0.0782800731832	-0.4834819586003	0.0513625773200	0.0011670833333	0.0000000000000	0.0000000000000
0.5119000000000	0.0095393924666	0.0880238329253	-0.4812290884432	0.0515374240800	0.0013338333333	0.0000000000000	0.0000000000000
0.5120000000000	0.0094868347510	0.0977665926673	-0.4789761182861	0.0517132708400	0.0015005833333	0.0000000000000	0.0000000000000
0.5121000000000	0.0094339856449	0.1075083524094	-0.4767230481290	0.0518901176000	0.0016673333333	0.0000000000000	0.0000000000000
0.5122000000000	0.0093808408574	0.1172491121514	-0.4744698779719	0.0520679643600	0.0018340833333	0.0000000000000	0.0000000000000
0.5123000000000	0.0093273960610	0.1269888718935	-0.4722166078148	0.0522468111200	0.0020008333333	0.0000000000000	0.0000000000000
0.5124000000000	0.0092736468900	0.1367276316355	-0.4699632376577	0.0524266578800	0.0021675833333	0.0000000000000	0.0000000000000
0.5125000000000	0.0092195889408	0.1464653913776	-0.4677097675006	0.0526075046400	0.0023343333333	0.0000000000000	0.0000000000000
0.5126000000000	0.0091652177716	0.1562021511196	-0.4654561973435	0.0527893514000	0.0025010833333	0.0000000000000	0.0000000000000
0.5127000000000	0.0091105288020	0.1659379108617	-0.4632025271864	0.0529721981600	0.0026678333333	0.0000000000000	0.0000000000000
0.5128000000000	0.0090555174139	0.1756726706037	-0.4609487570293	0.0531560449200	0.0028345833333	0.0000000000000	0.0000000000000
0.5129000000000	0.0090001789520	0.1854064303458	-0.4586948868722	0.0533408916800	0.0030013333333	0.0000000000000	0.0000000000000
0.5130000000000	0.0089445086241	0.1951391900878	-0.4564409167151	0.0535267384400	0.0031680833333	0.0000000000000	0.0000000000000
0.5131000000000	0.0088885015999	0.2048709498299	-0.4541868465580	0.0537135852000	0.0033348333333	0.0000000000000	0.0000000000000
0.5132000000000	0.0088321529116	0.2146017095719	-0.4519326764009	0.0539014319600	0.0035015833333	0.0000000000000	0.0000000000000
0.5133000000000	0.0087754576533	0.2243314693140	-0.4496784062438	0.0540902787200	0.0036683333333	0.0000000000000	0.0000000000000
0.5134000000000	0.0087184107810	0.2340602290560	-0.4474240360867	0.0542801254800	0.0038350833333	0.0000000000000	0.0000000000000
0.5135000000000	0.0086610071122	0.2437879887981	-0.4451695659296	0.0544709722400	0.0040018333333	0.0000000000000	0.0000000000000
0.5136000000000	0.0086032415262	0.2535147485401	-0.4429149957725	0.0546628190000	0.0041685833333	0.0000000000000	0.0000000000000
0.5137000000000	0.0085451087644	0.2632405082822	-0.4406603256154	0.0548556657600	0.0043353333333	0.0000000000000	0.0000000000000
0.5138000000000	0.0084866034307	0.2729652680242	-0.4384055554583	0.0550495125200	0.0045020833333	0.0000000000000	0.0000000000000
0.5139000000000	0.0084277200918	0.2826890277663	-0.4361506853012	0.0552443592800	0.0046688333333	0.0000000000000	0.0000000000000
0.5140000000000	0.0083684531766	0.2924117875083	-0.4338957151441	0.0554402060400	0.0048355833333	0.0000000000000	0.0000000000000
0.5141000000000	0.0083087969766	0.3021335472504	-0.4316406449870	0.0556370528000	0.0050023333333	0.0000000000000	0.0000000000000
0.5142000000000	0.0082487457455	0.3118543069924	-0.4293854748299	0.0558348995600	0.0051690833333	0.0000000000000	0.0000000000000
0.5143000000000	0.0081882935988	0.3215740667345	-0.4271302046728	0.0560337463200	0.0053358333333	0.0000000000000	0.0000000000000
0.5144000000000	0.0081274345134	0.3312928264765	-0.4248748345157	0.0562335930800	0.0055025833333	0.0000000000000	0.0000000000000
0.5145000000000	0.0080661623277	0.3410105862186	-0.4226193643586	0.0564344398400	0.0056693333333	0.0000000000000	0.0000000000000
0.5146000000000	0.0080044707420	0.3507273459606	-0.4203637942015	0.0566362866000	0.0058360833333	0.0000000000000	0.0000000000000
0.5147000000000	0.0079423532191	0.3604431057027	-0.4181081240444	0.0568391333600	0.0060028333333	0.0000000000000	0.0000000000000
0.5148000000000	0.0078798030844	0.3701578654447	-0.4158523538873	0.0570429801200	0.0061695833333	0.0000000000000	0.0000000000000
0.5149000000000	0.0078168134251	0.3798716251868	-0.4135964837302	0.0572478268800	0.0063363333333	0.0000000000000	0.0000000000000
0.5150000000000	0.0077533771911	0.3895843849288	-0.4113405135731	0.0574536736400	0.0065030833333	0.0000000000000	0.0000000000000
0.5151000000000	0.0076894871950	0.3992961446709	-0.4090844434160	0.0576605204000	0.0066698333333	0.0000000000000	0.0000000000000
0.5152000000000	0.0076251361124	0.4090069044129	-0.4068282732589	0.0578683671600	0.0068365833333	0.0000000000000	0.0000000000000
0.5153000000000	0.0075603164814	0.4187166641550	-0.4045720031018	0.0580772139200	0.0070033333333	0.0000000000000	0.0000000000000
0.5154000000000	0.0074950206026	0.4284254238970	-0.4023156329447	0.0582870606800	0.0071700833333	0.0000000000000	0.0000000000000
0.5155000000000	0.0074292405387	0.4381331836391	-0.4000591627876	0.0584979074400	0.0073368333333	0.0000000000000	0.0000000000000
0.5156000000000	0.0073629681139	0.4478399433811	-0.3978025926305	0.0587097542000	0.0075035833333	0.0000000000000	0.0000000000000
0.5157000000000	0.0072961949139	0.4575457031232	-0.3955459224734	0.0589226009600	0.0076703333333	0.0000000000000	0.0000000000000
0.5158000000000	0.0072289122870	0.4672504628652	-0.3932891523163	0.0591364477200	0.0078370833333	0.0000000000000	0.0000000000000
0.5159000000000	0.0071611113442	0.4769542226073	-0.3910322821592	0.0593512944800	0.0080038333333	0.0000000000000	0.0000000000000
0.5160000000000	0.0070927830588	0.4866569823493	-0.3887753120021	0.0595671412400	0.0081705833333	0.0000000000000	0.0000000000000
0.5161000000000	0.0070239181660	0.4963587420914	-0.3865182418450	0.0597839880000	0.0083373333333	0.0000000000000	0.0000000000000
0.5162000000000	0.0069545071633	0.5060595018334	-0.3842610716879	0.0600018347600	0.0085040833333	0.0000000000000	0.0000000000000
0.5163000000000	0.0068845403107	0.5157592615755	-0.3820038015308	0.0602206815200	0.0086708333333	0.0000000000000	0.0000000000000
0.5164000000000	0.0068140076307	0.5254580213175	-0.3797464313737	0.0604405282800	0.0088375833333	0.0000000000000	0.0000000000000
0.5165000000000	0.0067428988080	0.5351557810596	-0.3774889612166	0.0606613750400	0.0090043333333	0.0000000000000	0.0000000000000
0.5166000000000	0.0066712032900	0.5448525408016	-0.3752313910595	0.0608832218000	0.0091710833333	0.0000000000000	0.0000000000000
0.5167000000000	0.0065989102870	0.5545483005437	-0.3729737209024	0.0611060685600	0.0093378333333	0.0000000000000	0.0000000000000
0.5168000000000	0.0065260087722	0.5642430602857	-0.3707159507453	0.0613299153200	0.0095045833333	0.0000000000000	0.0000000000000
0.5169000000000	0.0064524874827	0.5739368200278	-0.3684580805882	0.0615547620800	0.0096713333333	0.0000000000000	0.0000000000000
0.5170000000000	0.0063783348187	0.5836295797698	-0.3662001104311	0.0617806088400	0.0098380833333	0.0000000000000	0.0000000000000
0.5171000000000	0.0063035387437	0.5933213395119	-0.3639420402740	0.0620074556000	0.0100048333333	0.0000000000000	0.0000000000000
0.5172000000000	0.0062280867852	0.6030120992539	-0.3616838701169	0.0622353023600	0.0101715833333	0.0000000000000	0.0000000000000
0.5173000000000	0.0061519659350	0.6127018589960	-0.3594255999598	0.0624641491200	0.0103383333333	0.0000000000000	0.0000000000000
0.5174000000000	0.0060751626500	0.6223906187380	-0.3571672298027	0.0626939958800	0.0105050833333	0.0000000000000	0.0000000000000
0.5175000000000	0.0059976627520	0.6320783784801	-0.3549087596456	0.0629248426400	0.0106718333333	0.0000000000000	0.0000000000000
0.5176000000000	0.0059194513284	0.6417651382221	-0.3526501894885	0.0631566894000	0.0108385833333	0.0000000000000	0.0000000000000
0.5177000000000	0.0058405126228	0.6514508979642	-0.3503915193314	0.0633895361600	0.0110053333333	0.0000000000000	0.0000000000000
0.5178000000000	0.0057608300435	0.6611356577062	-0.3481327491743	0.0636233829200	0.0111720833333	0.0000000000000	0.0000000000000
0.5179000000000	0.0056803860588	0.6708194174483	-0.3458738790172	0.0638582296800	0.0113388333333	0.0000000000000	0.0000000000000
0.5180000000000	0.0055991620944	0.6805021771903	-0.3436149088601	0.0640940764400	0.0115055833333	0.0000000000000	0.0000000000000
0.5181000000000	0.0055171385209	0.6901839369324	-0.3413558387030	0.0643309232000	0.0116723333333	0.0000000000000	0.0000000000000
0.5182000000000	0.0054342945427	0.6998646966744	-0.3390966685459	0.0645687699600	0.0118390833333	0.0000000000000	0.0000000000000
0.5183000000000	0.0053506078772	0.7095444564165	-0.3368373983888	0.0648076167200	0.0120058333333	0.0000000000000	0.0000000000000
0.5184000000000	0.0052660546284	0.7192232161585	-0.3345780282317	0.0650474634800	0.0121725833333	0.0000000000000	0.0000000000000
0.5185000000000	0.0051806090615	0.7289009759006	-0.3323185580746	0.0652883102400	0.0123393333333	0.0000000000000	0.0000000000000
0.5186000000000	0.0050942435779	0.7385777356426	-0.3300589879175	0.0655301570000	0.0125060833333	0.0000000000000	0.0000000000000
0.5187000000000	0.0050069284826	0.7482534953847	-0.3277993177604	0.0657730037600	0.0126728333333	0.0000000000000	0.0000000000000
0.5188000000000	0.0049186319511	0.7579282551267	-0.3255395476033	0.0660168505200	0.0128395833333	0.0000000000000	0.0000000000000
0.5189000000000	0.0048293196888	0.7676020148688	-0.3232796774462	0.0662616972800	0.0130063333333	0.0000000000000	0.0000000000000
0.5190000000000	0.0047389545911	0.7772747746108	-0.3210197072891	0.0665075440400	0.0131730833333	0.0000000000000	0.0000000000000
0.5191000000000	0.0046474975000	0.7869465343529	-0.3187596371320	0.0667543908000	0.0133398333333	0.0000000000000	0.0000000000000

ξ	α	ϕ_1	ϕ'_1	ϕ''_1	ψ_1	ψ'_1	ψ''_1
0511100000000	0.0200000000	0.5110000000	0.0000000000	-8.4290000000	0.0000000000	0.0000000000	0.0491999999
0511110000000		0.5009884+1150	-8.4922743866	-8.4926887152	0.0471000974249	0.0482028809750	0.0402054902150
0511120000000		0.5009955+1702	-8.4941611374?	-8.4917288?3674	0.0474074204?35	0.0484152?2873	0.0402100078400
0511130000000		0.5009884+4895	-8.4957577425	-8.4914656?0304	0.0479238?74440	0.0486?30?58070	0.0402143515520
0511140000000		0.5009841+2050	-8.4971099337	-8.4913554+6730	0.0481454?7386	0.0488392?47296	0.0402184?48942
0511150000000		0.5009784+4230	-8.4986?805710	-8.4910?993825	0.0483?0?5?398	0.0491?59?213?	0.0402?1?92754
0511160000000		0.5009?47+8480	-8.4999291?984	-8.4904?868930	0.0485?7?61286	0.0491281?66625	0.0402240?46745
0511170000000		0.5009579+4440	-8.5010177438?	-8.4898?48?7501	0.0485?4?91940	0.0491507?1246	0.0402272?60609
0511180000000		0.5009473+9130	-8.5019?9?2388	-8.4892?An9101	0.0486?9?3?193	0.0491736?2030	0.0402298?3161
0511190000000		0.5009369+9810	-8.5011?477029	-8.4886?+13485	0.0488?4?4?085	0.0491967?0?03	0.0402321?3222
0511200000000		0.5009249+9110	-8.5012?A21190	-8.4880?37?999	0.0491?7?61449	0.0492200?69746	0.0402344?07041
0511210000000		0.5009115+7265	-8.5012?116?19	-8.4873?237565	0.0491?6?51091	0.0492436?8306	0.0402367?4943
0511220000000		0.5008981+4418	-8.5013?053239	-8.4866?A44016	0.0491?6?02158	0.0492674?1167	0.0402389?36294
0511230000000		0.5008843+2595	-8.5013?408679	-8.4859?370422	0.0491?3?23252	0.0492914?8459	0.0402407?96?57
0511240000000		0.5008703+6712	-8.5014?4?0?04	-8.4853?4?24291	0.0492?4?30256	0.0493156?7855	0.0402427?03501
0511250000000		0.5008565+3190	-8.5014?9?8341	-8.4846?9?75034	0.0492?7?37667	0.0493399?1545	0.0402444?31048
0511260000000		0.5008419+5754	-8.5014?853587	-8.4839?9?74?26	0.0493?3?04380	0.0493645?2527	0.0402463?31144
0511270000000		0.5008284+9744	-8.5015?A19171	-8.4832?3?07422	0.0493?9?71367	0.0493892?2273	0.0402480?9?98?
0511280000000		0.5008116+0187	-8.5015?4?6094	-8.4827?4?3?75	0.0493A?1?0?349	0.0494141?8757	0.0402?0?9?5302
0511290000000		0.5007955+4054	-8.5015?635?00	-8.4824?9?A6560	0.0494?2?07004	0.0494392?5730	0.0402514?2709
0511300000000		0.5007802+9413	-8.5015?4?3583	-8.4821?8?9133	0.0494?4?0?912	0.0494644?3814	0.0402533?3813?
0511310000000		0.5007641+6345	-8.5016?A?2507	-8.4816?176092	0.0494?9?04602	0.0494898?1617	0.0402551?26949
0511320000000		0.5007479+7A5A	-8.5016?73A10	-8.4812?3?3665	0.0495?5?3?188	0.0495154?1005	0.0402569?0?046
0511330000000		0.5007311+9A15	-8.5016?734594	-8.4810?3?6352	0.0495?8?0A?85	0.0495414?7965	0.0402588?01819
0511340000000		0.5007147+4713	-8.5016?483595	-8.4179?743159	0.0466442?4347	0.0495672?4817	0.0402600?35274
0511350000000		0.5004974+9280	-8.5016?934294	-8.4179?7A1528	0.0467122?55?15	0.0495934?4?64?	0.0402634?96504
0511360000000		0.5004?08+0A1A	-8.5017?003429	-8.4?1A31A8334	0.0467?2?5?364	0.0496149?79814	0.0402654?35?09
0511370000000		0.5004436+3705	-8.5017?A01824	-8.4?16?000?84	0.0468?6?47615	0.0496465?40460	0.0402674?83633
0511380000000		0.5004?63+2134	-8.5017?145?7?	-8.4?14?8A7953	0.0469?237A?99	0.0496734?85497	0.0402?0?05?4?
0511390000000		0.5004?38+A?5?	-8.5017?443501	-8.4?1?8136768	0.0469?7?5?8??	0.0497?07?1810	0.0402734?59679
0511400000000		0.5004112+4480	-8.5017?112160	-8.4?145415781	0.0501?04?4397	0.0497282?1415	0.0402777?35856
0511410000000		0.5003934+9655	-8.5017?459A10	-8.4?14?375156	0.0501114A1397	0.0497562?6?5C	0.0402814?38616
0511420000000		0.5003753+2708	-8.5018?004127	-8.4?14?478663	0.0501194?0997	0.0497?46?1434	0.0402864?10262
0511430000000		0.5003574+1A3C	-8.5018?453892	-8.4?16?95454	0.0501?7?5?544	0.0498135?6585	0.0402924?2131
0511440000000		0.5003392+1144	-8.5018?30?95	-8.4?1?9?08609	0.0501?5A?9393	0.0498430?7155	0.0402987?9?309
0511450000000		0.5003208+A170	-8.5018?443A59	-8.4?16?2?3967	0.0501?4?1A175	0.0498733?4?85	0.0403070?30839
0511460000000		0.5003023+0351	-8.5018?822502	-8.4?1A?76008	0.0501?3?13961	0.0499045?1048	0.0403172?95?2
0511470000000		0.5002836+A?19	-8.5018?A?3942	-8.4?171?79948	0.0501A2?0A077	0.0499362?745C	0.0403304?1599
0511480000000		0.5002647+9503	-8.5018?+?1318	-8.4?1A?928904	0.0501?1A?7599	0.0499707?40637	0.0403474?0A674
0511490000000		0.5002449+7421	-8.5019?4?4042	-8.4?2?57?2918	0.0501?191A340	0.0501006?9770	0.0403719?9092
0511500000000		0.5002269+7479	-8.5019?A15?2A	-8.4?2?4A?0141	0.0501?2?0+477	0.0501045?4373	0.0404079?2775
0511510000000		0.5002070+4894	-8.5019?156?01	-8.4?2A?5?0?87	0.0502?34A?7?8	0.0501088?+0335	0.0404?5?7?705

ξ	α	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
051100000000	0920000000	000000000000	051100000000	000000000000	000000000000	000000000000	000000000000
051110000000		050199860519	050594912100	-49208899719	045535788827	047101949653	048205798322
051120000000		[illegible]	050996085438	-49346749959	046274370984	047415205190	048422442298
051130000000		050299181774	050992116510	-49442537873	046938009750	047950743830	048650035788
051140000000		050398191478	050987341238	-49507858441	047225338394	048171877429	048867741851
051150000000		050496503058	050982029000	-49551872435	047445686099	048272763063	049113570135
051160000000		050594515344	050976354194	-49580765213	047779702320	048399342667	049139401081
051170000000		050691684275	050970049760	-49598794804	048125340093	048592143225	049166286543
051180000000		050788606597	050964404521	-49608957961	048189319147	048732207516	049194257673
051190000000		050864795120	050958289140	-49613391859	048277758581	048941026305	049223952907
051200000000		050960287655	050952150606	-49613649496	048379807785	049117959429	049253621027
051210000000		051107516552	050946026290	-49610869700	048509672130	049144867274	049285117752
051230000000		051126315614	050933913335	-49599381700	048846803880	049206404627	049352073734
051240000000		051135694925	050927956425	-49591796313	049108746616	[illegible]	049387476035
051250000000		051144695022	050922079460	-49583518456	049135895447	049286023423	049424775743
051260000000		051154046851	050916287394	-49574859055	049146090418	049330500361	049463726572
051270000000		051163201075	050910583430	-49565988099	049201420531	049378800425	049504476933
051280000000		051172278764	050904967588	-49557150291	049242007249	049431440647	049546974540
051290000000		051181300731	050899439670	-49548477719	049287062186	049488365550	049591487652
051300000000		051190247846	050893997656	-49540098195	049339831252	049549065158	049634710364
051310000000		051199180957	050888636570	-49532123713	049398094077	049616181715	049680876165
051320000000		051208040835	050883352895	-49524665037	049463237436	049687061481	049740414779
051330000000		051216748257	050878141490	-49517788250	049535793785	049764407420	049796059076
051340000000		051225603862	050872094809	-49511617622	049616309487	049846928348	049854054987
051350000000		051234308369	050867906530	-49506241663	049705381785	049935526890	049917724001
051360000000		051242942177	050862806373	-49501765715	049803631388	050103041962	049984958992
051370000000		051251545858	050857867890	-49498308713	049911736960	050113446579	050105499663
051380000000		051260119631	050852896946	-49496007040	050103041451	050124220905	050113384519
051390000000		051268623673	050847943465	-49495023655	050116044805	050135945419	050121032613
051400000000		051277078508	050842991420	-49495595732	050130267990	050146633331	050131157776
051410000000		051285483681	050838026650	-49497849441	050145803537	050162247377	050141904731
051420000000		051293878930	050833027516	-49502217020	050162759137	050176941238	050152560468
051430000000		051302164682	050827974350	-49509065457	050181929150	050192803993	050165172312
051440000000		051310308138	050822836123	-49518738250	050201358793	050210022063	050179474949
051450000000		051318660468	050817583625	-49532583196	050223293981	050228769275	050195936064
051460000000		051326749275	050812168319	-49551067254	050247172244	050249303782	050215232257
051470000000		051334643209	050806542265	-49575089205	050273219414	050271939418	050236389896
051480000000		051342849140	050800618661	-49609869990	050301446503	050297145438	050267057627
051490000000		051350884448	050794705730	-49657017415	050332749264	050325608121	050304103025
051500000000		[illegible]	[illegible]	[illegible]	050368914849	050358480784	050355790266
051510000000		051356600488	050779698450	-49834343510	050404410778	050397205462	050432811133

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
0511000000000	0493500000000	0511000000000	0000000000000	-5025000000	0000000000000	0000000000000	0493499399632
0511100000000		0509984841150	-492273445241	-402069769440	0471757974470	0483549170700	0493597607577
0511200000000		0509965841445	-494161254110	-5017287046?	0477128411456	0487187650?3	0493679635997
0511300000000		0509306484628	-495705418010	-5014640075?	0481414450806	0491090?1660	0493751151?6
0511400000000		050?0511e1937	-497110975??	-5012564295?	0482949?75166	0491468?349?	0493815?5273
0511500000000		0508764443810	-499280025?0	-5010847695?	0484486421827	0491853?8646	0493877?36366
0511600000000		0509476434?9	-498929070745	-4947?689001	0486660417036	0492243?3855	0493927797??
0511700000000		050857?94050	-5010183112?	-4085?258334	0489048?0205	0492638?0670	0493977020?6
0511800000000		0508473?24??	-501094?9?58	-407?6112057	0491188?0?5?	0493036724??	0494022707??
0511900000000		05?035?0?6?5	-50?1160035??	-4965?032661	04?151?4?935	0493443?1741	0494065?6059
0512000000000		0508240?6729	-50?2279?8??	-495860?9645	0491877?4986	0493851?0981	0494105?4298?
0512100000000		050?011484130	-50?12835??21	-4955738?974	0492?29?4?415	0494264?360?	0494143?6605
0512200000000		0508983?3080	-50?13357?107	-4947?159129	049273052001	0494680?9927	04941?019682
0512300000000		05088?7e?55e	-50?13702614?	-4943?005916	049321?5?872	0495100?7515	04942??30?45
0512400000000		050?7070?1227	-50?142075369	-4939693650S	0493750?0703	04955220?471	04942464?477
0512500000000		050?5?3?9690	-50?145?80795	-493?5015400	0494?2?4?745	04959495?165	049427?0?26?
0512600000000		05?2?1??4659	-50?149?90667	-493?7416292	0494940?21726	04963768?102	0494307?4?4?
0512700000000		05?826406??0	-50152645?33	-4931413?123	04958993?505	049?69112?420	0494339277??
0512800000000		050811?3?4?	-501550?4?70	-49294154618	04963?0?5?982	04972463417?	0494369?1229
0512900000000		0507953?4068	-501589381?8	-4927727673?	04970497784?	0497689?447?	049439?076147
0513000000000		0507794?067?	-5016123991?	-4926?2?3639	04978?967736	04981260?608	0494420?5024
0513100000000		0507631?593?	-5016381040?	-49251749200	04986?415500	04985707250?	0494459?3721
0513200000000		0507466?1915	-50166283307	-49242714613	04995?9?2340	049901782492	0494480?9312
0513300000000		05072?1?6?25	-50168672251	-4923?912232	0501047?0?53	049946068860	0494520?422?
0513400000000		050712909372	-50171008757	-4923?279228	05011447?489	0499920?5923	0494558?4266
0513500000000		05069?5691030	-50173304394	-492?8798612	050124?24072	050103?04428	049454?93055
0513600000000		05067824?07?	-5017559044?	-4929827969?	05013?5?7577	05010830924?	049462?20775
0513700000000		05066059?440	-50177?7650?	-4925900295?	05014?9?5146	050113047?2?	0494665450874
0513800000000		050642?92736	-5018019564?	-49233675832	0501574?8549	05011779352?	04947?057140
0513900000000		0506245?1710	-5018?557007	-492?94??892	05016985?983	05012247069?	04947?94?904
0514000000000		050606188990	-50184097393	-4924?106838	0501829944??	05012740?4512	04948??30067
0514100000000		05058758484?	-50187527309	-49259147540	05019??4887	0501321?2152	04948744961?
0514200000000		0505?6??935?	-5019??9?14?	-4927?143508	050208?6709?	05013701154?	0494949?6922
0514300000000		050549??8?70	-5019300064?	-49206816411	050232???9??	05014204813	04950358439?
0514400000000		05053??42773	-5019601384?	-4931100776?	05023?1?1161	0501470?4702	0495131?5790
0514500000000		050510?08408	-5019925378?	-4934?163618	0502?2125907	05015228?512	0495250?0592
0514600000000		050490182237	-50202797663	-49370609568	0502676?2278	050157?10975	049?2410984?
0514700000000		0504697?3209	-50206690572	-4941114??15	0502?3445512	0501631?9972	04956075726?
0514800000000		0504488?873?	-5021105803?	-494?2082458	0503059?3889	050168847349	049586479255
0514900000000		05042744?153	-5021 37207	-4953169?714	0503174?7262	05017488?161	0496227454??
0515000000000		05040550901?	-50221778879	-4962?749941	0503?595?05?	050181349497	0496762666??
0515100000000		05038305?68?	-5022867975?	-4977?2?3?54	0503?3?7747	05018840397?	0497?6411?37?

ξ	α	ϕ_{11}	ϕ'_{11}	ϕ''_{11}	ψ_{11}	ψ'_{11}	ψ''_{11}
051100000000	049350000000	050000000000	051100000000	000000000000	000000000000	000000000000	000000000000
051110000000		049909606252	050994012100	-049266831138	045584424930	047178440930	048436129567
051120000000		050149794626	050994090379	-049346761944	046448404523	047726744086	048739424015
051130000000		050299141777	050992118310	-049442579977	047144150062	048166340174	049113756257
051140000000		050398121421	050987390932	-049507062311	047339342199	048300705483	049155354794
051150000000		050496607610	050982027455	-049562083323	047774460565	048477605301	049198747644
051160000000		050594514715	050976348768	-049581143731	048194447391	048698804433	049243051633
051170000000		050749144940	050970439540	-049599419034	048219448118	048966400090	049291000465
051180000000		050758507826	050954386286	-049609094861	048331297567	049128101919	049339950074
051190000000		051084470075	050958259400	-049614420579	048477727191	049164049305	049390465462
051200000000		050940248744	050952103320	-049605442571	048642947840	049206503362	049443032719
051210000000		051107515127	050945955140	-049613677317	048809120060	049295516602	049490049208
051220000000		051116384077	050939436404	-049609482724	049117120902	049306246733	049556331666
051230000000		051126312037	050933765105	-049604370459	049150440111	049354042184	049616111411
051240000000		051135610489	050927751445	-049595262660	049190324553	049429545945	049678441599
051250000000		051144647355	050921801795	-049591775384	049234756019	049500600590	049743492517
051260000000		051154056636	050915916277	-049585245704	049290456374	049578341790	049811463784
051270000000		051163148669	050910096090	-049578957566	049352647412	049663047510	049882574421
051280000000		051172247939	050904335506	-049573184521	049429406585	049754940359	049957088445
051290000000		051181292746	050898630400	-049548067507	049503925080	049854507865	050103920183
051300000000		051190240728	050892071320	-049563870451	049594670536	049562142960	050111752231
051310000000		051199142362	050887749645	-049560704645	049694440540	050107852697	050123116413
051320000000		051207297812	050881751050	-049559031092	049810631256	050120307673	050129067607
051330000000		051216747409	050876160788	-049568794124	049937640510	050133714351	050139257727
051340000000		051225561062	050870569311	-049560313042	050107847204	050146163433	050149948317
051350000000		051234147730	050864951220	-049547847063	050123432907	050143060325	050160811083
051360000000		051242769930	050859284673	-049560460678	050140629375	050160317109	050172035435
051370000000		051251341292	050853548430	-049578182468	050150846143	050196142124	050184074191
051380000000		051259470673	050847710627	-049597933808	050180227812	050217315024	050195076183
051390000000		051268318125	050841742675	-049634435630	050203044155	050237601214	050213139041
051400000000		051276744054	050835600267	-049654090651	050227924107	050259247579	050224012444
051410000000		051285199446	050829245120	-049648284320	050255004497	050243776413	050246084667
051420000000		051243724451	050822415108	-049678267039	050284454360	050330447944	050265174520
051430000000		051301460730	050815456480	-049715441628	050317059675	050337140412	050244100493
051440000000		051309140504	050804277454	-049761388724	050352251055	050367207101	050313744103
051450000000		051317440730	050800393600	-049818425862	050390449425	050399442839	050341047142
051460000000		051325406327	050791861033	-049689431369	050412350057	050435779604	050375723561
051470000000		051333499610	050792837300	-049980487347	050477050108	050475201438	050413464463
051480000000		051341283611	050772100120	-050109412849	050527424393	050514445047	050465111674
051490000000		051348710403	050760456340	-050125503089	050581454550	050568604658	050524015698
051500000000		051356446378	050746824700	-050147009800	050641414487	050625704595	050617576694
051510000000		[illegible]	050730588695	[illegible]	050707414550	050697330247	[illegible]

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
051100000000	049300000000	051100000000	000000000000	-052800000000	000000000000	000000000000	049372299805
051110000000		050936191150	-049227345666	-050206493390	047201059551	048055419510	049410050431
051120000000		050895541610	-049416130703	-050172878548	047814441663	048821485736	049420015646
051130000000		[illegible]	[illegible]	-050140051930	048184765493	049124067610	049426702921
051140000000		050804181560	-049711142250	-050125677601	048339040755	049167845412	049436097259
051150000000		050975443720	-049824092045	-050108732845	048520710535	049211804122	049442783918
051160000000		050957041098	-049929429277	-049948890851	048784762215	049256302853	049444890011
051170000000		050957406354	-050101856410	-049613426097	049103370282	049301541644	049454514559
051180000000		050473604831	-050109498140	-049737476127	049135036621	049347282263	049454734282
051190000000		050355703345	-050116661106	-049686986957	049172049929	049393469100	049464406363
051200000000		050023497450	-050122479764	-049588702564	049214821985	049440103551	049467181935
051210000000		050911484430	-050126469117	-049530486682	049260960531	049487326760	049473560775
051220000000		050898351562	-050135821013	-049491156596	049314601305	049534886352	049477599723
051230000000		050884796448	-050138116169	-049438062417	049367340687	049582841020	049481009687
051240000000		050870711429	-050142319148	-049462714061	049429852335	049631162956	049485960045
051250000000		050856166456	-050146186679	-049371461137	049604049551	049679944045	049488961160
051260000000		050841485544	-050149771079	-049345436425	049554480967	049726961509	049492544001
051270000000		050826315640	-050153112177	-049393467951	049649904500	049778409760	049493053669
051280000000		050810247699	-050156253195	-049304049264	049720272410	049828110360	049499197740
051290000000		050795177940	-050159221325	-049289686945	049805848690	049874831290	049502920244
051300000000		050779308218	-050162056795	-049277436677	049895920765	049928614362	049503067720
051310000000		050762775320	-050164778689	-049267345444	049951939675	049979401379	049504725733
051320000000		050746175208	-050167424393	-049261070907	050109142134	050103046138	049512453779
051330000000		050729168698	-050170007484	-049256673419	050119730852	050108143270	049516167321
051340000000		050712274701	-050172569897	-049254404517	050130821982	050113307943	049517021574
051350000000		050694701374	-050175109193	-049255084389	050142018375	050118500214	049523656422
051360000000		050677082318	-050177675593	-049257865769	050154841343	050123841361	049527727105
051370000000		050659140470	-050180274055	-049263077350	050167135581	050129105862	049532409525
051380000000		050640865372	-050182945542	-049270834487	050180372041	050134407346	049530968315
051390000000		050622594458	-050185499128	-049281281941	050194094360	050139005169	049542193206
051400000000		050603581576	-050188580492	-049294462763	050208950933	050145342074	049547094493
051410000000		050584640400	-050191602433	-049311276303	050223152013	050150048246	049554469453
051420000000		050565594939	-050191816702	-049331573946	050238622975	050156498515	049562712584
051430000000		050545106615	-050192247090	-049356123632	050254430729	050162009759	049571302006
051440000000		050525864401	-050201958117	-049365772711	050270994209	050167060564	049581043745
051450000000		050505589000	-050205961712	-049491640264	050288011147	050173747726	049594056526
051460000000		050484566672	-050210417331	-049445159951	050305700620	050179740263	049511039777
051470000000		050463375410	-050215330091	-049519746752	050323996760	050185949126	049531486714
051480000000		050441590160	-050220860678	-049588614741	050342904451	050192466469	049652072643
051490000000		050419198730	-050227169590	-049679165554	050362482462	050199103999	049697182107
051500000000		050396100158	-050234562284	-049805119790	050382759759	050206482206	049753015750
051510000000		050372294473	-050243430118	-049907775618	050403788686	050214346953	049847088345

ζ	Ω	ϕ_{11}	ϕ'_{11}	ϕ''_{11}	ψ_{11}	ψ'_{11}	ψ''_{11}
051100000000	054000000000	050000000000	051100000000	000000000000	000000000000	000000000000	000000000000
051110000000		049939660524	050998842100	-049200691784	045647571365	047203999366	044411976650
051120000000		050199764625	050994690352	-049346767391	046348750490	047830540383	048845784586
051130000000		050299101770	050992118170	-040442599114	047187600762	048190148770	049130007147
051140000000		050596181335	050987339774	-049508009524	047480676799	048343764827	049177548319
051150000000		050496502721	050982026305	-049562179178	047891372185	048545926025	049427140116
051160000000		050594514425	050976356301	-049581315784	048149900439	048798724943	040278801731
051170000000		050691384335	050970434690	-049509702777	048250680113	049110428552	040332572148
051180000000		050788506564	050964377097	-049610364307	048378624056	049146449261	040388813746
051190000000		050844764038	050958245420	-049615469905	048545069520	049188400568	040446702720
051200000000		050940244627	050952081628	-049616606616	044757005764	049235677924	040507735297
051210000000		051107514459	050945922605	-049[illegible]3391	049101833325	049289752771	040570224112
051220000000		051116943036	050934789089	-049611401446	049133861407	049350069018	040635901783
051230000000		051126310437	050933698750	-049606437998	049172150010	049416983676	040704118188
051240000000		051135617151	050927658568	-049601201621	049217490717	049490999726	040775347195
051250000000		051144463773	050921675500	-049595528429	049290584043	049572186160	040849684931
051260000000		051154[illegible]	050915747594	-049589975866	049392178297	049660997817	040927756041
051270000000		051163178957	050909974680	-049584842608	049403044774	049757740490	050100861554
051280000000		051172248469	050904048206	-049580426934	049484041926	049862840244	050107375460
051290000000		051181240651	050898262890	-049576957076	049574904755	049976097305	050118910669
051300000000		051190213853	050892505097	-049574475533	049679434902	050109940475	050127705364
051310000000		051139110271	050886764705	-049573826185	049706148195	050123291553	050137603751
051320000000		051207989147	050861022969	-049574485545	049926417199	050137409717	050148056819
051330000000		051216780711	050875265250	-049577491606	050107150904	050152883340	050159124359
051340000000		051225644322	050869466672	-049582445203	050123250420	050169367400	050170876123
051350000000		051234109868	050863607480	-049590028328	050141060490	050187074054	050183395162
051360000000		051242796150	050857456476	-049600558485	050160701404	050206085294	050196779881
051370000000		051251292671	050851585791	-049614482764	050182317594	050226402723	050211149613
051380000000		051259787364	050845353783	-049632328499	050206049730	050248372866	050226448566
051390000000		0512[illegible]52	050838024445	-049654738846	050232035930	050271615811	050243456071
051400000000		051276595016	050832241124	-049682499467	050260864469	050297305116	050261794152
051410000000		051284893047	050825254035	-049715597932	050291612770	050324234618	050281046848
051420000000		051293058837	050817883650	-049758288197	050325538202	050353527160	050304079321
051430000000		051301179327	050810058735	-049809206880	050362296426	050395178164	050329976366
051440000000		051309298124	050801861568	-049871535507	050402504699	050419493066	050357591726
051450000000		051317210455	050792582415	-049948274593	050446294048	050496894757	050390141767
051460000000		051325026903	050782633860	-050104370371	050493991405	050497706692	050428248938
051470000000		051332889030	050771630649	-050116419661	050545072355	050542727650	050473024335
051480000000		051340514913	050751232711	-050131984783	050602707179	050592451413	050530393631
051490000000		[illegible]	[illegible]	-050142785519	050664748700	050649311930	050603278364
051500000000		051355468145	050728391416	-050182110827	050732858047	050714417755	050703479962
051510000000		051362568216	050708284325	-050227344509	050807967685	050791326670	050856099181

ζ	u	Φ_1	Φ_1'	Φ_1''	ψ_1	ψ_1'	ψ_1''
05110000000	04945000000	05110 [illegible]	[illegible]00000	-50250000000	00000000000	00000000000	04049999784
05111000000		05099881150	-49227346573	-50206380599	047227198541	048456351949	040462352985
05112000000		05099541549	-49416136703	-50192887711	047916871872	048924136446	040472367586
05113000000		05099068325	-49573457175	-50146640947	048207861191	049140103559	040482290713
05114000000		050884181054	-49711192882	-50195716868	048372304342	049188847319	040490409167
05115000000		050970[illegible]	-49826193440	-50168706249	048585990430	049238279580	040498131273
05116000000		050067498479	-49929908936	-49949463613	048849107352	049288441801	040504099876
05117000000		050057[illegible]8770	-50101886145	-49894051263	0491[illegible]4300534	049339246512	040511326219
05118000000		050047204249	-50109743387	-49749470069	049154782361	049390001761	040517196271
05119000000		050035948[illegible]	-50116726301	-49659314916	049194454867	049442584045	040522474160
05120000000		050023995[illegible]	-50122971342	-49591844996	049241836867	049495215377	040527816869
05121000000		050011362910	-5 28593684	-49534353007	049293812439	049548238680	040532468807
05122000000		05089874550	-50133687225	-49485487030	049351000360	049601740777	040537270656
05123000000		050884490468	-50138331925	-49444341503	049413879948	04965[illegible]86330	040541456637
05124000000		050870690851	-50142594984	-49409282355	049482151939	049710045963	040545858749
05125000000		050856142276	-50146835214	-49390413661	049555894655	049764043465	040549906105
05126000000		050841352908	-50150203082	-49354723106	049635135915	049820042944	040553826363
05127000000		050826108725	-50153841076	-49334005683	049719874110	049875041195	040557433842
05128000000		050810600957	-50156896645	-49317182416	049810280721	049931547231	040561393646
05129000000		050794742030	-50159995422	-49303755140	049906910315	049987932165	040565086047
05130000000		050778605815	-50162982375	-49293529893	050100987942	050104458382	040568777297
05131000000		050762185150	-50165875290	-49286276829	050111514765	05011[illegible]149398	040572469112
05132000000		050745492720	-50168717372	-49281972409	050122822859	050115907913	040576214719
05133000000		050728414950	-50171521264	-49280109715	050134498235	050121602756	040580036711
05134000000		050711[illegible]	[illegible]	-4928[illegible]	05014[illegible]62705	0501275090[illegible]	040583902553
05135000000		050693588248	-50177154185	-49284037417	050160201804	050133372977	040588102307
05136000000		050675606027	-50180038205	-49291389173	050173838711	050139471858	040592440400
05137000000		050657584456	-50182990771	-49300665107	050188057891	050145253004	040597085022
05138000000		050639084289	-50186061461	-49312934329	050202884923	050151214830	040602097858
05139000000		050620344050	-50189259426	-49328405486	050218532360	050157266880	040607594216
05140000000		050601236659	-50192641296	-49347402763	050234340236	050163348744	040613474556
05141000000		050581518500	-50196219680	-49370324911	050250077551	050169543855	040620573555
05142000000		050561984428	-50200062778	-49397784130	050268251473	050175783546	040629415421
05143000000		050541607345	-50204191016	-49430425085	050286138457	050182117907	040637422573
05144000000		050521194184	-50208692282	-49469415285	050304478410	050188541723	040648458500
05145000000		050500[illegible]0940	-50213803233	-49516183824	050323844356	050195006973	040661716088
05146000000		050478306307	-50219048380	-49572832052	050343702759	050201786750	040678039974
05147000000		050456106000	-50225110189	-4964263[illegible]	050364224345	050208647557	040699027357
05148000000		050433387955	-50231962263	-49730773217	050385444415	050215790280	040726006153
05149000000		[illegible]	[illegible]	-49846211000	050407399217	050223240375	040765817980
05150000000		050385340198	-50249035242	-50100696799	050430108672	050231170877	040823095130
05151000000		050359912118	-50260144268	-50125087675	050459440037	050239819937	040921048[illegible]

ξ	α	ϕ_{II}	ψ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.0000000000	0.0000000000	0.0000000000
0.0111000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0111100000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0112000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0112500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0113000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0113500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0114000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0114500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0115000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0115500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0116000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0116500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0117000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0117500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0118000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0118500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0119000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0119500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0120000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0120500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0121000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0121500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0122000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0122500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0123000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0123500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0124000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0124500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0125000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0125500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0126000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0126500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0127000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0127500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0128000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0128500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0129000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0129500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0130000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0130500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0131000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0131500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0132000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0132500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0133000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0133500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0134000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0134500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0135000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0135500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0136000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0136500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0137000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0137500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0138000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0138500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0139000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0139500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0140000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0140500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0141000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0141500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0142000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0142500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0143000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0143500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0144000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0144500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0145000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0145500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0146000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0146500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0147000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0147500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0148000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0148500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0149000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0149500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0150000000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0150500000		0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

ξ	α	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
0811[illegible]	0[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0[illegible]9712
0811[illegible]		[illegible]	-[illegible]7348230	-[illegible]8942	0[illegible]9955	[illegible]	0[illegible]71578
0811[illegible]		[illegible]	-[illegible]5923[illegible]	-[illegible]2167[illegible]	0[illegible]9312	[illegible]	0[illegible]23425
0811[illegible]		[illegible]	-[illegible]4670[illegible]	-[illegible]6720079	0[illegible]4499	[illegible]	0[illegible]53957
0811[illegible]		[illegible]	-[illegible]7807[illegible]	-[illegible]2386	0[illegible]528	[illegible]	0[illegible]44352
0811[illegible]		[illegible]	-[illegible]1204	-[illegible]3A440	0[illegible]6227	[illegible]	0[illegible]69202
0811[illegible]		[illegible]	-[illegible]7593	0[illegible]2161	0[illegible]7963	[illegible]	0[illegible]26784
0811[illegible]		[illegible]	-[illegible]9369	0[illegible]6096	0[illegible]2072	[illegible]	0[illegible]55620
0811[illegible]		[illegible]	-[illegible]1255	0[illegible]7246	0[illegible]8576	[illegible]	0[illegible]71760
0811[illegible]		[illegible]	-[illegible]2528	0[illegible]A116	0[illegible]8695	[illegible]	0[illegible]59779
0811[illegible]		[illegible]	-[illegible]3364	0[illegible]518	0[illegible]5324	[illegible]	0[illegible]92955
0811[illegible]		[illegible]	-[illegible]3612	0[illegible]4478	0[illegible]54C5	[illegible]	0[illegible]28809
0811[illegible]		[illegible]	-[illegible]7602	0[illegible]9472	0[illegible]9505	[illegible]	0[illegible]17826
0811[illegible]		[illegible]	-[illegible]5884	0[illegible]2095	0[illegible]99CC	[illegible]	0[illegible]02893
0811[illegible]		[illegible]	-[illegible]8763	0[illegible]1291	0[illegible]12CC	[illegible]	0[illegible]21498
0811[illegible]		[illegible]	-[illegible]4774	0[illegible]9830	0[illegible]9736	[illegible]	0[illegible]08295
0811[illegible]		[illegible]	-[illegible]7633	0[illegible]A049	0[illegible]3614	[illegible]	0[illegible]93099
0811[illegible]		[illegible]	-[illegible]4887	0[illegible]8495	0[illegible]3403	[illegible]	0[illegible]93615
0811[illegible]		[illegible]	-[illegible]4910	0[illegible]0095	0[illegible]4998	[illegible]	0[illegible]73385
0811[illegible]		[illegible]	-[illegible]8722	0[illegible]4451	0[illegible]3865	[illegible]	0[illegible]16623
0811[illegible]		[illegible]	-[illegible]2360	0[illegible]7400	0[illegible]3018	[illegible]	0[illegible]77220
0811[illegible]		[illegible]	-[illegible]9907	0[illegible]8887	0[illegible]8938	[illegible]	0[illegible]67620
0811[illegible]		[illegible]	-[illegible]3810	0[illegible]267A	0[illegible]8238	[illegible]	0[illegible]14216
0811[illegible]		[illegible]	-[illegible]1832	0[illegible]A170	0[illegible]33C8	[illegible]	0[illegible]61702
0811[illegible]		[illegible]	-[illegible]2706	0[illegible]4926	0[illegible]1543	[illegible]	0[illegible]09714
0811[illegible]		[illegible]	-[illegible]2701	0[illegible]2980	0[illegible]6220	[illegible]	0[illegible]42887
0811[illegible]		[illegible]	-[illegible]4304	0[illegible]7597	0[illegible]4729	[illegible]	0[illegible]83734012
0811[illegible]		[illegible]	-[illegible]3819	0[illegible]1342	0[illegible]2407	[illegible]	0[illegible]21214
0811[illegible]		[illegible]	-[illegible]2760	0[illegible]4709	0[illegible]5845	[illegible]	0[illegible]50603
0811[illegible]		[illegible]	-[illegible]2166	0[illegible]1881	0[illegible]4021	[illegible]	0[illegible]38079
0811[illegible]		[illegible]	-[illegible]9026	0[illegible]A335	0[illegible]1872	[illegible]	0[illegible]76149
0811[illegible]		[illegible]	-[illegible]2785	0[illegible]0584	0[illegible]4305	[illegible]	0[illegible]93129
0811[illegible]		[illegible]	-[illegible]7148	0[illegible]1371	0[illegible]3099	[illegible]	0[illegible]29065
0811[illegible]		[illegible]	-[illegible]6209	0[illegible]1698	0[illegible]4642	[illegible]	0[illegible]10676
0811[illegible]		[illegible]	-[illegible]7706	0[illegible]4074	0[illegible]6363	[illegible]	0[illegible]50126
0811[illegible]		[illegible]	-[illegible]4152	0[illegible]4711	0[illegible]7916	[illegible]	0[illegible]40401
0811[illegible]		[illegible]	-[illegible]6743	0[illegible]4638	0[illegible]5515	[illegible]	0[illegible]23369
0811[illegible]		[illegible]	-[illegible]4634	0[illegible]4492	0[illegible]4551	[illegible]	0[illegible]42985
0811[illegible]		[illegible]	-[illegible]4737	0[illegible]2A43	0[illegible]7887	[illegible]	0[illegible]43247
0811[illegible]		[illegible]	-[illegible]4621	0[illegible]5719	0[illegible]1970	[illegible]	0[illegible]00A05
0811[illegible]		[illegible]	-[illegible]4085	0[illegible]4962	0[illegible]2065	[illegible]	0[illegible]05464
0811[illegible]		[illegible]	-[illegible]7797	0[illegible]1440	0[illegible]2069	[illegible]	0[illegible]40502

ξ	α	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
0.110000000	0.000000000	0.000000000	0.011000000	0.000000000	0.000000000	0.000000000	0.000000000
0.111000000		-0.000000000	0.009999912	-0.002666665	0.000100133	0.000000000	0.000000000
0.112000000		-0.000000000	0.009999200	-0.002666667	0.000400533	0.000000000	0.000000000
0.113000000		-0.000000000	0.009992117	-0.002666667	0.000900533	0.000000000	0.000000000
0.114000000		-0.000000000	0.009987337	-0.002666667	0.001600533	0.000000000	0.000000000
0.115000000		-0.000000000	0.009980220	-0.002666667	0.002500533	0.000000000	0.000000000
0.116000000		-0.000000000	0.009976333	-0.002666667	0.003600533	0.000000000	0.000000000
0.117000000		-0.000000000	0.009970409	-0.002666667	0.004900533	0.000000000	0.000000000
0.118000000		-0.000000000	0.009963933	-0.002666667	0.006400533	0.000000000	0.000000000
0.119000000		-0.000000000	0.009958173	-0.002666667	0.008100533	0.000000000	0.000000000
0.120000000		-0.000000000	0.009951907	-0.002666667	0.010000533	0.000000000	0.000000000
0.121000000		-0.000000000	0.009945950	-0.002666667	0.012100533	0.000000000	0.000000000
0.122000000		-0.000000000	0.009939636	-0.002666667	0.014400533	0.000000000	0.000000000
0.123000000		-0.000000000	0.009933941	-0.002666667	0.016900533	0.000000000	0.000000000
0.124000000		-0.000000000	0.009927162	-0.002666667	0.019600533	0.000000000	0.000000000
0.125000000		-0.000000000	0.009921001	-0.002666667	0.022500533	0.000000000	0.000000000
0.126000000		-0.000000000	0.009914847	-0.002666667	0.025600533	0.000000000	0.000000000
0.127000000		-0.000000000	0.009908693	-0.002666667	0.028900533	0.000000000	0.000000000
0.128000000		-0.000000000	0.009902515	-0.002666667	0.032400533	0.000000000	0.000000000
0.129000000		-0.000000000	0.009896301	-0.002666667	0.036100533	0.000000000	0.000000000
0.130000000		-0.000000000	0.009890018	-0.002666667	0.040000533	0.000000000	0.000000000
0.131000000		-0.000000000	0.009883685	-0.002666667	0.044100533	0.000000000	0.000000000
0.132000000		-0.000000000	0.009877140	-0.002666667	0.048400533	0.000000000	0.000000000
0.133000000		-0.000000000	0.009870471	-0.002666667	0.052900533	0.000000000	0.000000000
0.134000000		-0.000000000	0.009863587	-0.002666667	0.057600533	0.000000000	0.000000000
0.135000000		-0.000000000	0.009856444	-0.002666667	0.062500533	0.000000000	0.000000000
0.136000000		-0.000000000	0.009848073	-0.002666667	0.067600533	0.000000000	0.000000000
0.137000000		-0.000000000	0.009841117	-0.002666667	0.072900533	0.000000000	0.000000000
0.138000000		-0.000000000	0.009832785	-0.002666667	0.078400533	0.000000000	0.000000000
0.139000000		-0.000000000	0.009823847	-0.002666667	0.084100533	0.000000000	0.000000000
0.140000000		-0.000000000	0.009814930	-0.002666667	0.090000533	0.000000000	0.000000000
0.141000000		-0.000000000	0.009803975	-0.002666667	0.096100533	0.000000000	0.000000000
0.142000000		-0.000000000	0.009792457	-0.002666667	0.102400533	0.000000000	0.000000000
0.143000000		-0.000000000	0.009780217	-0.002666667	0.108900533	0.000000000	0.000000000
0.144000000		-0.000000000	0.009766196	-0.002666667	0.115600533	0.000000000	0.000000000
0.145000000		-0.000000000	0.009750048	-0.002666667	0.122500533	0.000000000	0.000000000
0.146000000		-0.000000000	0.009733458	-0.002666667	0.129600533	0.000000000	0.000000000
0.147000000		-0.000000000	0.009713511	-0.002666667	0.136900533	0.000000000	0.000000000
0.148000000		-0.000000000	0.009693957	-0.002666667	0.144400533	0.000000000	0.000000000
0.149000000		-0.000000000	0.009673255	-0.002666667	0.152100533	0.000000000	0.000000000
0.150000000		-0.000000000	0.009630915	-0.002666667	0.160000533	0.000000000	0.000000000
0.151000000		-0.000000000	0.009589420	-0.002666667	0.168100533	0.000000000	0.000000000

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
0.5110000000	0.4987500000	0.5110000000	0.0000000000	-0.0025000000	0.0000000000	0.0000000000	0.0000000000
0.5111000000		0.5099888889	-0.0022350000	-0.0020089549	0.0035831739	0.0032792760	0.0029139484
0.5112000000		0.5089555556	-0.0019917357	-0.0017929499	0.0028425000	0.0025932320	0.0023727194
0.5113000000		0.5079000000	-0.0017585938	-0.0016147859	0.0023333333	0.0021350361	0.0019752201
0.5114000000		0.5068222222	-0.0015356250	-0.0014259621	0.0019557144	0.0017861706	0.0016311470
0.5115000000		0.5057222222	-0.0013225000	-0.0012509225	0.0016111111	0.0014732525	0.0013377067
0.5116000000		0.5045999999	-0.0011190625	-0.0010955941	0.0013020000	0.0011920000	0.0010884185
0.5117000000		0.5034555556	-0.0009253125	-0.0009043910	0.0010269136	0.0009405136	0.0008588185
0.5118000000		0.5022888889	-0.0007411875	-0.0007271537	0.0007800000	0.0007142857	0.0006521739
0.5119000000		0.5010999999	-0.0005666406	-0.0005625000	0.0005611111	0.0005142857	0.0004695652
0.5120000000		0.4998888889	-0.0004016016	-0.0004102500	0.0003702222	0.0003391304	0.0003095652
0.5121000000		0.4986555556	-0.0002460938	-0.0002704688	0.0002072222	0.0001898551	0.0001733333
0.5122000000		0.4973999999	-0.0000999844	-0.0001429688	0.0000722222	0.0000661836	0.0000604348
0.5123000000		0.4961222222	0.0000367188	-0.0000276563	-0.0000388889	-0.0000356122	-0.0000325217
0.5124000000		0.4948222222	0.0001625000	0.0000753125	-0.0001000000	-0.0000916327	-0.0000836957
0.5125000000		0.4935000000	0.0002792969	0.0001664063	-0.0001500000	-0.0001374490	-0.0001255435
0.5126000000		0.4921555556	0.0003871094	0.0002414063	-0.0001888889	-0.0001730612	-0.0001580435
0.5127000000		0.4907888889	0.0004859375	0.0003003906	-0.0002166667	-0.0001985714	-0.0001813913
0.5128000000		0.4893999999	0.0005757813	0.0003533203	-0.0002333333	-0.0002138776	-0.0001954348
0.5129000000		0.4879888889	0.0006566406	0.0004001953	-0.0002388889	-0.0002189796	-0.0002002174
0.5130000000		0.4865555556	0.0007285156	0.0004410156	-0.0002333333	-0.0002138776	-0.0001954348
0.5131000000		0.4851000000	0.0007914063	0.0004757813	-0.0002166667	-0.0001985714	-0.0001813913
0.5132000000		0.4836222222	0.0008453125	0.0005044922	-0.0001888889	-0.0001730612	-0.0001580435
0.5133000000		0.4821222222	0.0008902344	0.0005271484	-0.0001500000	-0.0001374490	-0.0001255435
0.5134000000		0.4806000000	0.0009261719	0.0005437500	-0.0001000000	-0.0000916327	-0.0000836957
0.5135000000		0.4790555556	0.0009531250	0.0005542969	-0.0000388889	-0.0000356122	-0.0000325217
0.5136000000		0.4774888889	0.0009710938	0.0005587891	0.0000722222	0.0000661836	0.0000604348
0.5137000000		0.4759000000	0.0009800781	0.0005572266	0.0002072222	0.0001898551	0.0001733333
0.5138000000		0.4742888889	0.0009800781	0.0005496094	0.0003702222	0.0003391304	0.0003095652
0.5139000000		0.4726555556	0.0009710938	0.0005359375	0.0005611111	0.0005142857	0.0004695652
0.5140000000		0.4710000000	0.0009531250	0.0005162109	0.0007800000	0.0007142857	0.0006521739
0.5141000000		0.4693222222	0.0009261719	0.0004904297	0.0010269136	0.0009405136	0.0008588185
0.5142000000		0.4676222222	0.0008902344	0.0004585938	0.0013020000	0.0011920000	0.0010884185
0.5143000000		0.4659000000	0.0008453125	0.0004207031	0.0016111111	0.0014732525	0.0013377067
0.5144000000		0.4641555556	0.0007914063	0.0003767578	0.0019557144	0.0017861706	0.0016311470
0.5145000000		0.4623888889	0.0007285156	0.0003267578	0.0023333333	0.0021350361	0.0019752201
0.5146000000		0.4606000000	0.0006566406	0.0002707031	0.0027440000	0.0025196327	0.0023555556
0.5147000000		0.4587888889	0.0005757813	0.0002085938	0.0031888889	0.0029408163	0.0027777778
0.5148000000		0.4569555556	0.0004859375	0.0001404297	0.0036666667	0.0033979592	0.0032173913
0.5149000000		0.4551000000	0.0003871094	0.0000664063	0.0041777778	0.0038913265	0.0036956522
0.5150000000		0.4532222222	0.0002792969	-0.0000136719	0.0047222222	0.0044208333	0.0042173913
0.5151000000		0.4513222222	0.0001625000	-0.0000976563	0.0052999999	0.0049846939	0.0047826087
0.5152000000		0.4494000000	0.0000367188	-0.0001875000	0.0059111111	0.0055826531	0.0053913043
0.5153000000		0.4474555556	-0.0001000000	-0.0002832031	0.0065555556	0.0062142857	0.0060434783
0.5154000000		0.4454888889	-0.0002460938	-0.0003847656	0.0072333333	0.0068797959	0.0067391304
0.5155000000		0.4435000000	-0.0004016016	-0.0004921875	0.0079444444	0.0075795918	0.0074782609
0.5156000000		0.4414888889	-0.0005666406	-0.0006054688	0.0086888889	0.0083132653	0.0082608696
0.5157000000		0.4394555556	-0.0007411875	-0.0007246094	0.0094666667	0.0090812245	0.0090869565
0.5158000000		0.4374000000	-0.0009253125	-0.0008496094	0.0102777778	0.0098832653	0.0099565217
0.5159000000		0.4353222222	-0.0011190625	-0.0009804688	0.0111222222	0.0107193878	0.0108695652
0.5160000000		0.4332222222	-0.0013225000	-0.0011171875	0.0120000000	0.0115896327	0.0118260870
0.5161000000		0.4311000000	-0.0015356250	-0.0012597656	0.0129111111	0.0124940000	0.0128260870
0.5162000000		0.4289555556	-0.0017585938	-0.0014082031	0.0138555556	0.0134326531	0.0138695652
0.5163000000		0.4267888889	-0.0019917357	-0.0015625000	0.0148333333	0.0144055102	0.0149565217
0.5164000000		0.4246000000	-0.0022350000	-0.0017226563	0.0158444444	0.0154126531	0.0160869565
0.5165000000		0.4223888889	-0.0024883789	-0.0018886719	0.0168888889	0.0164540816	0.0172608696
0.5166000000		0.4201555556	-0.0027519531	-0.0020605469	0.0179666667	0.0175297959	0.0184782609
0.5167000000		0.4179000000	-0.0030256250	-0.0022382813	0.0190777778	0.0186397959	0.0197391304
0.5168000000		0.4156222222	-0.0033093750	-0.0024218750	0.0202222222	0.0197840816	0.0210434783
0.5169000000		0.4133222222	-0.0036031250	-0.0026113281	0.0213999999	0.0209626531	0.0223913043
0.5170000000		0.4110000000	-0.0039068750	-0.0028066406	0.0226111111	0.0221755102	0.0237826087
0.5171000000		0.4086555556	-0.0042206250	-0.0030078125	0.0238555556	0.0234226531	0.0252173913
0.5172000000		0.4062888889	-0.0045443750	-0.0032148438	0.0251333333	0.0247040816	0.0266956522
0.5173000000		0.4039000000	-0.0048781250	-0.0034277344	0.0264444444	0.0260197959	0.0282173913
0.5174000000		0.4014888889	-0.0052218750	-0.0036464844	0.0277888889	0.0273697959	0.0297826087
0.5175000000		0.3990555556	-0.0055756250	-0.0038710938	0.0291666667	0.0287540816	0.0313913043
0.5176000000		0.3966000000	-0.0059393750	-0.0041015625	0.0305777778	0.0301726531	0.0330434783
0.5177000000		0.3941222222	-0.0063131250	-0.0043378906	0.0320222222	0.0316255102	0.0347391304
0.5178000000		0.3916222222	-0.0066968750	-0.0045800781	0.0334999999	0.0331126531	0.0364782609
0.5179000000		0.3891000000	-0.0070906250	-0.0048281250	0.0350111111	0.0346340816	0.0382608696
0.5180000000		0.3865555556	-0.0074943750	-0.0050820313	0.0365555556	0.0361897959	0.0400869565
0.5181000000		0.3839888889	-0.0079081250	-0.0053417969	0.0381333333	0.0377797959	0.0419565217
0.5182000000		0.3814000000	-0.0083318750	-0.0056074219	0.0397444444	0.0394040816	0.0438695652
0.5183000000		0.3787888889	-0.0087656250	-0.0058789063	0.0413888889	0.0410626531	0.0458260870
0.5184000000		0.3761555556	-0.0092093750	-0.0061562500	0.0430666667	0.0427555102	0.0478260870
0.5185000000		0.3735000000	-0.0096631250	-0.0064394531	0.0447777778	0.0444826531	0.0498695652
0.5186000000		0.3708222222	-0.0101268750	-0.0067285156	0.0465222222	0.0462441837	0.0519565217
0.5187000000		0.3681222222	-0.0106006250	-0.0070234375	0.0482999999	0.0480400816	0.0540869565
0.5188000000		0.3654000000	-0.0110843750	-0.0073242188	0.0501111111	0.0498702041	0.0562608696
0.5189000000		0.3626555556	-0.0115781250	-0.0076308594	0.0519555556	0.0517346939	0.0584782609
0.5190000000		0.3598888889	-0.0120818750	-0.0079433594	0.0538333333	0.0536334694	0.0607391304
0.5191000000		0.3571000000	-0.0125956250	-0.0082617188	0.0557444444	0.0555665306	0.0630434783
0.5192000000		0.3542888889	-0.0131193750	-0.0085859375	0.0576888889	0.0575339796	0.0653913043
0.5193000000		0.3514555556	-0.0136531250	-0.0089160156	0.0596666667	0.0595357143	0.0677826087
0.5194000000		0.3486000000	-0.0141968750	-0.0092519531	0.0616777778	0.0615717347	0.0702173913
0.5195000000		0.3457222222	-0.0147506250	-0.0095937500	0.0637222222	0.0636421429	0.0726956522
0.5196000000		0.3428222222	-0.0153143750	-0.0099414063	0.0657999999	0.0657468367	0.0752173913
0.5197000000		0.3399000000	-0.0158881250	-0.0102949219	0.0679111111	0.0678858163	0.0777826087
0.5198000000		0.3369555556	-0.0164718750	-0.0106542969	0.0700555556	0.0700591837	0.0803913043
0.5199000000		0.3339888889	-0.0170656250	-0.0110195313	0.0722333333	0.0722669388	0.0830434783
0.5200000000		0.3310000000	-0.0176693750	-0.0113906250	0.0744444444	0.0745090816	0.0857391304

ξ	α	ϕ_{11}	ϕ'_{11}	ϕ''_{11}	ψ_{11}	ψ'_{11}	ψ''_{11}
.5110000000	.4570000000	0.0000000000	0.0110000000	.0000000000	.0000000000	.0000000000	.0000000000
.5111000000		0501799607559	0300996912100	-.0920653700	.0451168244	0473860045	0497220259119
.5112000000		0501997224617	0309920701117	-.0934921593	.0464032059	0480413028	0491479207/91
.5113000000		0502991417	030392116955	-.0344276793	.0473283011	04897/40531	04922/512472
.5114000000		050398621073	030937435719	-.0950892500	.0477860043	04860137971	0493127092 49
.5115000000		0504969917 47	030982016185	-.0355302272	.0481559010	0409653A9875	049397/494353
.5116000000		05059941190	030976824396	-.0350282985 4	.0492728956	0441337/7664	049447900/92
.5117000000		050691069180	03077239335	-.0360219909	.0484386435	0441912 79847	04958199735
.5118000000		05070485534	03096493056	-.036142317 76	.0486425/5101	0447943026	04967989153
.5119000000		05080/07645	030956126559	-.0362116486	.0489454922	0449323559	04979171059
.5120000000		050980867	03095193269	-.0362473847	.0491324/993	04901 2/81251	0498/A2856
.5121000000		051107508561	03094264 3835	-.036241 8396	.0491793016	04450723105	04999/836544
.5122000000		0511149731 5	030934972396	-.0362652030	.049234235	04941249913 9	050111256212
.5123000000		05112629	03093510855	-.036246924 93	.0493012/319	04479300370	05012320848
.5124000000		0511135095 77	030932093979	-.0362706617 9	.049395 9	[illegible]	050133466079
.5125000000		0511144012920	030942566870	-.036285 54 89	.049473 5 47	0501001 29	050144466075
.5126000000		0511134069	030914253189	-.036315997 82	.0495812 9	[illegible]	05015424 391
.5127000000		05115 312	030907935315	-.036367204	.049 52	0501	050173140
.5128000000		0511724 52 77	0309015 2031	-.094444 39152	.0494 9 44	0501 954	050171313105
.5129000000		051101448 3	0308902640	-.09652 65920	.0501007/48	050117 758	050200214 06
.5130000000		05119045	030883402312	-.0966974	.050118943	0501073 797	0502233 07374
.5131000000		051119	030884 17375	-.096081868 9	.050139349	050219 44 62	05024 56 01
.5132000000		0512 27097 4	030874416435	-.0971212440 0	.0501420 5	050280025148	050250780 74
.5133000000		051316 074	03086 7 56	-.0974140 399	.0501 79	050367357524	05027400 4104
.5134000000		0512250430	030859766 76	-.0977715231 3	.0502156 311	050296173346	05029 479787
.5135000000		05123301 3	030851749300	-.0982034 921	.0502 067/9	05039 07097	05032 220686
.5136000000		05124207720	030843 1 3	-.0987209689	.05028 1127 9	05034024440	050343 743
.5137000000		0512500 66 4	03083481535	-.093376 4106	.05031894299	05039 0 4 9	05036 909174
.5138000000		05125874 34	030824617375	-.01 0493 50	.0503 03 1	0504 01 6	0503950923
.5139000000		05126646477	03081 1 3 27	-.010 57 9	.0504098 8 1	0504 0 5	050424 77 4
.5140000000		0512750430 57	030804493395	-.0119606011	.05044 5	05051 093 4	0504 5542 411
.5141000000		05128306920	030790151425	-.0131741 4	.0505096 312	05056415759	05049 217 17
.5142000000		0512 0 9 43	030770269340	-.0146146749	.0505 4 8 129	050617 7 43	05052 837 3
.5143000000		05129902748	030750833145	-.0163 23 1	[illegible]	0506 401 7	05057100 /85
.5144000000		0513064 49E2	030743 20475	-.01839258 1	.050/ 33/1	050791 4909	0506179448
.5145000000		05131 90374	030723910335	-.0208878173	.0507797 29	050704 7205	050674421
.5146000000		05132018770	030701526 2	-.0239478 44	.0508 273 71	0508 49992	050735774741
.5147000000		0513274106	030672774	-.0277667 1	.0509 32 9	05092 42 25	05081 42 41
.5148000000		051733 1384 4	030653 38 22	-.0326511060	.0511051 170	05110 4 5	05091 8842 0
.5149000000		0513402 7	030610 07710	-.0391240927	.05111 9	0511 27 17	05111 3 3488
.5150000000		05134 694110	030564 27360	-.0481844069	.0511277 7 3	051113 87 6	05111 86 397
.5151000000		051351606367	030514 68 5	-.0620766175	.051160749	051194 4	0511 3618

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	-0.92273507795	-0.80206369642	0.47348218582	0.48719463095	0.49729891394
[illegible]	[illegible]	[illegible]	-0.94161779450	-0.81729547340	0.48144574619	0.49185986829	0.49740237545
[illegible]	[illegible]	[illegible]	-0.95736805470	-0.83134762477	0.48832794783	0.49528178698	0.49760468670
[illegible]	[illegible]	[illegible]	-0.97215322511	-0.84524965524	0.49528743944	0.49297443360	0.49774869317
[illegible]	[illegible]	[illegible]	-0.98826894915	-0.86010924029	0.48962483298	0.49397940215	0.49780363043
[illegible]	[illegible]	[illegible]	-0.99931084393	-0.87035851845	0.49133905944	0.49485040888	0.49795076252
[illegible]	[illegible]	[illegible]	-0.01010209361	-0.88045340771	0.49183447586	0.49533964220	0.49808672961
[illegible]	[illegible]	[illegible]	-0.01110607717	-0.89047212787	0.49241063086	0.49516416974	0.49813067416
[illegible]	[illegible]	[illegible]	-0.01170188793	-0.89675835893	0.49305683984	0.49469844925	0.49824874171
[illegible]	[illegible]	[illegible]	-0.01236216472	-0.90327227454	0.49436079424	0.49478131258	0.49832634387
[illegible]	[illegible]	[illegible]	-0.01324794151	-0.91250569096	0.49494539732	0.49494944478	0.49844021384
[illegible]	[illegible]	[illegible]	-0.01354863597	-0.91578361265	0.49535704705	0.49494941358	0.49848736884
[illegible]	[illegible]	[illegible]	-0.01398432050	-0.92027209636	0.49606297654	0.49510341697	0.49865414597
[illegible]	[illegible]	[illegible]	-0.01446552184	-0.92545722423	0.49674068012	0.49511421594	0.49886058018
[illegible]	[illegible]	[illegible]	-0.01469891372	-0.92844057474	0.49767691181	0.49512068094	0.49886027587
[illegible]	[illegible]	[illegible]	-0.01532626e3	-0.93748926707	0.50100203194	0.49512934906	0.49872486194
[illegible]	[illegible]	[illegible]	-0.01573x1x80	-0.94062107709	0.50113504x14	0.49513x81x04	0.49867x7944x5
[illegible]	[illegible]	[illegible]	-0.01761461492	-0.96039669818	0.50127847197	0.49514e912245	0.49888549782
[illegible]	[illegible]	[illegible]	-0.01634864x40	-0.98035610200	0.50142959327	0.49515781607	0.49886789358
[illegible]	[illegible]	[illegible]	-0.01764547e94	-0.98907 [illegible]	[illegible]	0.49516440734	0.49869394247
[illegible]	[illegible]	[illegible]	-0.01736548781	-0.99417852694	0.50175e0x4x2	0.49517254701	0.49869x411n3
[illegible]	[illegible]	[illegible]	-0.01778849x4	-0.99e4e4e34e86	0.50193748224	0.49518e7e4e6	0.49869x3x1015
[illegible]	[illegible]	[illegible]	-0.01884256x41	-0.[illegible]	0.50212491443	0.49519173324	0.49870x7x0x6
[illegible]	[illegible]	[illegible]	-0.01808484e82	-0.99406944088	0.50232004e88e	0.49520e1a69e4	0.49891255427
[illegible]	[illegible]	[illegible]	-0.01916604x33	-0.99484x6334x3	0.50252542x711	0.49520x4x4214	0.49910x79447
[illegible]	[illegible]	[illegible]	-0.01961967e42	-0.99567116791	0.50272063e583	0.49522x8e526a	0.49924863145
[illegible]	[illegible]	[illegible]	-0.02025[illegible]e3	-0.99x11302723	0.50319738081	0.49529[illegible]8a	0.49892e13274
[illegible]	[illegible]	[illegible]	-0.02017e9e0a78	-0.99n022e04e0	0.50343908764	0.49524e537b2	0.49952764[illegible]
[illegible]	[illegible]	[illegible]	-0.02012n30x86	-0.99x7e0e3130	0.50369013134	0.49525e3n0130	0.49953x83497
[illegible]	[illegible]	[illegible]	-0.02224e0x7e0	-0.99[illegible]e3e0	0.50395220520	0.49526e87845	0.49943x263e4
[illegible]	[illegible]	[illegible]	-0.02372337e72	-0.99406a1e0557	0.50422209031	0.49527511297	0.49944447e46
[illegible]	[illegible]	[illegible]	-0.02462332e09	-0.99e5e07e9e48	0.50449029e9e	0.49528e3e203e	0.49944e4e5x1c
[illegible]	[illegible]	[illegible]	-0.02560404e8e	-0.[illegible]	0.50479184e903	0.49529e4e1e3e	0.49953427747
[illegible]	[illegible]	[illegible]	-0.02e7e0e0e7e	-0.[illegible]	0.50509e0e3e11	0.49530e83611	0.49956a19493
[illegible]	[illegible]	[illegible]	-0.02e8e163e95	-0.[illegible]	0.50539e99e5e3	0.49531322748	0.49964857446
[illegible]	[illegible]	[illegible]	-0.02e943e2e47	-0.[illegible]	0.50571733113	0.49532459402e	0.49972e30368
[illegible]	[illegible]	[illegible]	-0.03105035e5	-0.[illegible]	0.50600451444	0.49533236569	0.49984172e86
[illegible]	[illegible]	[illegible]	-0.03218e4e2e2	-0.[illegible]	0.50634e2e4e2e	0.49534e6e2e09	0.50010013211[illegible]
[illegible]	[illegible]	[illegible]	-0.03351e07e89	-0.[illegible]	0.50673034e4e	0.49535e2e1e4e	0.50010[illegible]e7e
[illegible]	[illegible]	[illegible]	-0.03728104e13	-0.[illegible]	0.50708e8e2e2e	0.49536338420e	0.50010d14392
[illegible]	[illegible]	[illegible]	-0.03e2e163e95	-0.[illegible]	0.50739[illegible]	0.49537[illegible]	0.49964857446
[illegible]	[illegible]	[illegible]	-0.02e943e2e47	-0.[illegible]	0.50771733113	0.49538[illegible]	0.49972e30368
[illegible]	[illegible]	[illegible]	-0.03105035e5	-0.[illegible]	0.50800451444	0.49539[illegible]	0.49984[illegible]75376
[illegible]	[illegible]	[illegible]	-0.03218e4e2e2	-0.[illegible]	0.50834e2e4e2e	0.49540e6e2e09	0.50010d13211[illegible]
[illegible]	[illegible]	[illegible]	-0.03351e07e89	-0.[illegible]	0.50873034e4e	0.49541e2e1e4e	0.50010A04[illegible]
[illegible]	[illegible]	[illegible]	-0.03728104e13	-0.[illegible]	0.50908e8e2e2e	0.49542338420e	0.50010d14392

ξ	α	ϕ_{II}	ψ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
0811000000000	0487500000000	0511000000000	0000000000000	-0025000000000	0000000000000	0000000000000	0497000000000
0811100000000		0509988819400	-0492274513750	-0420000000000	0471783998000	0487000000000	0497700000001
0811200000000		0500000000000	-0491000000000	-0411720000000	0481627429170	0451000000000	0497700000000
0811300000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0811400000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0811500000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0811600000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0811700000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0811800000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0811900000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0812000000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0812100000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0812200000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0812300000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0812400000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0812500000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0812600000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0812700000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0812800000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0812900000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0813000000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0813100000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0813200000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0813300000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0813400000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0813500000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0813600000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0813700000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0813800000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0813900000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0814000000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0814100000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0814200000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0814300000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0814400000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0814500000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0814600000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0814700000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0814800000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0814900000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000
0815000000000		0500000000000	-0490750000000	-0401400000000	0481000000000	0450000000000	0498000000000

ξ	α	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
05110000000	0437500000	0000000000	0011000000	0000000000	0000000000	0000000000	0000000000
05111000000		0499390569	0.0995212100	-0920653A705	0461751e3191	0473e73866S	048771706198
05112000000		0501027421	[illegible]	-0734682566	04710240244	0401887748	049154490035
05113000000		0502394816	0.099241162	-09442604839	04738173159	0483845789	049243769208
05114000000		0503984910	0.098733444	-09508516981	047580487	0408150019	049332002709
05115000000		0504360915	0.098425174	-0955320840	0481471373	0491023610	049423386
05116000000		0505 [illegible]	[illegible]	-0956316280	04820234	0491437606	[illegible]
05117000000		0506416047	0.097538425	-09602747847	0484499427	0492070457	049623567446
05118000000		0507848928	0.096443470	-0961901145	0482079416	0492379069	049722450375
05119000000		[illegible]	[illegible]	[illegible]	0491724019	049370095	049637842785
05120000000		[illegible]	0.096518514	-0962652596	049101937284	049482461	049951022693
05121000000		0511173072	0.0963458751	-0962465031	0491911428	049640469	050109900764
05122000000		0511169315	0.095423816	-0962984979	04948093630	0494842085217	050114200361
05123000000		0511262294	0.0993427905	-09430976034	0493927416	04973108823	050132003073
05124000000		0511358910	0.0942045930	-09432744845	04948773376	04998 [illegible]	050143340423
05125000000		0511446261	0.0992191970	-09435809050	0495073350	050107249907	050159427404
05126000000		0511519073	0.094123757	-09440763046	049692748	0501993187	050173817490
05127000000		0511631047	0.0947937089	-09440122016	04978560487	0502170485	050183426840
05128000000		0511724669	0.0930090402	-0965860864	0499074289	0501417889	050214956393
05129000000		0511812371	0.0949481538	-0967208971	0501079703	05018072918	050221665246
05130000000		0511907327	0.0948785124	-09490633407	05017946524	0502041002	050237214317
05131000000		0511980731	[illegible]	-09713672445	0501892394	05023880	050257494403
05132000000		0512076841	0.0967329082	-09742319928	05017360733	05029760219	050277190098
05133000000		0512016364	0.0963619473	-0977741694	05024C8049	0502844228	050297907380
05134000000		0512249627	0.0945743602	-09819915413	05027104295	0503174892	050314669093
05135000000		0512334877	0.0944193500	-09870923018	05024853849	05039404302	050342924117
05136000000		0512418347	0.09840184071	-09931727739	0503011093	0503844	050367781918
05137000000		0512509947	0.0983042210	-0100767100	05038160927	0504201066	050394338184
05138000000		0512584220	0.0942063371	-010898134	05039407742	0504410865	050422950502
05139000000		0512660879	0.0980348086	-01189908	05049474294	0505090788	050453705492
05140000000		0512747126	0.079320466	-013087838	0504879403	050585772850	050497580014
05141000000		0512820808	0.078214325	-014492706	050535947	0506043028	050524882375
05142000000		0512903860	0.0767132092	-016157589	0506093108	05064078340	050564393303
05143000000		0512974451	0.0758026040	-01813957	05067069	05074 [illegible]	050610674474
05144000000		0513053406	0.0730722391	-020514 [illegible]	[illegible]	[illegible]	050661946983
05145000000		0513125815	0.0708839090	-023383941	05089504497	0508471980	050772043619
05146000000		0513195156	0.0693729405	-026909014	0509740424	0509925390	050784103804
05147000000		0513264143	0.0654751045	-031200305	051102043906	0511010474	050870059043
05148000000		0513323941	0.0620723481	-03691(8072	05111268900	05111472073	050971007974
05149000000		0513380399	0.0590149	-0443028763	05117916492	05110738860	051110018143
05150000000		0513441722	0.0531049606	-0647400061	0511957497	0511923171707	051127692607
05151000000		0513491951	0.0469456165	-07087477767	05118074099	05114837339	051159934449

ξ	a	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
[illegible]	04880000000	05113000000	00000000000	[illegible]	00000000000	00000000000	04973990000
+51110000000		05104863970	+4922785301	+0206003300	04740362400	[illegible]	04962106271
+51120000000		05099594130	+4941013806	+0172982000	04816294792	04916409376	04961083078
[illegible]		[illegible]	[illegible]	+0146066356	04836952710	04920615422	04965700271
+51140000000		[illegible]	+4971171487	+0126121080	04864187920	04935710663	04967248035
+51150000000		05087640605	[illegible]	[illegible]	04910410424	[illegible]	04968355155
+51160000000		05059505970	+5010418704	+0984854808	04920673364	04960741109	04969397403
+51170000000		[illegible]	+5011070370	+0972784057	04927160847	04940130410	04991477241
+51180000000		[illegible]	+5011787471	+0946032057	04931560187	04978669270	04992405219
+51190000000		05024751137	+5012081902	+0942216300	04949003791	04986039179	04993010601
+51210000000		[illegible]	[illegible]	+0957221476	04957170487	04997159430	04994080014
+51220000000		05029790470	+5013039402	+0963231180	04962397010	05010494004	04995401347
+51230000000		05028390160	+5014052046	+0980074017	04973572717	05011454944	04996218354
+51240000000		05026900910	+5014039347	+0947066006	04987047501	05013446705	04996433015
+51250000000		05025493759	+5015079011	+0905440056	04904037130	05013540384	04997310093
+51260000000		05083967077	+5015441055	+0944825087	05011290722	05014577759	04994657205
+51270000000		[illegible]	+5015062007	+0304202121	05027030410	05016503794	04998368015
+51280000000		05080705653	+5016341830	+0944305190	05014404136	05016430050	04994480019
+51290000000		05079128990	+5016795091	+0344852116	05041003810	05017940222	05010000030
+51300000000		05077424310	+5017050592	+0459102746	05047911947	05018300777	05010082591
+51310000000		05075679701	+5017715809	+0057800019	05004107161	05019354043	05010102240
+51320000000		05073807595	+5018202141	+0449509477	05021423850	05020170304	05010240057
+51330000000		05072039406	+5018794371	+0822317800	05027932701	05021514060	05010202106
+51340000000		05070139761	+5019204814	+0950132014	05024144490	05022400775	05010253424
+51350000000		05068170369	+5019514117	+0559103211	05028453936	05023430007	05010270023
+51360000000		05066178514	+5020490107	+0363604180	05030443800	05024502702	05010305117
+51370000000		05064107569	+5021003645	+0960701114	05033385299	05025444804	05010353670
+51380000000		05061059901	+5021410984	+0374551812	05035003085	05026242700	05010354177
+51390000000		05059473977	+5022083131	+0381200102	05034873004	05027714312	05010374793
+51400000000		05057013774	+5023439071	+0386091849	05041356375	05029204879	05010390050
+51410000000		05055140204	+5023373575	+0997639371	05044904018	05029340387	05010403101
+51420000000		05052534560	+5025305901	+0310765100	05047530396	05030405350	05010406239
+51430000000		05049901540	+5026329585	+0119199821	05050674284	05031244060	05010401431
+51440000000		05047244633	+5027706037	+0132603726	05053913850	05033054504	05010490071
+51450000000		05044370603	+5020194235	+0148333096	05057264830	05034043977	05010408735
+51460000000		05041403243	+5030742605	+0167037077	05060713270	05035832792	05010330460
+51470000000		05038240436	+5032041824	+0189733736	05062742735	05037671181	05010303705
+51480000000		05034980836	+5034107594	+0218012044	05067335953	05039705820	[illegible]
+51490000000		05031318172	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
+51500000000		05027406333	+5039714402	+0308401427	05078553931	05039147972	05010290347
+51510000000		05023358469	+5043002939	+0381148466	05079524729	05040817063	05010334875

ξ	a	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
.5112000000	0.4980000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5113000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5114000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5115000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5116000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5117000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5118000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5119000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5120000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5121000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5122000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5123000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5124000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5125000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5126000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5127000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5128000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5129000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5130000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5131000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5132000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5133000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5134000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5135000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5136000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5137000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5138000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5139000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5140000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5141000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5142000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5143000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5144000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5145000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5146000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5147000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5148000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5149000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.5150000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
0511000000000	[illegible]	0511200000000	[illegible]	[illegible]	[illegible]	[illegible]	0498799995e8
0511100000000		0509988713f8	-4993775554f9	-4920841087a4	0479483988968	0482863388890	0469041582e3
0511200000000		0509955513c1	-4941691791f1	-4901750114f6	0481792183e5	0491807183f1	0469249139e8
0511300000000		0509905522f2	-4936157374f5	-4901469289f7	0484060481620	0492791369f4	0469831442e6
0511400000000		0509884104401	-4937118741e6	-4901262459e4	0487286077e4	0493A62616f1	0469594067e1
0511500000000		0509876434f7e	-4937425730e0	-4901096491a4	0491145812e2	0494A69e3841	0469741029e8
0511600000000		0509867e57a6a	-4933222731	-4900921694e8	0491A60442e3	0495A1095f33A	0469875192e1
0511700000000		0509857e20e3f	-4931023796e6	-4900527728f4	0492774287f4	0496A3495e5f	0469994857ef3
0511800000000		0509847180e69	-4931094512e	-4900764754f6	0492987e49a3	0497A4000187	0469101127969
0511900000000		0509835778e5	-4931174354e6	-4900690422f2	0493A023e4e3	0498A5e5f10f	0469102190763
0512000000000		0509826e2130	-4931243037e8	-4900631527e40	0494719348f9	0499A43e8467	0469103183815
0512100000000		0509819e5a05	-4931302721e4	-4900564063f03	0495736593e5	0501072e0750	0469104114e46
0512200000000		0509797e41a97	-4931356184e9	-4900546647e42	0496e6366480	0501176e6952	0469104989e44
0512300000000		0509783782e9f	-4931413531f7	-4900518365f09	0498092e8865	0501381e7222	0469105814e96
0512400000000		0509769e2a64	-4931469e6e39	-4900497371e26	0499427e7611	0501498181e8	0469106590e97
0512500000000		0509758512e4e	-4931511944f2	-4900483849e38	0501e866e6e4	0501499959e09	0469107322e47
0512600000000		0509739147e4e	-4931560e5e4e	-4900476988e23	0501e941e6131	0501502e127e	0469108015116
0512700000000		0509723328e9e	-4931607534f9	-4900476e4e76	0501e7e48159	0501711258e7	0469108466e9e7
0512800000000		0509707515e6f	-4931655e2f2	-4900481764e53	0501e8413e14	0501812013137	0469109276e94
0512900000000		0509690229e1e	-4931700e6e63	-4900497777e72	0501977155e92	0501909979951	0469109849e10
0513000000000		0509672929e8f	-4931754311e3	-4900500e5e77	0502107011e47	0502079e81779	0469110379e98
0513100000000		0509655180e0e	-4931804284e9e	-4900532818e37	0502179e6e47	0502189e5189	0469110871e43
0513200000000		0509636796e7e	-4931861073f3	-4900561590e79	0502280e43e91	0502394194e82	0469111312e16
0513300000000		0509617994e1e	-4931914e75e2	-4900595059e2	0502431e1e4e	0502597317772	0469111714e95
0513400000000		0509498e8e11	-4931986e5e44	-4900637e1187	0502774e8e69	0502e44e95742	0469112053e94
0513500000000		0509478324e2e	-4932044e5374	-4900681227e38	0503128e81e9	0503299799289	0469112447e48
0513600000000		0509457e40e5e	-4932118e6e95	-4900741762e02	0503599435e91	0503796e43159	0469112560e94
0513700000000		0509439e01e9e	-4932193e3e82	-4900547770e79	0503647e9637e	0503892228539	0469112708e41
0513800000000		0509413570e8e	-4932276e51e9	-4900787107e84	0503958e1e91	0503e95e02139	0469112771e40
0513900000000		0509392e8e81e	-4932371e3e17	-4900907377e30	0504359737e53	0503e07e80784	0469112738e85
0514000000000		0509366e7e37e	-4932471e7e81	-4901094708e06	0504e96837210	0503316e46157	0469112592e43
0514100000000		0509340813e4e	-4932582e7e91	-4901161759e30	0504e90c68e3	0503972e5211	0469112315e91
0514200000000		0509319385e1e	-4932706e5e7a	-4901284e10100	0505222e5772	0503955e4118	0469111883e85
0514300000000		0509286674e0e	-4932846e09e4e	-4901424728e3	0505967e7219	0503946e47081	0469111270e94
0514400000000		0509257530e6e	-4932999e5277e	-4901587594e6	0505e22e5e84	0503e67e80957	0469110430e14
0514500000000		0509226e8e4e2e	-4933154e52e80	-4901778274e50	0506388e4130	0503717e8509	0469109352e13
0514600000000		0509194e90e11	-4933347374e1	-4902004714e13	0506e4e97164e1	0503e2611e11	0469107952e45
0514700000000		0509159778e1e	-4933561e0779	-4902278e8e02	0507e53761e15	0503e93e9569e	0469106180e12
0514800000000		0509122948e1e2	-4933e0e5e0e8e	-4902e02e3e80	0507e5[illegible]	[illegible]	[illegible]
0514900000000		0509085e48e79	-4934e08e4e4e1	-4903e0e173e43	0507e61e9e9e	0503e14112e69	0469101181e13
0515000000000		0509041e9e7e0	-4934e2522e51	-4903e6738e56	0508784e328e	0503e94e46870	0469774978e4
0515100000000		0509e48975f7e	-4934e2e44e4e	-4904e5347718	0508726e36e24	0503e93e3e4e	0469137467e9

ξ	a	$\phi_{\parallel}$	$\phi'_{\parallel}$	$\phi''_{\parallel}$	$\psi_{\parallel}$	$\psi'_{\parallel}$	$\psi''_{\parallel}$
0.0000000000	1.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0100000000		0.0049999583	0.0099983333	-0.0149962500	0.0016666250	0.0049983333	0.0099950000
0.0200000000		0.0099986667	0.0199933333	-0.0299850000	0.0066665000	0.0199866667	0.0399733333
0.0300000000		0.0149962500	0.0299850000	-0.0449662500	0.0149962500	0.0449662500	0.0899100000
0.0400000000		0.0199933333	0.0399733333	-0.0599400000	0.0266533333	0.0799200000	0.1598133333
0.0500000000		0.0249895833	0.0499583333	-0.0749062500	0.0416562500	0.1248437500	0.2496875000
0.0600000000		0.0299850000	0.0599400000	-0.0898650000	0.0599400000	0.1797300000	0.3594600000
0.0700000000		0.0349795833	0.0699183333	-0.1048162500	0.0815562500	0.2445937500	0.4891875000
0.0800000000		0.0399733333	0.0798933333	-0.1197600000	0.1065600000	0.3196800000	0.6393600000
0.0900000000		0.0449662500	0.0898650000	-0.1346962500	0.1349662500	0.4048987500	0.8097975000
0.1000000000		0.0499583333	0.0998333333	-0.1496250000	0.1666250000	0.4998750000	0.9997500000
0.1100000000		0.0549495833	0.1098000000	-0.1645462500	0.2015062500	0.6046687500	1.2093375000
0.1200000000		0.0599400000	0.1197600000	-0.1794600000	0.2397600000	0.7192800000	1.4385600000
0.1300000000		0.0649295833	0.1297183333	-0.1943662500	0.2813062500	0.8439187500	1.6878375000
0.1400000000		0.0699183333	0.1396733333	-0.2092650000	0.3261150000	0.9784350000	1.9568700000
0.1500000000		0.0749062500	0.1496250000	-0.2241562500	0.3741562500	1.1228437500	2.2456875000
0.1600000000		0.0798933333	0.1595733333	-0.2390400000	0.4254000000	1.2771200000	2.5542400000
0.1700000000		0.0848795833	0.1695183333	-0.2539162500	0.4798062500	1.4412187500	2.8824375000
0.1800000000		0.0898650000	0.1794600000	-0.2687850000	0.5374350000	1.6151850000	3.2303700000
0.1900000000		0.0948495833	0.1893983333	-0.2836462500	0.5982562500	1.7990437500	3.5980375000
0.2000000000		0.0998333333	0.1993333333	-0.2985000000	0.6622800000	1.9927200000	3.9854400000
0.2100000000		0.1048162500	0.2092650000	-0.3133462500	0.7294962500	2.1962812500	4.3925775000
0.2200000000		0.1097983333	0.2191933333	-0.3281850000	0.7999050000	2.4097150000	4.8194500000
0.2300000000		0.1147795833	0.2291183333	-0.3430162500	0.8735062500	2.6330187500	5.2660575000
0.2400000000		0.1197600000	0.2390400000	-0.3578400000	0.9502800000	2.8662000000	5.7324000000
0.2500000000		0.1247395833	0.2489583333	-0.3726562500	1.0302562500	3.1092812500	6.2185775000
0.2600000000		0.1297183333	0.2588733333	-0.3874650000	1.1134350000	3.3622650000	6.7245900000
0.2700000000		0.1346962500	0.2687850000	-0.4022662500	1.1998162500	3.6251587500	7.2504375000
0.2800000000		0.1396733333	0.2786933333	-0.4170600000	1.2894000000	3.8979600000	7.7961200000
0.2900000000		0.1446495833	0.2885983333	-0.4318462500	1.3821862500	4.1806687500	8.3616375000
0.3000000000		0.1496250000	0.2985000000	-0.4466250000	1.4781750000	4.4732850000	8.9470000000
0.3100000000		0.1545995833	0.3083983333	-0.4613962500	1.5773662500	4.7758187500	9.5522175000
0.3200000000		0.1595733333	0.3182933333	-0.4761600000	1.6797600000	5.0882800000	10.1772800000
0.3300000000		0.1645462500	0.3281850000	-0.4909162500	1.7853562500	5.4106687500	10.8221775000
0.3400000000		0.1695183333	0.3380733333	-0.5056650000	1.8941550000	5.7429850000	11.4869100000
0.3500000000		0.1744895833	0.3479583333	-0.5204062500	2.0061562500	6.0852287500	12.1714775000
0.3600000000		0.1794600000	0.3578400000	-0.5351400000	2.1213600000	6.4374000000	12.8758800000
0.3700000000		0.1844295833	0.3677183333	-0.5498662500	2.2397662500	6.7994987500	13.6001175000
0.3800000000		0.1893983333	0.3775933333	-0.5645850000	2.3613750000	7.1715250000	14.3441900000
0.3900000000		0.1943662500	0.3874650000	-0.5792962500	2.4861862500	7.5534787500	15.1080975000
0.4000000000		0.1993333333	0.3973333333	-0.5940000000	2.6142000000	7.9453600000	15.8918400000
0.4100000000		0.2042995833	0.4071983333	-0.6086962500	2.7454162500	8.3471687500	16.6954175000
0.4200000000		0.2092650000	0.4170600000	-0.6233850000	2.8798350000	8.7589050000	17.5188300000
0.4300000000		0.2142295833	0.4269183333	-0.6380662500	3.0174562500	9.1805687500	18.3620775000
0.4400000000		0.2191933333	0.4367733333	-0.6527400000	3.1582800000	9.6121600000	19.2251600000	
0.4500000000		0.2241562500	0.4466250000	-0.6674062500	3.3023062500	10.0536787500	20.1080775000
0.4600000000		0.2291183333	0.4564733333	-0.6820650000	3.4495350000	10.5051250000	21.0108300000
0.4700000000		0.2340795833	0.4663183333	-0.6967162500	3.5999662500	10.9664987500	21.9334175000
0.4800000000		0.2390400000	0.4761600000	-0.7113600000	3.7536000000	11.4378000000	22.8758400000
0.4900000000		0.2439995833	0.4859983333	-0.7259962500	3.9104362500	11.9190287500	23.8380975000
0.5000000000		0.2489583333	0.4958333333	-0.7406250000	4.0704750000	12.4101850000	24.8201900000
0.5100000000		0.2539162500	0.5056650000	-0.7552462500	4.2337162500	12.9112687500	25.8221175000
0.5200000000		0.2588733333	0.5154933333	-0.7698600000	4.4001600000	13.4222800000	26.8438800000
0.5300000000		0.2638295833	0.5253183333	-0.7844662500	4.5698062500	13.9432187500	27.8854775000
0.5400000000		0.2687850000	0.5351400000	-0.7990650000	4.7426550000	14.4740850000	28.9469100000
0.5500000000		0.2737395833	0.5449583333	-0.8136562500	4.9187062500	15.0148787500	30.0281775000
0.5600000000		0.2786933333	0.5547733333	-0.8282400000	5.0979600000	15.5656000000	31.1292800000
0.5700000000		0.2836462500	0.5645850000	-0.8428162500	5.2804162500	16.1262487500	32.2502175000
0.5800000000		0.2885983333	0.5743933333	-0.8573850000	5.4660750000	16.6968250000	33.3909900000
0.5900000000		0.2935495833	0.5841983333	-0.8719462500	5.6549362500	17.2773287500	34.5515975000
0.6000000000		0.2984999583	0.5939999583	-0.8864999583	5.8469999583	17.8677599583	35.7320399583
0.6100000000		0.3034495833	0.6037983333	-0.9010462500	6.0422662500	18.4681187500	36.9323175000
0.6200000000		0.3083983333	0.6135933333	-0.9155850000	6.2407350000	19.0784050000	38.1524300000
0.6300000000		0.3133462500	0.6233850000	-0.9301162500	6.4424062500	19.6986187500	39.3923775000
0.6400000000		0.3182933333	0.6331733333	-0.9446400000	6.6472800000	20.3287600000	40.6521600000
0.6500000000		0.3232395833	0.6429583333	-0.9591562500	6.8553562500	20.9688287500	41.9317775000
0.6600000000		0.3281850000	0.6527400000	-0.9736650000	7.0666350000	21.6188250000	43.2312300000
0.6700000000		0.3331295833	0.6625183333	-0.9881662500	7.2811162500	22.2787487500	44.5505175000
0.6800000000		0.3380733333	0.6722933333	-1.0026600000	7.4988000000	22.9486000000	45.8896400000
0.6900000000		0.3430162500	0.6820650000	-1.0171462500	7.7196862500	23.6283787500	47.2485975000
0.7000000000		0.3479583333	0.6918333333	-1.0316250000	7.9437750000	24.3180850000	48.6273900000
0.7100000000		0.3528995833	0.7015983333	-1.0460962500	8.1710662500	25.0177187500	50.0260175000
0.7200000000		0.3578400000	0.7113600000	-1.0605600000	8.4015600000	25.7272800000	51.4444800000
0.7300000000		0.3627795833	0.7211183333	-1.0750162500	8.6352562500	26.4467687500	52.8827775000
0.7400000000		0.3677183333	0.7308733333	-1.0894650000	8.8721550000	27.1761850000	54.3409100000
0.7500000000		0.3726562500	0.7406250000	-1.1039062500	9.1122562500	27.9155287500	55.8188775000
0.7600000000		0.3775933333	0.7503733333	-1.1183400000	9.3555600000	28.6648000000	57.3166800000
0.7700000000		0.3825295833	0.7601183333	-1.1327662500	9.6020662500	29.4239987500	58.8343175000
0.7800000000		0.3874650000	0.7698600000	-1.1471850000	9.8517750000	30.1931250000	60.3717900000
0.7900000000		0.3923995833	0.7795983333	-1.1615962500	10.1046862500	30.9721787500	61.9290975000
0.8000000000		0.3973333333	0.7893333333	-1.1760000000	10.3608000000	31.7611600000	63.5062400000
0.8100000000		0.4022662500	0.7990650000	-1.1903962500	10.6201162500	32.5600687500	65.1032175000
0.8200000000		0.4071983333	0.8087933333	-1.2047850000	10.8826350000	33.3689050000	66.7200300000
0.8300000000		0.4121295833	0.8185183333	-1.2191662500	11.1483562500	34.1876687500	68.3566775000
0.8400000000		0.4170600000	0.8282400000	-1.2335400000	11.4172800000	35.0163600000	70.0131600000
0.8500000000		0.4219895833	0.8379583333	-1.2479062500	11.6894062500	35.8549787500	71.6894775000
0.8600000000		0.4269183333	0.8476733333	-1.2622650000	11.9647350000	36.7035250000	73.3856300000
0.8700000000		0.4318462500	0.8573850000	-1.2766162500	12.2432662500	37.5619987500	75.1016175000
0.8800000000		0.4367733333	0.8670933333	-1.2909600000	12.5250000000	38.4304000000	76.8374400000
0.8900000000		0.4416995833	0.8767983333	-1.3052962500	12.8099362500	39.3087287500	78.5930975000
0.9000000000		0.4466250000	0.8865000000	-1.3196250000	13.0980750000	40.1969850000	80.3685900000
0.9100000000		0.4515495833	0.8961983333	-1.3339462500	13.3894162500	41.0951687500	82.1639175000
0.9200000000		0.4564733333	0.9058933333	-1.3482600000	13.6839600000	42.0032800000	83.9790800000
0.9300000000		0.4613962500	0.9155850000	-1.3625662500	13.9817062500	42.9213187500	85.8140775000
0.9400000000		0.4663183333	0.9252733333	-1.3768650000	14.2826550000	43.8492850000	87.6689100000
0.9500000000		0.4712395833	0.9349583333	-1.3911562500	14.5868062500	44.7871787500	89.5435775000
0.9600000000		0.4761600000	0.9446400000	-1.4054400000	14.8941600000	45.7350000000	91.4380800000
0.9700000000		0.4810795833	0.9543183333	-1.4197162500	15.2047162500	46.6927487500	93.3524175000
0.9800000000		0.4859983333	0.9639933333	-1.4339850000	15.5184750000	47.6604250000	95.2865900000
0.9900000000		0.4909162500	0.9736650000	-1.4482462500	15.8354362500	48.6380287500	97.2405975000
1.0000000000		0.4958333333	0.9833333333	-1.4625000000	16.1556000000	49.6255600000	99.2144400000

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
0.0		0.0	0.0	0.0	0.0	0.0	0.0
0.1		0.0998...	-0.4922...	-0.0204...	0.0474...	0.0...	0.0...
0.2		0.0995...	-0.0941...	-0.0173...	0.0918...	0.0...	0.0...
0.3		0.0986...	-0.1457...	-0.0146...	0.0841...	0.0...	0.0...
0.4		0.0973...	-0.1946...	-0.0126...	0.0737...	0.0...	0.0...
0.5		0.0955...	-0.2403...	-0.0110...	0.0611...	0.0...	0.0...
0.6		0.0933...	-0.2825...	-0.0098...	0.0468...	0.0...	0.0...
0.7		0.0907...	-0.3210...	-0.0089...	0.0312...	0.0...	0.0...
0.8		0.0878...	-0.3557...	-0.0081...	0.0147...	0.0...	0.0...
0.9		0.0845...	-0.3866...	-0.0074...	-0.0026...	0.0...	0.0...
1.0		0.0809...	-0.4137...	-0.0067...	-0.0196...	0.0...	0.0...
1.1		0.0770...	-0.4373...	-0.0059...	-0.0358...	0.0...	0.0...
1.2		0.0728...	-0.4576...	-0.0049...	-0.0506...	0.0...	0.0...
1.3		0.0683...	-0.4750...	-0.0036...	-0.0634...	0.0...	0.0...
1.4		0.0636...	-0.4899...	-0.0020...	-0.0737...	0.0...	0.0...
1.5		0.0587...	-0.5027...	0.0000...	-0.0810...	0.0...	0.0...
1.6		0.0536...	-0.5138...	0.0026...	-0.0850...	0.0...	0.0...
1.7		0.0484...	-0.5236...	0.0059...	-0.0855...	0.0...	0.0...
1.8		0.0431...	-0.5324...	0.0098...	-0.0825...	0.0...	0.0...
1.9		0.0377...	-0.5405...	0.0143...	-0.0762...	0.0...	0.0...
2.0		0.0323...	-0.5480...	0.0193...	-0.0670...	0.0...	0.0...
2.1		0.0268...	-0.5551...	0.0247...	-0.0554...	0.0...	0.0...
2.2		0.0213...	-0.5618...	0.0304...	-0.0420...	0.0...	0.0...
2.3		0.0158...	-0.5681...	0.0363...	-0.0274...	0.0...	0.0...
2.4		0.0103...	-0.5738...	0.0422...	-0.0123...	0.0...	0.0...
2.5		0.0048...	-0.5787...	0.0479...	0.0026...	0.0...	0.0...
2.6		-0.0006...	-0.5826...	0.0531...	0.0170...	0.0...	0.0...
2.7		-0.0060...	-0.5853...	0.0575...	0.0305...	0.0...	0.0...
2.8		-0.0113...	-0.5865...	0.0608...	0.0426...	0.0...	0.0...
2.9		-0.0165...	-0.5858...	0.0625...	0.0527...	0.0...	0.0...
3.0		-0.0216...	-0.5829...	0.0621...	0.0602...	0.0...	0.0...
3.1		-0.0265...	-0.5773...	0.0592...	0.0646...	0.0...	0.0...
3.2		-0.0312...	-0.5686...	0.0534...	0.0654...	0.0...	0.0...
3.3		-0.0356...	-0.5563...	0.0443...	0.0622...	0.0...	0.0...
3.4		-0.0396...	-0.5398...	0.0317...	0.0547...	0.0...	0.0...
3.5		-0.0432...	-0.5186...	0.0154...	0.0427...	0.0...	0.0...
3.6		-0.0463...	-0.4921...	-0.0047...	0.0262...	0.0...	0.0...
3.7		-0.0488...	-0.4595...	-0.0299...	0.0052...	0.0...	0.0...
3.8		-0.0507...	-0.4199...	-0.0611...	-0.0208...	0.0...	0.0...
3.9		-0.0519...	-0.3723...	-0.0987...	-0.0516...	0.0...	0.0...
4.0		-0.0523...	-0.3154...	-0.1432...	-0.0875...	0.0...	0.0...
4.1		-0.0519...	-0.2477...	-0.1952...	-0.1277...	0.0...	0.0...
4.2		-0.0505...	-0.1674...	-0.2551...	-0.1710...	0.0...	0.0...
4.3		-0.0480...	-0.0723...	-0.3230...	-0.2151...	0.0...	0.0...
4.4		-0.0443...	0.0397...	-0.3990...	-0.2581...	0.0...	0.0...
4.5		-0.0393...	0.1615...	-0.4829...	-0.2977...	0.0...	0.0...
4.6		-0.0329...	0.2997...	-0.5742...	-0.3312...	0.0...	0.0...
4.7		-0.0250...	0.4538...	-0.6722...	-0.3555...	0.0...	0.0...
4.8		-0.0154...	0.6227...	-0.7757...	-0.3668...	0.0...	0.0...
4.9		-0.0040...	0.8044...	-0.8827...	-0.3601...	0.0...	0.0...
5.0		0.0093...	0.9966...	-0.9901...	-0.3290...	0.0...	0.0...

ξ	α	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
0511000000000	[illegible]	[illegible]	[illegible]	0000000000000	0000000000000	0000000000000	0000000000000
0511100000000		0489996000000	0500000000000	-0492065426455	0481885338900	0478897650400	0489157580411
0511200000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0511300000000		0502994116419	0500992174400	-0494479419955	0476176473994	0488299000000	0492669653411
0511400000000		0500998129788	0500987932000	-0495088059540	0481002754494	0487648853301	0490395044394
0511500000000		0500996590300	0500982000416	-0495537940448	0481085300799	0491914485270	0490505744815
0511600000000		0500994509400	0500976904791	-0495862150411	0483860964863	0491997157300	0490620328549
0511700000000		0500991588128	0500970955300	-0496044844879	0485976040960	0489748600022	0490739962506
0511800000000		0500988375049	0500964538300	-0496177722945	0488824155283	0489394640008	0490864419188
0511900000000		[illegible]	0500958014400	-0496244168433	0490191774304	0489187497241	0490993468078
0512000000000		0500980175078	0500951710956	-0496232183122	0490148831617	0489584815078	0491124851890
0512100000000		0501107503000	0500945977225	-0496144634887	0490276797927	0489444693025	0491268463388
0512200000000		0501114925018	0500938915100	-0496040372968	0490278347457	0489298799734	0491414435895
0512300000000		0501126242080	0500932408475	-0495448899901	0490338041933	0489097721450	0491556318191
0512400000000		0501135574219	0500926089484	-0495057442831	0490485846019	0488889375000	0491672461900
0512500000000		0501144804458	0500915484475	-0494578686468	0490202040544	0488597247313	0491788976680
0512600000000		0501153946820	0500912054192	-0494097017417	0489790333290	0488247040475	0491896222671
0512700000000		0501163042434	0500906140455	-0493642098442	0489486648340	0487864598059	0492024544548
0512800000000		0501172089090	0500899005705	-0493090300479	0489107676460	0487484919594	0492143132118
0512900000000		0501181045878	0500892063970	-0492569428427	0488728117947	0487117215087	0492262926092
0513000000000		0501189979477	0500886447170	-0492075076472	0488414187119	0486744597957	0492383137843
0513100000000		0501198797671	0500876084214	-0491538425098	0488077694804	0486377494182	0492505887912
0513200000000		0501207446948	0500868752487	-0490879308412	0487722809484	0485998695201	0492628450747
0513300000000		0501216111879	0500860118270	-0490291483941	0487385191004	0485597482279	0492753260482
0513400000000		0501224844869	0500850888382	-0489559310843	0487040948174	0485183488836	0492879852398
0513500000000		0501233137269	0500840879405	-0489031000101	0486713673974	0484769503068	0493006299749
0513600000000		0501241443969	0500830222070	-0488429050969	0486373113004	0484376490351	0493135618460
0513700000000		0501249728948	0500818811305	-0487725804999	0486049390104	0483983774304	0493266480619
0513800000000		0501257988078	0500805448574	-0487034951885	0485697984847	0483591193528	0493400538103
0513900000000		0501265877430	0500791851705	-0486494811140	0485359009935	0483209992380	0493536984295
0514000000000		0501273679455	0500775403474	-0486166552054	0485078699360	0482866968881	0493676318643
0514100000000		0501281349100	0500758047494	-0486046504801	0485687510624	0482716392424	0493819967343
0514200000000		0501288827120	0500738212020	-0486210390013	0485722541162	0482782667452	0493967160781
0514300000000		0501296101991	0500715823580	-0486538574842	0485894213215	0482981983490	0494120125240
0514400000000		0501303173979	0500690519581	-0486772270736	0485883105533	0483092914201	0494277932965
0514500000000		0501309896932	0500661187485	-0486912948094	0485898807874	0483184821054	0494438003644
0514600000000		0501316381199	0500627420471	-0486926866782	0485894892964	0483268868377	0494602610988
0514700000000		0501322427074	0500588220405	-0486846632855	0485790446273	0483318468359	0494770210280
0514800000000		0501328082294	0500544239004	-0486509675406	0485638199819	0483198176434	0494941136567840
0514900000000		0501333476339	0500496478494	-0486082720484	0485467224963	0483032297568	0495128462013
0515000000000		0501337773647	0500418896492	-0485794507899	0485191941868	0482894892669	0495344889990
0515100000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
0.5110000000	0.4990000000	0.5110000000	0.0000000000	-0.0050000000	0.0000000000	0.0000000000	0.0489999995A8
0.5111000000		0.5009984831988	-0.0027934988	-0.0020641242S	0.0474840799A0	0.0489126299530	0.0489240737B9
0.5112000000		0.5009953811A1	-0.0941692227A0	-0.0017901849Z	0.0481832915A7	0.0491848255549	0.0499459347B6
0.5113000000		0.5009900021B8	-0.0957575448A5	-0.0014698461B4	0.0488157149B9	0.0492803A733A	0.0499645792B5
0.5114000000		0.5009841436B8	-0.0971181659B3	-0.0012A278150	0.0487846141A3	0.0497996743A7	0.0499812110B8
0.5115000000		0.5009764237B8	-0.0982942410C5	-0.0010997C2741	0.0491171854C8	0.0498765A3207	0.0499962406B0
0.5116000000		0.5009676C31A4	-0.0992327777A1	-0.0009629396B3	0.0491A9817966	0.0499348788A11	0.0500100960B0
0.5117000000		0.5009578316A0	-0.1002394048A8	-0.0008578A1807	0.0492425969A3	0.0499050A2952	0.0501022576A1
0.5118000000		0.5009471734A6	-0.1011078245B0	-0.0007649694S0	0.0493055390A3	0.0497813A2A34	0.0501034255A9
0.5119000000		0.5009356A67A8	-0.1017456258B	-0.0006A2592AA5	0.0493B89U1A12	0.0498883B0A0A0	0.0501045119B0
0.5120000000		0.5009236A3B17	-0.1024980A19	-0.0005A4CC91A7	0.0494B2600A69	0.0499BF3A2679	0.0501055267A6
0.5121000000		0.5009109786A8	-0.1030877A97	-0.0004B7156A21	0.0495B7C03760	0.0501096364A7	0.0501064778B5
0.5122000000		0.5008976C33A8	-0.1036057376	-0.0003550A41A50	0.0497A1903755	0.0501063990A8	0.0501073711A3
0.5123000000		0.5008837377B8	-0.1041A1859A6	-0.0002652A4908	0.0498776A8765	0.0501131A9152	0.0501082119A3
0.5124000000		0.5008686A37A4	-0.1045A6A5F1B3	-0.0001785A4A12	0.0499652A9263	0.0501A1970659	0.0501090035A6
0.5125000000		0.5008544127B8	-0.1049A1A8A7A6	-0.0000A27A493	0.0501111599B5	0.0501599197Z9	0.0501097481A4
0.5126000000		0.5008390A312B	-0.1052A6B6A2B0	-0.0048A50A7912	0.0501270A7280	0.0501A3A1882B	0.0501104516A1
0.5127000000		0.5008231B5B7B	-0.1056A1134A1A	-0.0048A2221A45	0.0501B3044190	0.0501780A7A81	0.0501111112A1
0.5128000000		0.5007A9A13A7B	-0.1059A1705A4A	-0.0050A7520A1	0.0501B1178020	0.0501079A2180	0.0501123047A8
0.5129000000		0.5007725784A1B	-0.1061769534B7	-0.0052A6A4A24	0.0502A1A83270	0.0502C0590161	0.0501128337A3
0.5130000000		0.5007507134B9	-0.1063A1A2A1A3	-0.0054A156A04	0.0502229U2A84	0.0502A0918B2F	0.0501133A75B5
0.5131000000		0.5007362A8709	-0.1064A1A0A1A7	-0.0056A0B0A7A8	0.0502A2A0A2A0	0.0502A0A0A0A0	0.0501137A5A3
0.5132000000		0.5007172A1A0B	-0.1063A1A1A1A1	-0.0057A1A0A1A1	0.0502A1A0A1A1	0.0502A0A0A0A1	0.0501141A42B5
0.5133000000		0.5006970169C8	-0.1062A1A1A1A1	-0.0058A0A1A0A0	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501144A63A3
0.5134000000		0.5006773761B5	-0.1060A1A1A1A1	-0.0059A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501147318A4
0.5135000000		0.5006563254A1B	-0.1057A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501149005A9
0.5136000000		0.5006345A8A1B	-0.1053A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501150126A8
0.5137000000		0.5006119A0208	-0.1049A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501150187B7
0.5138000000		0.5005884617A1B	-0.1044A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501149183A2
0.5139000000		0.5005643A8A7B	-0.1039A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501147096A8
0.5140000000		0.5005396A8A7B	-0.1033A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501144793A2
0.5141000000		0.5005138248A7B	-0.1026A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501141793A2
0.5142000000		0.5004873A8A1B	-0.1019A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501137268A8
0.5143000000		0.5004603A8A1B	-0.1011A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501132463A3
0.5144000000		0.5004327A8A1B	-0.1003A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501126806A3
0.5145000000		0.5004044A8A1B	-0.0994A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501120A36A1
0.5146000000		0.5003756A8A1B	-0.0985A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501113963A1
0.5147000000		0.5003463A8A1B	-0.0975A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501106A39A5
0.5148000000		0.5003165A8A1B	-0.0964A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501098A44A6
0.5149000000		0.5002863A8A1B	-0.0953A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501090A75A2
0.5150000000		0.5002556A8A1B	-0.0941A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501082A1BA8
0.5151000000		0.5002246A8A1B	-0.0929A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501073A1BA8
0.5152000000		0.5001932A8A1B	-0.0916A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501064A1BA8
0.5153000000		0.5001615A8A1B	-0.0903A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501055A1BA8
0.5154000000		0.5001296A8A1B	-0.0889A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501046A1BA8
0.5155000000		0.5000975A8A1B	-0.0875A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501037A1BA8
0.5156000000		0.5000653A8A1B	-0.0860A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501028A1BA8
0.5157000000		0.5000331A8A1B	-0.0845A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501019A1BA8
0.5158000000		0.5000009A8A1B	-0.0829A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501010A1BA8
0.5159000000		0.4999687A8A1B	-0.0813A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0501001A1BA8
0.5160000000		0.4999365A8A1B	-0.0796A1A1A1A1	-0.0060A0A1A0A1	0.0502A0A1A0A1	0.0502A0A0A0A1	0.0500992A1BA8

ξ	α	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.1000000000		-0.4999960000	0.0000000000	-0.4920654206	0.0000000000	0.0000000000	0.0000000000
0.2000000000		-0.4999840000	0.0000000000	-0.4841308411	0.0000000000	0.0000000000	0.0000000000
0.3000000000		-0.4999640000	0.0000000000	-0.4761962617	0.0000000000	0.0000000000	0.0000000000
0.4000000000		-0.4999360000	0.0000000000	-0.4682616822	0.0000000000	0.0000000000	0.0000000000
0.5000000000		-0.4999000000	0.0000000000	-0.4603271028	0.0000000000	0.0000000000	0.0000000000
0.6000000000		-0.4998560000	0.0000000000	-0.4523925234	0.0000000000	0.0000000000	0.0000000000
0.7000000000		-0.4998040000	0.0000000000	-0.4444579439	0.0000000000	0.0000000000	0.0000000000
0.8000000000		-0.4997440000	0.0000000000	-0.4365233645	0.0000000000	0.0000000000	0.0000000000
0.9000000000		-0.4996760000	0.0000000000	-0.4285887850	0.0000000000	0.0000000000	0.0000000000
1.0000000000		-0.4996000000	0.0000000000	-0.4206542056	0.0000000000	0.0000000000	0.0000000000
1.1000000000		-0.4995160000	-0.0000012100	-0.4127196262	0.0000000000	0.0000000000	0.0000000000
1.2000000000		-0.4994240000	-0.0000066880	-0.4047850467	0.0000000000	0.0000000000	0.0000000000
1.3000000000		-0.4993240000	-0.0000092115	-0.3968504673	0.0000000000	0.0000000000	0.0000000000
1.4000000000		-0.4992160000	-0.0000098712	-0.3889158879	0.0000000000	0.0000000000	0.0000000000
1.5000000000		-0.4990960000	-0.0000082063	-0.3809813084	0.0000000000	0.0000000000	0.0000000000
1.6000000000		-0.4989680000	-0.0000076033	-0.3730467290	0.0000000000	0.0000000000	0.0000000000
1.7000000000		-0.4988320000	-0.0000070953	-0.3651121495	0.0000000000	0.0000000000	0.0000000000
1.8000000000		-0.4986880000	-0.0000064344	-0.3571775701	0.0000000000	0.0000000000	0.0000000000
1.9000000000		-0.4985360000	-0.0000058099	-0.3492429907	0.0000000000	0.0000000000	0.0000000000
2.0000000000		-0.4983760000	-0.0000052624	-0.3413084112	0.0000000000	0.0000000000	0.0000000000
2.1000000000		-0.4982080000	-0.0000047072	-0.3333738318	0.0000000000	0.0000000000	0.0000000000
2.2000000000		-0.4980320000	-0.0000041192	-0.3254392523	0.0000000000	0.0000000000	0.0000000000
2.3000000000		-0.4978480000	-0.0000034421	-0.3175046729	0.0000000000	0.0000000000	0.0000000000
2.4000000000		-0.4976560000	-0.0000027487	-0.3095700935	0.0000000000	0.0000000000	0.0000000000
2.5000000000		-0.4974560000	-0.0000019860	-0.3016355140	0.0000000000	0.0000000000	0.0000000000
2.6000000000		-0.4972480000	-0.0000012823	-0.2937009346	0.0000000000	0.0000000000	0.0000000000
2.7000000000		-0.4970320000	-0.0000006036	-0.2857663551	0.0000000000	0.0000000000	0.0000000000
2.8000000000		-0.4968080000	-0.0000000000	-0.2778317757	0.0000000000	0.0000000000	0.0000000000
2.9000000000		-0.4965760000	-0.0000000000	-0.2698971963	0.0000000000	0.0000000000	0.0000000000
3.0000000000		-0.4963360000	-0.0000000000	-0.2619626168	0.0000000000	0.0000000000	0.0000000000
3.1000000000		-0.4960880000	-0.0000000000	-0.2540280374	0.0000000000	0.0000000000	0.0000000000
3.2000000000		-0.4958320000	-0.0000000000	-0.2460934579	0.0000000000	0.0000000000	0.0000000000
3.3000000000		-0.4955680000	-0.0000000000	-0.2381588785	0.0000000000	0.0000000000	0.0000000000
3.4000000000		-0.4952960000	-0.0000000000	-0.2302242991	0.0000000000	0.0000000000	0.0000000000
3.5000000000		-0.4950160000	-0.0000000000	-0.2222897196	0.0000000000	0.0000000000	0.0000000000
3.6000000000		-0.4947280000	-0.0000000000	-0.2143551402	0.0000000000	0.0000000000	0.0000000000
3.7000000000		-0.4944320000	-0.0000000000	-0.2064205607	0.0000000000	0.0000000000	0.0000000000
3.8000000000		-0.4941280000	-0.0000000000	-0.1984859813	0.0000000000	0.0000000000	0.0000000000
3.9000000000		-0.4938160000	-0.0000000000	-0.1905514019	0.0000000000	0.0000000000	0.0000000000
4.0000000000		-0.4934960000	-0.0000000000	-0.1826168224	0.0000000000	0.0000000000	0.0000000000
4.1000000000		-0.4931680000	-0.0000000000	-0.1746822430	0.0000000000	0.0000000000	0.0000000000
4.2000000000		-0.4928320000	-0.0000000000	-0.1667476636	0.0000000000	0.0000000000	0.0000000000
4.3000000000		-0.4924880000	-0.0000000000	-0.1588130841	0.0000000000	0.0000000000	0.0000000000
4.4000000000		-0.4921360000	-0.0000000000	-0.1508785047	0.0000000000	0.0000000000	0.0000000000
4.5000000000		-0.4917760000	-0.0000000000	-0.1429439252	0.0000000000	0.0000000000	0.0000000000
4.6000000000		-0.4914080000	-0.0000000000	-0.1350093458	0.0000000000	0.0000000000	0.0000000000
4.7000000000		-0.4910320000	-0.0000000000	-0.1270747664	0.0000000000	0.0000000000	0.0000000000
4.8000000000		-0.4906480000	-0.0000000000	-0.1191401869	0.0000000000	0.0000000000	0.0000000000
4.9000000000		-0.4902560000	-0.0000000000	-0.1112056075	0.0000000000	0.0000000000	0.0000000000
5.0000000000		-0.4898560000	-0.0000000000	-0.1032710280	0.0000000000	0.0000000000	0.0000000000

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
0511000000000	-.9916000000	0511000000000	0000000000000	-8025000000000	0000000000000	0000000000000	0409099995A3
0511100000000		0506988910A0	-4927756373	-8020641396S	0474771647A3	0489937A9930	0409349818A5
0511200000000		0500955811A0	-4981692529S	-8017302249S	0481853291A4	0491A4879169	0409564451I0
0511300000000		0500990621A0	-4957596A395	-8014695596O	0484903389A4	0497A34A2529	0409752967A7
0511400000000		0500884137774	-4971103818A6	-8012429447S	0487528d7670	0495A1670709	0409921131A1
0511500000000		0500874A93A0	-4978964673A0	-8010972975I	0491184a74AA	0494A1898A80	05A1007309A4
0511600000000		0500967600345	-4995284A291	-8006381108C	0491717A4822	0495A32A8243	05A1021181A9
0511700000000		0500957814A05	-5010291644A4	-8008444582A	0492751613A7	0499A84081565	05A1033936A9
0511800000000		0500947170001	-5011084014A5	-4976572997A	0493A8854071	0497A0C043A31	05A1044743I9
0511900000000		0500935740A3A	-5011768602A	-4969356389S	0493A32423B4	049F8A165918	05A1056725B5
0512000000000		0500923644A30	-5012847194A1	-4963527097O	0494A8042649	0501041764A8	05A1064981A3
0512100000000		0500910944A1A	-5013083064A8	-4958A731954	0495A3524450	0501148A4310	05A1076599217
0512200000000		0500897582234	-5013612709S	-4955237118B	0497A97b23A6	0501316463b0	05A1085616B5
0512300000000		0500883699A3A	-5014145078I9	-4952498779I0	0498A68a7635	0501925A5C7A	05A1094106A2
0512400000000		0500869288470	-5014654A4491	-4950568274O	0499A491996A	0501A35A6B3P	05A1102096A3
0512500000000		0500853A0790	-5015163224A2	-4948936269G1	050112390649	0501A9616094	05A1109606A4
0512600000000		0500838A4884	-5015664A9A70	-4946A466728	0501284179A4	0501A4797461	05A1114694A7
0512700000000		0500823047060	-5016164214A6	-4944872478B	0501855436G1	0501A9A8693	05A1123332A7
0512800000000		0500806679180	-5016636A4092	-4942972095S3	0501A38U97C3	0501A4A02672	05A1129530A3
0512900000000		0500789A59A0	-5017158A016	-4940107387I3	0501A31A9199	0501A9597959	05A1135311A0
0513000000000		0500772396A6A	-5017689A9A0	-4938031181I7	0502A3719185	0502109073904	05A1140596A6
0513100000000		0500754A01224	-5018204A8A0	-4935595A4A4A	0502953759I7	0502993A97B7	05A1145464A9
0513200000000		0500735985A8A	-5018773972I2	-4933783594B4	0502A81A0540	0502998149B9	05A1149740A2
0513300000000		0500716940A0A	-5019303857I6	-4930A17a832	0502721429A7	0502A43A4239	05A115356382
0513400000000		0500697218A2A	-5020028597I3	-4928713316J3	0502872605A0	0502956884239	05A1156A6A91
0513500000000		0500A76884A6A	-5020735A831	-4926237A0960	0502993529102	0502A84A4574	05A1159105B9
0513600000000		0500A55784A84	-5021478A6A85	-4923897A4990	0502A0958165	0502A0A3746A	05A1160778B8
0513700000000		0500A33845A8A	-5022260A799	-4921277593J7	0503795427B6	0502A01677677	05A1161572A8
0513800000000		0500A11274I9	-5023187304A	-4918104A0426	0500A92A9795	0502A92A1785	05A1161334A0
0513900000000		0500987A4391A	-5024161A8A0	-4916019A3995	0500A0201590	0502A9A01062	05A1159918A1
0514000000000		0500962772197	-5025129728A5	-4913066A157	0504722693O4	0502A64A3529	05A1157104A5
0514100000000		0500936945228	-5026A082A03	-4910957559A0	0505A94A84A5	0502A4038632	05A1156706A7
0514200000000		0500909184A8	-5027778229A3	-4913AA01A77	0505990A79234	0502A4592A91	05A1146289A7
0514300000000		0500881582A4A	-5029148511A	-4915175545B	0505754039B0	0502A6998209	05A1137620A0
0514400000000		0500851543A81	-5030781231J	-4916918773J5	0506120635Z4	0502992795A9	05A1126162A9
0514500000000		0500819938A91	-5032541A07	-4918958A482	0506A9856780	0502A3A72699	05A1113674A0
0514600000000		0500786805A1A	-5034589A490	-4921977014B2	0506A8752A44	0502994A9264	05A109243143
0514700000000		0500750781A6A	-5036A82A2A2	-4924300A6A3	0507987509A0	0502A0A39A02F	05A1068423A3
0514800000000		0500712627927	-5038A4119777	-4927855A0A97	0507A4A0A1C0	0502A1A75220	05A1031434A4
0514900000000		0500717A893A	-5040A479A61	-4932A667A31	0508119128B0	0502A9A01023	04A0992717A0
0515000000000		0500727A95A3F	-5038A0930A7A	-4939128847A	0508A5C00854	0502A47A5A50	04A0A07067A5
0515100000000		0500179A79A2	-5050347A0A74	-4948A99A947	0508A9C51795	0502A4A979774	04A0861279A7

ζ	α	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
0.511000000000	0.499100000000	0.000000000000	0.511000000000	0.000000000000	0.000000000000	0.000000000000	0.000000000000
0.511100000000		0.499996000000	0.509988012000	-0.992064543975	0.046191871633	0.047447962760	0.048936336899
0.511200000000		0.501999200000	0.509960056875	-0.993467644852	0.047248429140	0.048144965551	0.049192302248
0.511300000000		0.503998101611	0.509892115715	-0.994429400295	0.047927032613	0.048492587929	0.049295765783
0.511400000000		0.505988120759	0.509784331410	-0.995085850470	0.048102548897	0.048742041704	0.049233921741
0.511500000000		0.507969600440	0.509642005810	-0.995536866967	0.048227946934	0.048912107946	0.049516741670
0.511600000000		0.509940000000	0.509476502342	-0.995943618782	0.048354703855	0.049187709816	0.049834268242
0.511700000000		0.511898193920	0.509287855047	-0.996047567997	0.048570146704	0.049291210077	0.049756990346
0.511800000000		0.513843573868	0.509076433042	-0.996118106147	0.048841345981	0.049399101739	0.049883842948
0.511900000000		0.515774846638	0.508844003965	-0.996270398448	0.049174118770	0.049496159487	0.050101619944
0.512000000000		0.517690172058	0.508591498923	-0.996330702342	0.049172216907	0.049794608887	0.050115387343
0.512100000000		0.519587502560	0.508317466480	-0.996376884330	0.049291894330	0.049949117875	0.050129711494
0.512200000000		0.521466923000	0.508021960085	-0.996420227913	0.049306547312	0.049796236951	0.050144821049
0.512300000000		0.523326281424	0.507703404105	-0.996470350482	0.049316449869	0.049944552338	0.050160149696
0.512400000000		0.525164573480	0.507362000301	-0.996535564917	0.049448708919	0.050111674508	0.050176734485
0.512500000000		0.526980801294	0.506998429395	-0.996623869200	0.049615536720	0.050179186691	0.050193218861
0.512600000000		0.528773534043	0.506612742406	-0.996748241980	0.049795637571	0.050199343189	0.050210850248
0.512700000000		0.530541169580	0.506205028305	-0.996889867432	0.049916839640	0.050199343520	0.050229284583
0.512800000000		0.532282490019	0.505775890971	-0.997100116433	0.050110664933	0.050146238905	0.050248982447
0.512900000000		0.533996719740	0.505324863870	-0.997354633670	0.050190949782	0.050232091289	0.050268161844
0.513000000000		0.535683134180	0.504852927744	-0.997671341180	0.050144562907	0.050299025978	0.050290764581
0.513100000000		0.537340928442	0.504359928442	-0.998059928442	0.050181073929	0.050280181251	0.050312421249
0.513200000000		0.538969239872	0.503845863744	-0.998529239872	0.050216867941	0.050374807079	0.050335991015
0.513300000000		0.540566955785	0.503309330972	-0.999093309972	0.050258344304	0.050374344304	0.050360899596
0.513400000000		0.542131991146	0.502751535492	-0.999765354992	0.050286248988	0.050387773988	0.050387287594
0.513500000000		0.543663770734	0.502171610407	-0.999539671184	0.050859486720A	0.050415728087	0.050415728087
0.513600000000		0.545161419488	0.501569009989	-1.000119009989	0.050834553594	0.050867907350	0.050445917483
0.513700000000		0.546624688837	0.500945629105	-1.000810588480	0.050414310952	0.050714010952	0.050477201991
0.513800000000		0.548052781174	0.500300384247	-1.001590320458	0.050468234471	0.050549497169	0.050511967682
0.513900000000		0.549445784819	0.499633740785	-1.002542599996	0.050627220810	0.050414016558	0.050548980293
0.514000000000		0.550802773869	0.498945772807	-1.003721482671	0.050591651709	0.050491249544	0.050588497893
0.514100000000		0.552123747129	0.498236919759	-1.005193187730	0.050641847124	0.050791378189	0.050633038440
0.514200000000		0.553407705382	0.497507333792	-1.007018037483	0.050798672900	0.050880036780	0.050681579418
0.514300000000		0.554653648500	0.496757104695	-1.009247531286	0.050892174015	0.050799836280	0.050735979819
0.514400000000		0.555860577398	0.495986368091	-1.011928278211	0.050913031510	0.050967368211	0.050796329681
0.514500000000		0.557027493170	0.495195479190	-1.015032589151	0.051101180446	0.051070548750	0.050865673484
0.514600000000		0.558153397801	0.494384899287	-1.018773509160	0.051111958881	0.051120897699	0.050946231884
0.514700000000		0.559237293192	0.493554708425	-1.023247304453	0.051236331846	0.051190213250	0.051042054494
0.514800000000		0.560278179962	0.492705023162	-1.028547239436	0.051396374092	0.051190011178	0.051159575486
0.514900000000		0.561275058743	0.491836023722	-1.034773000430	0.051496374092	0.051190011178	0.051301715084
0.515000000000		0.562226771908	0.490948193482	-1.042086652900	0.051649494299	0.051193147317	0.051474719486
0.515100000000		0.563132371261	0.490042186892	-1.050312616892	0.051782223503	0.051779882480	0.051682729446

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ξ	α	$\phi_{\parallel}$	$\phi'_{\parallel}$	$\phi''_{\parallel}$	$\psi_{\parallel}$	$\psi'_{\parallel}$	$\psi''_{\parallel}$
0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0100000000		0.0099996667	0.0999866733	-0.9920543389	0.0001333333	0.0019946667	0.0298462865
0.0200000000		0.0199973333	0.1999466933	-0.9734867111	0.0010666667	0.0079573333	0.1178191979
0.0300000000		0.0299910000	0.2998650067	-0.9434867111	0.0035999111	0.0178213333	0.2594194158
0.0400000000		0.0399786667	0.3997334933	-0.9034867111	0.0085333333	0.0315733333	0.4484973816
0.0500000000		0.0499583333	0.4995437500	-0.8550867111	0.0166666667	0.0491666667	0.6429900158
0.0600000000		0.0599279333	0.5992731333	-0.8050867111	0.0287999111	0.0705973333	0.8370656858
0.0700000000		0.0698853333	0.6989200533	-0.7550867111	0.0457333333	0.0953333333	1.0277087603
0.0800000000		0.0798293333	0.7984826667	-0.7050867111	0.0682666667	0.1232000000	1.2119238098
0.0900000000		0.0897690000	0.8979594000	-0.6550867111	0.0970666667	0.1538666667	1.3866041384
0.1000000000		0.0997030000	0.9973486667	-0.6050867111	0.1328000000	0.1869333333	1.5488980481
0.1100000000		0.1096300000	1.0966488667	-0.5550867111	0.1765333333	0.2220000000	1.6961400384
0.1200000000		0.1195486667	1.1958488667	-0.5050867111	0.2288000000	0.2586666667	1.8259369548
0.1300000000		0.1294576667	1.2949488667	-0.4550867111	0.2896000000	0.2965333333	1.9362355488
0.1400000000		0.1393556667	1.3939488667	-0.4050867111	0.3594666667	0.3352000000	2.0253139549
0.1500000000		0.1492413333	1.4928488667	-0.3550867111	0.4384000000	0.3742666667	2.0918027365
0.1600000000		0.1591133333	1.5916488667	-0.3050867111	0.5264000000	0.4133333333	2.1347116551
0.1700000000		0.1689703333	1.6903488667	-0.2550867111	0.6234666667	0.4520000000	2.1535113644
0.1800000000		0.1788110000	1.7889488667	-0.2050867111	0.7296000000	0.4898666667	2.1481146209
0.1900000000		0.1886340000	1.8874488667	-0.1550867111	0.8448000000	0.5265333333	2.1190161908
0.2000000000		0.1984380000	1.9858488667	-0.1050867111	0.9690666667	0.5616000000	2.0671817707
0.2100000000		0.2082216667	2.0841488667	-0.0550867111	1.1024000000	0.5946666667	1.9941702698
0.2200000000		0.2179836667	2.1823488667	-0.0050867111	1.2448000000	0.6253333333	1.9021146209
0.2300000000		0.2277226667	2.2804488667	0.0449132889	1.3962666667	0.6532000000	1.7936161908
0.2400000000		0.2374373333	2.3784488667	0.0949132889	1.5568000000	0.6778666667	1.6717817707
0.2500000000		0.2471263333	2.4763488667	0.1449132889	1.7264000000	0.6989333333	1.5402698044
0.2600000000		0.2567883333	2.5741488667	0.1949132889	1.9050666667	0.7160000000	1.4032355488
0.2700000000		0.2664220000	2.6718488667	0.2449132889	2.0928000000	0.7286666667	1.2650369548
0.2800000000		0.2760260000	2.7694488667	0.2949132889	2.2896000000	0.7365333333	1.1300139549
0.2900000000		0.2855990000	2.8669488667	0.3449132889	2.4954666667	0.7392000000	1.0024027365
0.3000000000		0.2951396667	2.9643488667	0.3949132889	2.7104000000	0.7362666667	0.8863102738
0.3100000000		0.3046466667	3.0616488667	0.4449132889	2.9344000000	0.7273333333	0.7855116551
0.3200000000		0.3141186667	3.1588488667	0.4949132889	3.1674666667	0.7120000000	0.7032113644
0.3300000000		0.3235543333	3.2559488667	0.5449132889	3.4096000000	0.6898666667	0.6422146209
0.3400000000		0.3329523333	3.3529488667	0.5949132889	3.6608000000	0.6605333333	0.6048161908
0.3500000000		0.3423113333	3.4498488667	0.6449132889	3.9210666667	0.6234000000	0.5929817707
0.3600000000		0.3516300000	3.5466488667	0.6949132889	4.1904000000	0.5778666667	0.6080355488
0.3700000000		0.3609070000	3.6433488667	0.7449132889	4.4688000000	0.5233333333	0.6508369548
0.3800000000		0.3701410000	3.7399488667	0.7949132889	4.7562666667	0.4592000000	0.7217139549
0.3900000000		0.3793306667	3.8364488667	0.8449132889	5.0528000000	0.3849333333	0.8204027365
0.4000000000		0.3884746667	3.9328488667	0.8949132889	5.3584000000	0.2999333333	0.9463102738
0.4100000000		0.3975716667	4.0291488667	0.9449132889	5.6730666667	0.2036000000	1.0984116551
0.4200000000		0.4066203333	4.1253488667	0.9949132889	5.9968000000	0.0953333333	1.2754113644
0.4300000000		0.4156193333	4.2214488667	1.0449132889	6.3296000000	-0.0253333333	1.4757146209
0.4400000000		0.4245673333	4.3174488667	1.0949132889	6.6714666667	-0.1568000000	1.6976161908
0.4500000000		0.4334630000	4.4133488667	1.1449132889	7.0224000000	-0.2997333333	1.9392817707
0.4600000000		0.4423050000	4.5091488667	1.1949132889	7.3824000000	-0.4541333333	2.1987698044
0.4700000000		0.4510919333	4.6048488667	1.2449132889	7.7514666667	-0.6200000000	2.4729355488
0.4800000000		0.4598225000	4.7004488667	1.2949132889	8.1296000000	-0.7973333333	2.7585369548
0.4900000000		0.4684953333	4.7959488667	1.3449132889	8.5168000000	-0.9861333333	3.0512139549
0.5000000000		0.4771090000	4.8913488667	1.3949132889	8.9130666667	-1.1864000000	3.3455027365
0.5100000000		0.4856621667	4.9866488667	1.4449132889	9.3184000000	-1.3981333333	3.6348102738
0.5200000000		0.4941534000	5.0818488667	1.4949132889	9.7328000000	-1.6213333333	3.9113116551
0.5300000000		0.5025813333	5.1769488667	1.5449132889	10.1562666667	-1.8560000000	4.1661113644
0.5400000000		0.5109446667	5.2719488667	1.5949132889	10.5888000000	-2.1021333333	4.3888146209
0.5500000000		0.5192420000	5.3668488667	1.6449132889	11.0304000000	-2.3597333333	4.5681161908
0.5600000000		0.5274720000	5.4616488667	1.6949132889	11.4810666667	-2.6288000000	4.6917817707
0.5700000000		0.5356333333	5.5563488667	1.7449132889	11.9408000000	-2.9093333333	4.7468698044
0.5800000000		0.5437246667	5.6509488667	1.7949132889	12.4096000000	-3.2013333333	4.7200355488
0.5900000000		0.5517446667	5.7454488667	1.8449132889	12.8874666667	-3.5048000000	4.6000369548
0.6000000000		0.5596920000	5.8398488667	1.8949132889	13.3744000000	-3.8197333333	4.3779139549
0.6100000000		0.5675653333	5.9341488667	1.9449132889	13.8704000000	-4.1461333333	4.0490027365
0.6200000000		0.5753633333	6.0283488667	1.9949132889	14.3754666667	-4.4840000000	3.6134102738
0.6300000000		0.5830846667	6.1224488667	2.0449132889	14.8896000000	-4.8333333333	3.0774116551
0.6400000000		0.5907280000	6.2164488667	2.0949132889	15.4128000000	-5.1941333333	2.4525113644
0.6500000000		0.5982920000	6.3103488667	2.1449132889	15.9450666667	-5.5664000000	1.7538146209
0.6600000000		0.6057753333	6.4041488667	2.1949132889	16.4864000000	-5.9501333333	1.0000161908
0.6700000000		0.6131766667	6.4978488667	2.2449132889	17.0368000000	-6.3453333333	0.2124817707
0.6800000000		0.6204946667	6.5914488667	2.2949132889	17.5962666667	-6.7520000000	-0.5000301956
0.6900000000		0.6277280000	6.6849488667	2.3449132889	18.1648000000	-7.1701333333	-1.1900355488
0.7000000000		0.6348753333	6.7783488667	2.3949132889	18.7424000000	-7.5997333333	-1.8500369548
0.7100000000		0.6419353333	6.8716488667	2.4449132889	19.3290666667	-8.0408000000	-2.4679139549
0.7200000000		0.6489066667	6.9648488667	2.4949132889	19.9248000000	-8.4933333333	-3.0327027365
0.7300000000		0.6557880000	7.0579488667	2.5449132889	20.5296000000	-8.9573333333	-3.5344102738
0.7400000000		0.6625780000	7.1509488667	2.5949132889	21.1434666667	-9.4328000000	-3.9645116551
0.7500000000		0.6692753333	7.2438488667	2.6449132889	21.7664000000	-9.9197333333	-4.3161113644
0.7600000000		0.6758786667	7.3366488667	2.6949132889	22.3984000000	-10.4181333333	-4.5841146209
0.7700000000		0.6823866667	7.4293488667	2.7449132889	23.0394666667	-10.9280000000	-4.7657161908
0.7800000000		0.6887980000	7.5219488667	2.7949132889	23.6896000000	-11.4493333333	-4.8605817707
0.7900000000		0.6951113333	7.6144488667	2.8449132889	24.3488000000	-11.9821333333	-4.8708698044
0.8000000000		0.7013253333	7.7068488667	2.8949132889	25.0170666667	-12.5264000000	-4.8000355488
0.8100000000		0.7074386667	7.7991488667	2.9449132889	25.6944000000	-13.0821333333	-4.6532369548
0.8200000000		0.7134500000	7.8913488667	2.9949132889	26.3808000000	-13.6493333333	-4.4373139549
0.8300000000		0.7193580000	7.9834488667	3.0449132889	27.0762666667	-14.2280000000	-4.1600027365
0.8400000000		0.7251613333	8.0754488667	3.0949132889	27.7808000000	-14.8181333333	-3.8297102738
0.8500000000		0.7308586667	8.1673488667	3.1449132889	28.4944000000	-15.4197333333	-3.4553116551
0.8600000000		0.7364486667	8.2591488667	3.1949132889	29.2170666667	-16.0328000000	-3.0460113644
0.8700000000		0.7419300000	8.3508488667	3.2449132889	29.9488000000	-16.6573333333	-2.6111146209
0.8800000000		0.7473013333	8.4424488667	3.2949132889	30.6896000000	-17.2933333333	-2.1600161908
0.8900000000		0.7525613333	8.5339488667	3.3449132889	31.4394666667	-17.9408000000	-1.7020817707
0.9000000000		0.7577086667	8.6253488667	3.3949132889	32.1984000000	-18.5997333333	-1.2465698044
0.9100000000		0.7627420000	8.7166488667	3.4449132889	32.9664000000	-19.2701333333	-0.8026355488
0.9200000000		0.7676600000	8.8078488667	3.4949132889	33.7434666667	-19.9520000000	-0.3793369548
0.9300000000		0.7724613333	8.8989488667	3.5449132889	34.5296000000	-20.6453333333	0.0143860451
0.9400000000		0.7771446667	8.9899488667	3.5949132889	35.3248000000	-21.3501333333	0.3772872635
0.9500000000		0.7817086667	9.0808488667	3.6449132889	36.1290666667	-22.0664000000	0.7116897262
0.9600000000		0.7861520000	9.1716488667	3.6949132889	36.9424000000	-22.7941333333	1.0139883956
0.9700000000		0.7904733333	9.2623488667	3.7449132889	37.7648000000	-23.5333333333	1.2997139549
0.9800000000		0.7946713333	9.3529488667	3.7949132889	38.5962666667	-24.2840000000	1.5653972635
0.9900000000		0.7987446667	9.4434488667	3.8449132889	39.4368000000	-25.0461333333	1.8085883956
1.0000000000		0.8026920000	9.5338488667	3.8949132889	40.2864000000	-25.8197333333	2.0277102738

ξ	α	ϕ_1	ϕ'_1	ϕ''_1	ψ_1	ψ'_1	ψ''_1
0.5111000000	[illegible]	0.5111000000	0.0000000000	-0.4053000000	0.0000000000	0.0000000000	0.4092999984
0.5111100000		0.5109488100	-0.4922737313	-0.4020841517	0.0474492145	0.0880435806	0.4093555309
0.5112000000		0.5099853114	-0.4941693144	-0.4017307029	0.0481994011	0.0891996866	0.4093774658
0.5113000000		0.5089680000	-0.4957581305	-0.4014997345	0.0484995771	0.0895871291	0.4093967316
0.5114000000		0.5079841000	-0.4971394147	-0.4012632887	0.0487949346	0.0899824383	0.4094101917
0.5115000000		0.5069744000	-0.4983956470	-0.4010076466	0.0491810916	0.0903893678	0.4094294662
0.5116000000		0.5059387000	-0.4995324084	-0.4007310641	0.0495417548	0.0907911138	0.4094436922
0.5117000000		0.5048778000	-0.5005554641	-0.4004283500	0.0498803500	0.0911878350	0.4094569588
0.5118000000		0.5037918000	-0.5014676778	-0.4000965527	0.0501957481	0.0915794676	0.4094694662
0.5119000000		0.5026807000	-0.5022717197	-0.3997326000	0.0504859000	0.0919659000	0.4094812000
0.5120000000		0.5015445000	-0.5029700000	-0.3993333000	0.0507500000	0.0923471000	0.4094921000
0.5121000000		0.5003832000	-0.5035647000	-0.3988956000	0.0509875000	0.0927230000	0.4095021000
0.5122000000		0.4991968000	-0.5040577000	-0.3984163000	0.0511977000	0.0930935000	0.4095113000
0.5123000000		0.4979853000	-0.5044508000	-0.3978917000	0.0513800000	0.0934586000	0.4095196000
0.5124000000		0.4967487000	-0.5047459000	-0.3973179000	0.0515338000	0.0938182000	0.4095271000
0.5125000000		0.4954870000	-0.5049450000	-0.3966908000	0.0516585000	0.0941723000	0.4095338000
0.5126000000		0.4942002000	-0.5050500000	-0.3960060000	0.0517536000	0.0945208000	0.4095397000
0.5127000000		0.4928883000	-0.5050628000	-0.3952589000	0.0518186000	0.0948637000	0.4095449000
0.5128000000		0.4915513000	-0.5049853000	-0.3944447000	0.0518529000	0.0952009000	0.4095494000
0.5129000000		0.4901892000	-0.5048195000	-0.3935583000	0.0518560000	0.0955324000	0.4095532000
0.5130000000		0.4888020000	-0.5045673000	-0.3925944000	0.0518272000	0.0958581000	0.4095563000
0.5131000000		0.4873897000	-0.5042307000	-0.3915474000	0.0517660000	0.0961780000	0.4095588000
0.5132000000		0.4859523000	-0.5038116000	-0.3904115000	0.0516718000	0.0964921000	0.4095607000
0.5133000000		0.4844898000	-0.5033119000	-0.3891806000	0.0515439000	0.0968003000	0.4095621000
0.5134000000		0.4830023000	-0.5027335000	-0.3878484000	0.0513817000	0.0971026000	0.4095630000
0.5135000000		0.4814898000	-0.5020782000	-0.3864083000	0.0511844000	0.0973990000	0.4095635000
0.5136000000		0.4799523000	-0.5013479000	-0.3848534000	0.0509513000	0.0976894000	0.4095636000
0.5137000000		0.4783898000	-0.5005444000	-0.3831762000	0.0506815000	0.0979739000	0.4095634000
0.5138000000		0.4768023000	-0.4996695000	-0.3813690000	0.0503741000	0.0982525000	0.4095630000
0.5139000000		0.4751898000	-0.4987250000	-0.3794239000	0.0500282000	0.0985252000	0.4095624000
0.5140000000		0.4735523000	-0.4977127000	-0.3773327000	0.0496429000	0.0987921000	0.4095617000
0.5141000000		0.4718898000	-0.4966344000	-0.3750870000	0.0492172000	0.0990532000	0.4095610000
0.5142000000		0.4702023000	-0.4954919000	-0.3726781000	0.0487501000	0.0993085000	0.4095604000
0.5143000000		0.4684898000	-0.4942870000	-0.3700971000	0.0482405000	0.0995581000	0.4095600000
0.5144000000		0.4667523000	-0.4930214000	-0.3673351000	0.0476873000	0.0998020000	0.4095599000
0.5145000000		0.4649898000	-0.4916968000	-0.3643830000	0.0470892000	0.1000402000	0.4095602000
0.5146000000		0.4632023000	-0.4903150000	-0.3612315000	0.0464449000	0.1002728000	0.4095610000
0.5147000000		0.4613898000	-0.4888777000	-0.3578710000	0.0457530000	0.1004998000	0.4095624000
0.5148000000		0.4595523000	-0.4873867000	-0.3542917000	0.0450119000	0.1007212000	0.4095645000
0.5149000000		0.4576898000	-0.4858437000	-0.3504837000	0.0442200000	0.1009370000	0.4095674000
0.5150000000		0.4558023000	-0.4842504000	-0.3464369000	0.0433755000	0.1011473000	0.4095712000
0.5151000000		0.4538898000	-0.4826086000	-0.3421411000	0.0424766000	0.1013520000	0.4095760000
0.5152000000		0.4519523000	-0.4809199000	-0.3375854000	0.0415214000	0.1015512000	0.4095819000
0.5153000000		0.4499898000	-0.4791861000	-0.3327585000	0.0405080000	0.1017449000	0.4095891000
0.5154000000		0.4480023000	-0.4774088000	-0.3276484000	0.0394343000	0.1019331000	0.4095978000
0.5155000000		0.4459898000	-0.4755896000	-0.3222424000	0.0382981000	0.1021158000	0.4096081000
0.5156000000		0.4439523000	-0.4737302000	-0.3165274000	0.0370971000	0.1022931000	0.4096202000
0.5157000000		0.4418898000	-0.4718322000	-0.3104900000	0.0358289000	0.1024650000	0.4096343000
0.5158000000		0.4398023000	-0.4698972000	-0.3041164000	0.0344910000	0.1026315000	0.4096506000
0.5159000000		0.4376898000	-0.4679268000	-0.2973923000	0.0330808000	0.1027926000	0.4096694000
0.5160000000		0.4355523000	-0.4659225000	-0.2903030000	0.0315955000	0.1029484000	0.4096910000
0.5161000000		0.4333898000	-0.4638858000	-0.2828333000	0.0300322000	0.1030988000	0.4097157000
0.5162000000		0.4312023000	-0.4618183000	-0.2749675000	0.0283879000	0.1032439000	0.4097440000
0.5163000000		0.4289898000	-0.4597214000	-0.2666894000	0.0266594000	0.1033836000	0.4097762000
0.5164000000		0.4267523000	-0.4575966000	-0.2579821000	0.0248433000	0.1035179000	0.4098128000
0.5165000000		0.4244898000	-0.4554454000	-0.2488284000	0.0229358000	0.1036468000	0.4098543000
0.5166000000		0.4222023000	-0.4532693000	-0.2392104000	0.0209329000	0.1037702000	0.4099013000
0.5167000000		0.4198898000	-0.4510698000	-0.2291102000	0.0188303000	0.1038881000	0.4099544000
0.5168000000		0.4175523000	-0.4488484000	-0.2185100000	0.0166234000	0.1040004000	0.4100142000
0.5169000000		0.4151898000	-0.4466066000	-0.2073911000	0.0143073000	0.1041071000	0.4100814000
0.5170000000		0.4128023000	-0.4443459000	-0.1957346000	0.0118767000	0.1042080000	0.4101567000
0.5171000000		0.4103898000	-0.4420677000	-0.1835211000	0.0093259000	0.1043031000	0.4102410000
0.5172000000		0.4079523000	-0.4397734000	-0.1707304000	0.0066488000	0.1043923000	0.4103353000
0.5173000000		0.4054898000	-0.4374645000	-0.1573412000	0.0038387000	0.1044755000	0.4104407000
0.5174000000		0.4030023000	-0.4351423000	-0.1433312000	0.0008881000	0.1045526000	0.4105584000
0.5175000000		0.4004898000	-0.4328083000	-0.1286771000	-0.0022104000	0.1046235000	0.4106899000
0.5176000000		0.3979523000	-0.4304638000	-0.1133545000	-0.0054000000	0.1046880000	0.4108369000
0.5177000000		0.3953898000	-0.4281102000	-0.0973376000	-0.0087284000	0.1047459000	0.4110012000
0.5178000000		0.3928023000	-0.4257490000	-0.0805990000	-0.0122000000	0.1047970000	0.4111850000
0.5179000000		0.3901898000	-0.4233816000	-0.0631096000	-0.0158191000	0.1048411000	0.4113907000
0.5180000000		0.3875523000	-0.4210095000	-0.0448390000	-0.0196000000	0.1048778000	0.4116213000
0.5181000000		0.3848898000	-0.4186341000	-0.0257553000	-0.0235577000	0.1049068000	0.4118805000
0.5182000000		0.3822023000	-0.4162569000	-0.0058254000	-0.0277000000	0.1049276000	0.4121726000
0.5183000000		0.3794898000	-0.4138793000	0.0149844000	-0.0320400000	0.1049396000	0.4125027000
0.5184000000		0.3767523000	-0.4115028000	0.0366000000	-0.0366000000	0.1049421000	0.4128772000
0.5185000000		0.3739898000	-0.4091287000	0.0589000000	-0.0414000000	0.1049341000	0.4133040000
0.5186000000		0.3712023000	-0.4067586000	0.0819000000	-0.0464000000	0.1049144000	0.4137930000
0.5187000000		0.3683898000	-0.4043940000	0.1057000000	-0.0516000000	0.1048817000	0.4143564000
0.5188000000		0.3655523000	-0.4020364000	0.1303000000	-0.0571000000	0.1048345000	0.4150090000
0.5189000000		0.3626898000	-0.3996874000	0.1557000000	-0.0629000000	0.1047710000	0.4157690000
0.5190000000		0.3598023000	-0.3973484000	0.1819000000	-0.0690000000	0.1046892000	0.4166581000
0.5191000000		0.3568898000	-0.3950209000	0.2089000000	-0.0754000000	0.1045867000	0.4177031000

ξ	α	$\Phi_{\parallel}$	$\Phi'_{\parallel}$	$\Phi''_{\parallel}$	$\Psi_{\parallel}$	$\Psi'_{\parallel}$	$\Psi''_{\parallel}$
0.5110000000	0.4894000000	0.0500000000	0.5110000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.5111000000		0.4999960000	0.5099881210	-0.0000543007	0.0481332000	0.0474981.9789	0.0489561.5710
0.5112000000		0.5001997240	0.5099960000	-0.0003688000	0.0481.75000	0.0481.82000	0.0481962.8600
0.5113000000		0.5003981410	0.5099021.5500	-0.0004429590	0.0474981.7000	0.0484300.5350	0.0483022.6600
0.5114000000		0.5005981.0700	0.5099873.1100	-0.0005088.7000	0.0481.0470.2300	0.0487922.8700	0.0484127.9900
0.5115000000		0.5007965.0900	0.5099820.4400	-0.0005599.1.300	0.0432022.4700	0.0491.82.0.7500	0.0485280.9000
0.5116000000		0.5009945.0900	0.5099792.3900	-0.0006045.4900	0.0433428.0000	0.0491887.2900	0.0485.48.0700
0.5117000000		0.5011918.2900	0.5099709.4600	-0.0006505.0175	0.0485827.4.9.45	0.0492967.2.2.29	0.0487732.7.95
0.5118000000		0.5013885.2.14	0.5099692.2.00	-0.0006.8.0.5.25	0.0484907.5.0.7	0.0493405.1.8.5	0.0490.3.6.6.9.7
0.5119000000		0.5015846.4.8.7	0.5099749.0.10	-0.0006276.7.08	0.0481.0.8.9.2.2	0.0493.7.5.9.2.0	0.0501.0.3.5.3.1.1
0.5120000000		0.5017801.4.0.5.0	0.5099514.7.8.2	-0.0006.3.1.7.7.3.5	0.0485.8.4.1.7.0.0	0.0485.8.4.1.7.0	0.0501.1.7.9.2.2.8.3
0.5121000000		0.5019750.1.0.8	0.5099459.1.4.3.0	-0.0006.3.8.9.4.0.0.0	0.0482.6.0.9.7.7.2.1	0.0495.7.7.0.2.8.4.0	0.0501.3.2.8.6.1.4.9
0.5122000000		0.5021.64.3.2.8.4.7	0.5099.38.99.4.8.2	-0.0006.4.3.7.0.8.2.2	0.0483.1.2.1.9.8.4.2	0.0498.1.3.7.9.4.1.0	0.0501.4.7.7.9.8.2.4.8
0.5123000000		0.5023.4.2.7.9.8.3.1	0.5099.2.8.3.8.1.0.5	-0.0006.4.9.2.6.1.4.5.0	0.0484.0.0.2.5.9.5.4.9	0.0499.4.0.4.0.8.7.6.0	0.0501.6.3.6.6.7.3.1.9
0.5124000000		0.5025.3.6.5.7.1.4.9.7	0.5099.2.5.6.0.8.0.4.9	-0.0006.5.6.4.4.8.9.9.0	0.0485.0.5.6.0.1.7.4.0	0.0501.0.1.1.2.8.5.1.8	0.0501.8.0.3.0.6.7.9.7
0.5125000000		0.5027.2.8.7.0.8.4.8	0.5099.1.9.0.1.4.5.0	-0.0006.6.0.7.1.9.7.4	0.0486.3.9.0.7.2.7.0.0	0.0501.8.3.0.0.6.0.0.7	0.0501.9.7.1.6.0.6.9.4
0.5126000000		0.5029.1.5.7.4.8.7.1.5.7	0.5099.0.1.2.7.4.0.8.9	-0.0006.7.8.8.3.1.0.3	0.0479.7.2.2.4.1.8.9.7	0.0501.9.7.5.4.5.7.8.8	0.0502.1.5.4.7.7.4.2.7
0.5127000000		0.5031.0.7.8.0.1.7	0.5099.0.5.7.1.0.3.4.5	-0.0006.9.5.6.6.1.4.9.3	0.0494.7.6.9.4.6.0.4.4	0.0501.9.4.1.2.8.3.5.3	0.0502.3.9.1.3.8.0.1.1
0.5128000000		0.5033.0.1.7.2.0.7.3.9	0.5099.8.4.4.8.1.7.0.0	-0.0007.1.7.1.4.8.2.3.0	0.0481.1.2.5.1.4.1.8.4	0.0502.0.5.9.7.0.7.4.0	0.0502.5.4.0.3.1.9.9.0
0.5129000000		0.5034.9.1.8.1.0.2.1.7	0.5099.1.3.4.0.8.7.9.0	-0.0007.4.0.2.1.3.9.3.7	0.0501.3.8.7.9.9.5.4.4	0.0502.9.4.9.7.1.4.9.0	0.0502.7.4.7.0.5.0.0.8
0.5130000000		0.5036.8.9.0.4.9.2.1	0.5099.3.7.4.2.2.0.4.4	-0.0007.7.7.7.3.2.4.4.4	0.0501.0.7.9.7.8.9.5.2	0.0502.9.6.5.1.3.5.2.0	0.0502.9.4.4.1.3.1.9.6
0.5131000000		0.5038.8.5.7.0.4.5.5	0.5099.2.3.7.0.4.4.5.5	-0.0008.1.0.7.4.9.8.5.5	0.0501.0.9.5.3.0.4.7.7	0.0502.8.4.2.8.7.6.4.8	0.0503.1.9.3.5.9.3.4.6
0.5132000000		0.5040.8.3.7.2.1.3.5.1	0.5099.0.7.4.3.5.3.5.0	-0.0008.2.9.2.4.1.5.5.5	0.0502.1.5.3.1.3.9.7.A	0.0503.0.1.9.3.4.5.2.0	0.0503.4.5.3.2.8.2.0.7
0.5133000000		0.5042.7.8.3.0.4.2.5.5	0.5098.9.8.4.8.4.1.7.0	-0.0008.5.5.4.1.8.5.5.5	0.0502.7.8.3.1.3.9.7.A	0.0503.0.1.9.3.4.5.2.0	0.0503.4.5.3.2.8.2.0.7
0.5134000000		0.5044.7.2.7.5.0.4.7	0.5098.8.7.4.9.4.7.4	-0.0008.8.2.3.0.0.8.7	0.0502.6.4.0.1.1.0.4	0.0503.2.0.7.9.4.0.1	0.0503.9.5.1.7.4.7.9.1
0.5135000000		0.5046.6.3.0.1.2.9.5.0	0.5098.3.8.6.7.4.9.9	-0.0010.8.1.8.2.9.4.1	0.0502.7.7.4.8.3.1.3	0.0503.1.1.9.9.0.1.1	0.0503.4.2.4.7.0.0.4.5
0.5136000000		0.5048.4.1.3.6.0.2.9.0	0.5098.2.7.0.6.9.4.7	-0.0011.8.0.3.3.7.8.8	0.0503.3.3.3.9.4.8.4	0.0504.0.1.8.2.1.3.9	0.0504.5.4.6.4.2.0.7
0.5137000000		0.5050.2.0.5.4.0.8.7.9	0.5098.1.4.7.0.4.4.9.5	-0.0012.9.6.1.4.0.0.8	0.0504.2.3.4.0.9.8.4.1	0.0505.0.2.8.2.3.4.2.8	0.0504.8.7.9.1.1.0.4.1
0.5138000000		0.5051.9.7.6.9.0.1.5.4	0.5098.0.1.4.7.9.4.4.7	-0.0014.3.2.0.2.8.0.7	0.0504.7.8.5.4.1.0.4.2	0.0505.0.0.7.7.5.4.0.7	0.0505.2.2.5.8.2.6.4.5
0.5139000000		0.5053.9.5.4.7.0.9.4	0.5097.8.5.6.8.7.0.9.5	-0.0015.9.1.4.3.1.7.9	0.0505.9.7.5.2.0.6.5.0	0.0505.8.9.4.3.6.6.0	0.0505.6.0.4.4.7.6.9.7
0.5140000000		0.5055.9.3.7.7.4.1.3.0	0.5097.6.9.1.9.2.7.4.7	-0.0017.7.8.5.5.9.5.5	0.0506.0.4.5.4.6.3.4.0	0.0505.8.7.9.2.3.0.4.7	0.0506.0.1.9.5.1.6.8.8
0.5141000000		0.5057.9.0.0.4.7.9.0	0.5097.5.0.9.1.9.1.5	-0.0019.9.8.6.8.0.1.3	0.0506.0.6.1.7.8.3.4.5	0.0506.0.7.4.2.1.2.0	0.0506.0.4.8.7.9.4.1.1
0.5142000000		0.5059.8.8.3.4.2.9.4	0.5097.3.9.0.4.4.5.5.8	-0.0022.8.8.2.2.9.5	0.0506.7.4.8.1.9.1.8.3	0.0506.9.4.2.0.8.9.0	0.0506.9.5.9.5.9.4.5
0.5143000000		0.5061.9.3.5.5.1.6.0.7	0.5097.0.4.6.4.4.0.8	-0.0025.4.6.8.4.9.6.2	0.0508.8.0.1.2.4.4.4.5	0.0508.8.9.0.4.9.3.4.8	0.0507.5.0.9.8.5.9.4.2
0.5144000000		0.5063.9.0.2.4.9.0.7.8	0.5096.7.6.0.9.2.4.8	-0.0029.3.5.2.5.0.4.5	0.0509.2.9.4.7.9.0.7	0.0509.4.7.7.9.6.7.9.0	0.0508.1.2.8.6.4.7.4.5
0.5145000000		0.5065.9.0.0.8.4.7.7.A	0.5096.4.6.0.3.7.9.7.0	-0.0033.7.9.7.9.4.8.7	0.0511.0.3.3.4.9.0.9.9	0.0511.0.8.2.3.1.9.0	0.0508.8.3.4.6.8.0.9.8
0.5146000000		0.5067.9.1.3.3.9.5.0.5	0.5096.0.4.8.3.5.1.9	-0.0039.2.3.2.3.0.1.0	0.0511.1.4.3.7.0.8.4.2	0.0511.1.4.4.9.7.0.2.2	0.0509.0.5.4.5.2.3.6.9
0.5147000000		0.5069.8.9.2.1.1.9.7.0	0.5095.8.7.1.6.4.1.7.0	-0.0045.0.9.5.5.4.9.1	0.0511.9.6.3.1.8.0.3.1	0.0511.9.6.1.2.9.2.2.9	0.0511.0.6.3.9.2.6.9.8
0.5148000000		0.5071.8.7.6.8.2.4.9.3	0.5095.5.1.6.0.7.9.7.A	-0.0054.8.2.3.7.4.0.7	0.0511.9.9.3.4.9.4.1.1	0.0511.3.8.1.9.0.0.7	0.0511.1.8.7.8.1.6.4.1
0.5149000000		0.5073.8.5.3.9.4.4.3	0.5094.9.7.0.1.0.7.4	-0.0066.0.3.2.1.0.3.7	0.0511.8.3.4.8.4.9.9.3	0.0511.2.2.7.0.0.1.8.8	0.0511.5.4.4.2.7.0.9.9
0.5150000000		0.5075.8.3.7.4.7.0.0.8	0.5093.8.3.9.9.4.9.1	-0.0081.9.4.0.0.4.0.9	0.0511.4.9.0.6.2.7.0.8	0.0511.4.2.7.3.7.2.8	0.0511.5.4.4.2.7.0.9.9
0.5151000000		0.5077.8.3.1.8.0.1.2.0	0.5092.9.0.0.8.3.1.4.1	-0.0110.4.3.6.1.9.0.8	0.0511.4.0.1.3.7.9.3.8	0.0511.9.4.1.5.9.4.8	0.0511.8.6.7.6.5.8.4.4

$\tilde{c}$	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
0511000000000	0499000000000	0511000000000	0000000000000	-0023000000000	0000000000000	0000000000000	0499000000000
0511100000000		0509000000000	-0500000000000	-0020000000000	0000000000000	0000000000000	0000000000000
0511200000000		0507000000000	-0500000000000	-0017000000000	0000000000000	0000000000000	0000000000000
0511300000000		0505000000000	-0500000000000	-0014000000000	0000000000000	0000000000000	0000000000000
0511400000000		0503000000000	-0500000000000	-0012000000000	0000000000000	0000000000000	0000000000000
0511500000000		0501000000000	-0500000000000	-0010000000000	0000000000000	0000000000000	0000000000000
0511600000000		0500000000000	-0500000000000	-0008000000000	0000000000000	0000000000000	0000000000000
0511700000000		0500000000000	-0500000000000	-0006000000000	0000000000000	0000000000000	0000000000000
0511800000000		0500000000000	-0500000000000	-0004000000000	0000000000000	0000000000000	0000000000000
0511900000000		0500000000000	-0500000000000	-0002000000000	0000000000000	0000000000000	0000000000000
0512000000000		0500000000000	-0500000000000	-0000000000000	0000000000000	0000000000000	0000000000000
0512100000000		0500000000000	-0500000000000	0002000000000	0000000000000	0000000000000	0000000000000
0512200000000		0500000000000	-0500000000000	0004000000000	0000000000000	0000000000000	0000000000000
0512300000000		0500000000000	-0500000000000	0006000000000	0000000000000	0000000000000	0000000000000
0512400000000		0500000000000	-0500000000000	0008000000000	0000000000000	0000000000000	0000000000000
0512500000000		0500000000000	-0500000000000	0010000000000	0000000000000	0000000000000	0000000000000
0512600000000		0500000000000	-0500000000000	0012000000000	0000000000000	0000000000000	0000000000000
0512700000000		0500000000000	-0500000000000	0014000000000	0000000000000	0000000000000	0000000000000
0512800000000		0500000000000	-0500000000000	0017000000000	0000000000000	0000000000000	0000000000000
0512900000000		0500000000000	-0500000000000	0020000000000	0000000000000	0000000000000	0000000000000
0513000000000		0500000000000	-0500000000000	0023000000000	0000000000000	0000000000000	0000000000000
0513100000000		0500000000000	-0500000000000	0026000000000	0000000000000	0000000000000	0000000000000

ξ	α	Φ_{11}	Φ'_{11}	Φ''_{11}	Ψ_{11}	Ψ'_{11}	Ψ''_{11}
051100000000	0499000000[illegible]	000000000000	[illegible]	000000000000	000000000000	000000000000	000000000000
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	−0933406872815	[illegible]	[illegible]	04919064418
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	−0950402055	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	04065517779
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	05013398630
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	05014938671
05123000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
05124000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
05130000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
05132000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
05133000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
05134000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
05135000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
05136000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
05137000000		[illegible]	[illegible]	−0013142087	[illegible]	[illegible]	[illegible]
05138000000		05195757374	[illegible]	−03148321987	[illegible]	[illegible]	[illegible]
05139000000		[illegible]	[illegible]	−0310162441	[illegible]	[illegible]	[illegible]
05140000000		[illegible]	[illegible]	−00160758902	[illegible]	[illegible]	[illegible]
05141000000		[illegible]	[illegible]	−00203263859	[illegible]	[illegible]	05069310849
05142000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
05143000000		[illegible]	[illegible]	−0326133499	[illegible]	[illegible]	0507588704
05144000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
05145000000		[illegible]	050402157495	−00344406950	[illegible]	[illegible]	[illegible]
05146000000		[illegible]	[illegible]	−00399928857	[illegible]	[illegible]	[illegible]
05147000000		05172091906	[illegible]	−00469023054	[illegible]	[illegible]	05107393131
05148000000		05192648584	[illegible]	−00557162949	[illegible]	[illegible]	0511197729
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	05113479924
05150000000		05173542609	050394782490	−00836467880	[illegible]	[illegible]	05115984338
05151000000		05193852992	050379014429	−01108358845	[illegible]	[illegible]	05118771967

ξ	a	ϕ_i	ϕ_i'	ϕ_i''	ψ_i	ψ_i'	ψ_i''
0511000000	0499800000	0511000000	0000000000	-5025000000	0000000000	0000000000	0499499995
0511100000		0500980810	-4995795766	-5020417131	0474797.8143	0469A391328	0449760709
0511200000		0500950114	-4981673679	-5017303A436	0481034[illegible]	0491050383	0499648650
0511300000		0500901018	-4987598[illegible]	-5014072[illegible]	0484880.5825	0492890329	0501018166
0511400000		0500841420	-4971202679	-5012405[illegible]12	0487850016	0493846616	0501035721
0511500000		0500764227	-4985984400	-5010984.978	0491736.5504	0498401760	0501051584
0511600000		0500675957	-4994270[illegible]	-4994099103	0491792J2242	0494884.26721	0501060635
0511700000		0500570057	-5010287460	-4985A764190	0492055[illegible]	0497161003	0501079374
0511800000		0500471840	-5011057[illegible]	-4976885203	0493225J41A	0498917A84C	0501091692
0511900000		0500357738	-5011790015	-4967765036	0494105.6746	0499.5090	0501103145
0512000000		0500247614	-5012407073	-4944045608	0405094729A5	0501045971	0501113834
0512100000		0500108767	-5013068345	-4955919705	0496196110	0501187955	0501127842
0512200000		0500974998	-5013604[illegible]	-4956030121	04970C9J45A	0501970119	0501139228
0512300000		0500[illegible]587	-5014107905	-4983454909	0498735[illegible]	050[illegible]01	0501142014
0512400000		0500691324	-5014713478	-4951710879	0501017/970	0501690476	0501150325
0512500000		0500541828	-5018223813	-4980718988	0501173215	050141404850	0501158058
0512600000		0500386869	-5016779041	-4950437303	0501940522	0501910557	0501165377
0512700000		0500227278	-5014297A[illegible]	-4950823604	0501819395	0501907101	0501172176
0512800000		0500064153	-5014749[illegible]	-4951861697	050171C[illegible]	[illegible]	[illegible]
0512900000		0507894211	-5017273747	-4953536280	0501912357	05020827820	0501104297
0513000000		0507710411	-5017621877	-4955840014	0502126039	05022013566	0501109545
0513100000		0507533048	-5018302A4	-4968800807	0502385[illegible]	0502350050	0501114304
0513200000		0507[illegible]	-5019009[illegible]	-4942019604	0502490/919	0502403207	0501119836
0513300000		0507185984	-5010608322	-4966722607	0502040/496	05023405840	0501201866
0513400000		050A9852279	-5020397108	-4971767083	050310207J8	0502A805604	0501204477
0513500000		050A7484002	-5021085140	-4977580482	05037[illegible]126	0502801114	0501206316
0513600000		050A533759A	-5021801[illegible]	-4984264040	[illegible]	[illegible]	[illegible]
0513700000		0504310837	-5022771138	-4991829144	0503961335	0503042819	0501209460
0513800000		050A07406	-5023579140	-5010043583	0504271082	0503143133	0501209411
0513900000		0508635890	-5024783444	-5011019190	0504893.472	0503985902	0501207382
0514000000		0505942048	-5024939747	-5012125105	0504028301	0503403850	0501197567
0514100000		050831601AC	-5027213A.9	-5013582114	0505274041	0503973013	0501190666
0514200000		0508377213	-502402204	-5014815603	0505432674	05039A44005	0501181243
0514300000		0504743460	-5030185727	-5016460291	0506C2A632	0503991743	0501168523
0514400000		0504433054	-503192271	-5018761816	050698A2A47	0503875429	0501152719
0514500000		0504104943	-5033848A47	-5020580941	05067774745	0503984A005	0501132142
0514600000		0503794911	-5034085343	-5023221424	0507182305	0504101448	0501105930
0514700000		0503382934	-5034852874	-5026041.47	0507390A142	0504810A052	0501077650
0514800000		0502983143	-5041301819	-5030374613	0508024301	0504315705	0501030092
0514900000		0502593048	-5044068961	-5035497801	05080614A5	0504416951	0499752300
0515000000		0502089328	-5048524358	-5042591440	0508907J89	0504810169	0499024487
0515100000		0501590708	-5049295878	-5053120.3	0509.9A7 3995	05079[illegible]	[illegible]

ξ	a	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
.6511000000	0.4994000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6512000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6513000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6514000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6515000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6516000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6517000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6518000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6519000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6520000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6521000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6522000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6523000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6524000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6525000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6526000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6527000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6528000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6529000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6530000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6531000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6532000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6533000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6534000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6535000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6536000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6537000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6538000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6539000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6540000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6541000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6542000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6543000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6544000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6545000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6546000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6547000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6548000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6549000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000
.6550000000		0.0000000000	0.0000000000	-0.0000000000	0.0000000000	0.0000000000	0.0000000000

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
0511000000000	0499600000000	[illegible]	[illegible]	-0125000000000	[illegible]	[illegible]	0499599999987
0511100000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0499863545?0
0511200000000		[illegible]	[illegible]	-0502773062753	0481095911007	0491971?7247	0501?0499697
0511300000000		[illegible]	[illegible]	-05014702?936	[illegible]	[illegible]	[illegible]
0511400000000		[illegible]	[illegible]	-0501263619?0	0487042?5113	049807?852529	05010446?371
0511500000000		[illegible]	[illegible]	-0501097?09?2	0491249?7755	0498??3?2475	05010?265335
0511600000000		[illegible]	[illegible]	-0500685915112	050181139?05	0496143?6332	050107728344
0511700000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	05010?073374
0511800000000		[illegible]	[illegible]	-0497096? 3?39	0?335929175	[illegible]	050110317949
0511900000000		[illegible]	-0501?7820?97	-049498765057	049410847190	0499??394700	050111474984
0512000000000		0500230?64?4	-0501?9482??95	[illegible]	[illegible]	[illegible]	0501125546?8
0512100000000		05001084206?	[illegible]	[illegible]	[illegible]	[illegible]	050113365397
0512200000000		0500974787??	[illegible]	-049502336095	[illegible]	[illegible]	0501149125?3
0512300000000		05008952416?	-0501?1?59?04	-049537011?24	[illegible]	[illegible]	050115399?04
0512400000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0501167?37?3
0512500000000		[illegible]	-0501?2??9795	-04951057?075	[illegible]	050163101816	050117019?44
0512600000000		[illegible]	[illegible]	-0495?455?03	[illegible]	[illegible]	0501177?5341
0512700000000		050022619?7?	[illegible]	[illegible]	[illegible]	[illegible]	050118437?78
0512800000000		[illegible]	[illegible]	-0495?9411?723	050172??9???	0501?8514564	05011??692??
0512900000000		[illegible]	[illegible]	[illegible]	050103?4?105	0502104?2797	05011?4525?8
0513000000000		[illegible]	-0501?6?34?03	-049?6??????22	[illegible]	0502?2?43?69	0501201757?4
0513100000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0513200000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0501210?75?8
0513300000000		[illegible]	[illegible]	-0496777837?65	[illegible]	[illegible]	[illegible]
0513400000000		[illegible]	[illegible]	-04772956?013	[illegible]	[illegible]	050121?57341
0513500000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	050121?04?87
0513600000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	050121?69??1
0513700000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	050121?17??4
0513800000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	050121?291??
0513900000000		[illegible]	[illegible]	-0501122947?52	[illegible]	[illegible]	050121?826??
0514000000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0501207474?6
0514100000000		[illegible]	[illegible]	-050136449?13	[illegible]	[illegible]	[illegible]
0514200000000		[illegible]	[illegible]	-05015107?620	[illegible]	[illegible]	05011?967344
0514300000000		[illegible]	[illegible]	-0501678?7?30	[illegible]	[illegible]	[illegible]
0514400000000		[illegible]	[illegible]	-0501?731??98	[illegible]	[illegible]	[illegible]
0514500000000		[illegible]	[illegible]	-050210023103	[illegible]	[illegible]	050113677546
0514600000000		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	05011??61796
0514700000000		[illegible]	[illegible]	-0502694659?0	[illegible]	[illegible]	0501072?5593
0514800000000		[illegible]	[illegible]	-0503099?0847	[illegible]	[illegible]	05010?70?644
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0518000000000		[illegible]	[illegible]	-05043?8?2?20	[illegible]	[illegible]	[illegible]
0518100000000		05015259?7?0?	[illegible]	-0505421136?4	[illegible]	[illegible]	0497816159?0

ξ	α	ϕ_{11}	ϕ'_{11}	ϕ''_{11}	ψ_{11}	ψ'_{11}	ψ''_{11}
051100000000	0499800000000	0000000000000	0511000000000	0000000000000	0000000000000	0000000000000	0000000000000
051110000000		0490996073680	0500008012170	-0932069840085	0481402161084	0476480085328	049877939444
051120000000		0501395724677	0500060080409	-093468780035	047191702441	0481903972780	049402868241
051130000000		[illegible]	0507462116370	-093479303060	047480049980	048484356498	049312016540
051140000000		0507948120664	0500871330478	-095089089797	048108102712	048080010250	049426111510
051150000000		050198609837	0500820032480	-049554125071	048413949775	049151091999	049545133784
051160000000		050894508A19	0500970984262	-049580810067	048378236394	049191609278	049669117389
051170000000		050491641058	050970839190	-0493095483896	048081524055	049248013898	049794159384
051180000000		[illegible]	[illegible]	-049810219021	048908862181	049584408580	049932401980
051190000000		[illegible]	[illegible]	-049640089879	0489190940844	049940983798	050407202773
051200000000		050080141898	050081448138	-049639374986	049181678704	0498644009508	05012172605
051210000000		0511175000848	050084526095	-049640870704	0480248639285	049409339622	[illegible]
051220000000		0511148921920	050090828111	-049646307872	0489371239902	049689740318	050152963746
051230000000		051126477371	050092336420	-049652689806	048413104365	050100547357	05016994357
051240000000		05113954792	050095768278	-049660801028	048521433827	050117808708	05018601475
051250000000		051144702430	0500191110505	-049671745485	048609389445	050137204830	050203922791
051260000000		[illegible]	[illegible]	-049680033078	048787147292	050198509492	050222417297
051270000000		05116509880	05000587480	-049704562453	048947201789	050181806894	050241856715
051280000000		05117268630	050008219795	-049728143807	050116142335	050207001139	050262204404
051290000000		051181002119	050000793884	-04975707969	0501981901 1	050291280294	0502839363
051300000000		051189471219	0500830379 9	-049794041832	[illegible]	[illegible]	050305933241
051310000000		05119868170	050080884790	-0498384C043	050181018603	050249905947	050324494797
051320000000		051207387318	050088623907	-04989184708	050422253128	05039769318	05035928441
051330000000		051915984780	050085701840	-049859756097	050287038060	050388418707	050380565791
051340000000		0519248505930	050087089384	-050103165085	050489569312	050880880170	050408952398
051350000000		0519329238030	050090817005	-050112135781	050398309844	050804187227	050437808510
051360000000		[illegible]	[illegible]	-05014269347	050387337971	050609880390	050469317503
051370000000		0512494128428	050811730480	-050139001976	050497110814	050842084230	050502955147
051380000000		051257489107	05070781 91 1	-050149620830	050493881084	0508871142351	0505340773
051390000000		051265387080	050701739080	-050108806854	050586008970	050804990581	050578062984
051400000000		051273087084	050740809815	-050108858869	050629064974	050700089601	050620365917
051410000000		0512808732068	050746280490	-05021016047	050008144830	050773400290	05060058308
051420000000		0512807968828	050791008587	-050237880033	050778047828	050803887757	050717157145
051430000000		05125530841008	050800800185	-050270781063	050847048990	050817981398	050774001811
051440000000		05190108369	050847839877	-050310069884	050982880077	050800301858	050837548978
051450000000		051908399848	050894277815	-050357841790	05110089817	051100540750	050910009107
051460000000		051914581A14	050805698475	-050419379708	051118C40250	051118078159	05099409009
051470000000		0519204838808	05080079829	-050887881867	05111903411392	051199502387	05110919808
051480000000		051924539848	050807888180	-050979391715	051115373777	051180082057	051121835301
051490000000		05193020073	0508130384278	-050700850887	051118131113	051180892027	051137798202
051500000000		05193014868	050885767580	-050870539818	05117480884	051167801710	051158852384
051510000000		05193724898	050907881280	-051113021005	05118200108	0511884962080	051190993944

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
05110000000	04997000000	05110000000	00000000000	-05020000000	00000000000	00000000000	04976999594
05111000000		05099869108	-04922735035	-05020419126	04748934770	04989761202	04999662907
05112000000		05099559108	-04901624121	-05017700405	04810751758	04919920086	05010195075
05113000000		05009060195	-04880841041	-05014701103	04844803446	04990217367	05010346018
05114000000		05008410285	-04971207247	-05012630909	04890294806	04948704680	05010575257
05115000000		05007641964	-04899936100	-05010985808	04412672091	04951760734	05010737210
05116000000		05006750342	-04906293123	-04996584392	04919304594	04969174893	05010985039
05117000000		05005760190	-05010239407	-04985796076	04928060760	04973142732	05011020928
05118000000		05004714717	-05011082031	-04977046403	04937932418	04984213055	05011146650
05119000000		05003572371	-05011705014	-04979076293	04949194814	04999410033	05011243534
05120000000		05002399868	-05012495924	-04964313206	04952014790	05010447752	05011372658
05121000000		05001082819	-05014076148	-04959853543	04960726419	05011914115	05011474614
05122000000		05009794796	-05013696806	-04956439409	04978659316	05012960467	05011570299
05123000000		05008349884	-05014206736	-04953940982	04969193880	05014131345	05011659764
05124000000		05006908079	-05016799845	-04952402349	05010394080	05015902235	05011744204
05125000000		05005404386	-05014255108	-04951418608	05011980084	05014470971	05011623104
05126000000		05003854284	-05018749848	-04951258104	05011608318	05017468684	05011896950
05127000000		05002252124	-05012829560	-04951779108	05011551341	05018850039	05011965708
05128000000		05000597477	-05016807160	-04952906281	05017463465	05020057653	05012029074
05129000000		05078891892	-05017343642	-04954804729	05010952048	05021268714	05012087459
05130000000		05077140018	-05019905419	-04957296513	05021713474	05029479957	05012139508
05131000000		05075311440	-05018401647	-04960454985	05020072199	05023492573	05012186506
05132000000		05073428513	-05019114914	-04964295822	05024484154	05025512039	05012225758
05133000000		05071488801	-05019779828	-04968840704	05029004073	05026137739	05012259900
05134000000		05069470304	-05020496327	-04974151049	05031674779	05027930959	05012292450
05135000000		05067309491	-05021247759	-04980273429	05034477744	05028849770	05012297402
05136000000		05064921914	-05022104700	-04987726378	05037398349	05029427694	05012301590
05137000000		05062986073	-05023901613	-04995209170	05040484337	05031059735	05012293584
05138000000		05060608104	-05024012812	-05010421742	05043900451	05032260933	05012271134
05139000000		05058154828	-05025104889	-05011441904	05044894597	05039404883	05012232024
05140000000		05055985764	-05026305954	-05012597087	05050301995	05034724477	05012172969
05141000000		05052800302	-05027629272	-05013810514	05053884372	05039940408	05012090537
05142000000		05050058040	-05029003548	-05015406771	05057490795	05039143884	05011979774
05143000000		05047068322	-05030717848	-05017122608	05061474245	05039613840	05011835209
05144000000		05043903019	-05032527021	-05019105693	05068164295	05039804590	05011648095
05145000000		05040953204	-05034549059	-05021423222	05069173705	05040069795	05011411804
05146000000		05036968868	-05036826007	-05024170684	05072980146	05041740772	05011110239
05147000000		05033177390	-05039040242	-05027493396	05077530930	05049947107	05010727144
05148000000		05029099995	-05042511841	-05031621309	05081871432	05049591502	05010436077
05149000000		05024899860	-05047691592	-05036954902	05084731016	05044927985	04995495799
05150000000		05019919024	-05048808364	-05044262205	05090895447	05049944492	04987510824
05151000000		05014700092	-05048710995	-05055303409	05099948030	05046464108	04979792654

ξ	α	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
0.0000000000	0.4997000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.0111000000		0.4999999548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.0222000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.0333000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.0444000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.0555000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.0666000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.0777000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.0888000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.0999000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.1110000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.1221000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.1332000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.1443000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.1554000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.1665000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.1776000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.1887000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.1998000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.2109000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.2220000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.2331000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.2442000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.2553000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.2664000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.2775000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.2886000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.2997000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.3108000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.3219000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.3330000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.3441000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.3552000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.3663000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.3774000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.3885000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.3996000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.4107000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.4218000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.4329000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.4440000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.4551000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.4662000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.4773000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.4884000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000
0.4995000000		0.4999998548	-0.0000000000	-0.9920656514	0.0000000000	0.0000000000	0.0000000000

ξ	σ	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
0511000000	[illegible]	0511000000	0000000000	-0245000000	0000000000	0000000000	0499799995
[illegible]		[illegible]	-4999795 [illegible]	[illegible]	04949442090	04898978238	05010069035
[illegible]		0500995081078	-4941624 [illegible]	-0117901119	04810950416	04929128448	05010300172
[illegible]		0500990001668	-4959582 [illegible]	-0147021 [illegible]	04845267276	0495 [illegible]	0501050319 [illegible]
[illegible]		0500854182229	-4971209 [illegible]	-0126417108	04881 [illegible]	049411205250	0501062 [illegible]
[illegible]		0500764191 [illegible]	-4982994 [illegible]	-0109927183	04912764197	049 [illegible]	05010847 [illegible]
[illegible]		0500675072 [illegible]	-4993013 [illegible]	-0396427578	04918494 [illegible]	0496 [illegible] 5532	0501099724 [illegible]
[illegible]		[illegible]	[illegible]	-0385 [illegible]	0492 [illegible]	[illegible] 176	05011134520
[illegible]		0500471932 [illegible]	[illegible]	-0377128 [illegible]	[illegible]	[illegible] 1612	0501124152 [illegible]
[illegible]		050035717 [illegible]	[illegible] 77	-037008 [illegible]	[illegible]	[illegible] 5938	05011379570
[illegible]		050029508 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		0500108 [illegible]	[illegible]	-0960025 [illegible]	[illegible]	[illegible]	05011592749 [illegible]
[illegible]		05009749577 [illegible]	[illegible]	-0956047209 [illegible]	[illegible] 1511	[illegible] 20994	050116092 [illegible]
[illegible]		[illegible]	-50142164 [illegible]	-0984201208 [illegible]	0499011322 [illegible]	[illegible] 9339	0501179553 [illegible]
[illegible]		[illegible]	[illegible]	-0982602707 [illegible]	[illegible]	[illegible]	05011 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	-095167 [illegible]	[illegible]	[illegible]	0501201853 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible] 0317	05012 [illegible]
[illegible]		[illegible]	[illegible]	-0953927 [illegible]	[illegible]	[illegible]	0501219116 [illegible]
[illegible]		[illegible]	[illegible]	-0955440501 [illegible]	[illegible]	[illegible] 1236	0501220957 [illegible]
[illegible]		[illegible]	[illegible]	[illegible] 72	[illegible] 9975	[illegible] 5622	050122 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	05012 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible] 9339	0501234659 [illegible]
[illegible]		[illegible]	[illegible] 77	[illegible]	[illegible]	[illegible] 7027	05012379 [illegible]
[illegible]		[illegible]	[illegible]	-0975372113 [illegible]	[illegible]	[illegible]	0501240929 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	-09969252 [illegible]	[illegible]	[illegible]	05012 [illegible]
[illegible]		[illegible]	[illegible]	-0100613778 [illegible]	[illegible]	[illegible]	05012378 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	05012 [illegible]
[illegible]		[illegible]	[illegible]	-0128377 [illegible]	[illegible]	[illegible]	05012270334 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	050121 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible] 7033	[illegible]	[illegible]	05011906 [illegible]
[illegible]		[illegible]	[illegible]	[illegible] 31	[illegible]	[illegible]	050117 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	050114 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	050111 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	050107222 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	05010197 [illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

ξ	α	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
0511000000000	0499600000000	0000000000000	0511000000000	0000000000000	0000000000000	0000000000000	0000000000000
0511100000000		0499996000488	0511008912100	-0492065650	0481A38537705	0A30e06A234E	0481U83A240
0511200000000		0501937944A4	0500066888741	-0493448834772	0471744442345	0482C0794884	0482C0794884
0511300000000		0502991A1A4E	0500621152A0	-0484900788	0474898013:9	0484A8A43927	04831A516940C
0511400000000		0502981204A2	0500A7530000	-0495U0617947	0481104156A6	0488A3107958	04834992444
0511500000000		0500456900A7	0500A2001750	-0495U422A151	0484183d9930	0491937A161C	0495648066A2
0511600000000		0502945C8514	0500763936B9	-0495849E8F48	0483B2C339B2	0491056A6854	0488B3156941
0511700000000		0502A1841352	0500732330A5	-0496C8757E91	0481402779A5	0492A5A473A	0490147869A4
0511800000000		0502385A9972	0500A4301104	-0496197444A7	0485774c22A8	0493568a9119A	0499518254A6
0511900000000		0502840784A0	0500A7095340	-0496293304B3	0481736479C4	0498A1C83871	0491C04358A9
0512000000000		0502801A7A81	0500A1A23101	-0496A33C4101	0491858a35A8	0495A7B83687	0491247614A6
0512100000000		0511075000C0	0500a5232A00	-0496421904A1	0482497316A2	0497C9a14160	0491394861A0
0512200000000		0511164702241	0500387781B4	-0496A085B28	0483279A1757	0495B2a4B978	0491957406A4
0512300000000		0511264794A7	0500422575B0	-0496568035B22	0484931773241	0491C91516A2	0491724609A2
0512400000000		0511365A531A	0500355471A6	-0496639327D0	0485728434A3	0491394230E	0491894864B
0512500000000		0511447A4A0A	0500318079A35	-0496756259B9	0486A28A3965	0491C1A1397E	0491C1C538A0
0512600000000		0511537A4A5A	0500121473B5	-0496A7927124	0486137C0481	0491C1A19C1620	0491227043A6
0512700000000		0511636C1417	0500B5104A0A	-0497106606B6	0485873A4A5C	0491C1a8501402	0491246884A8
0512800000000		0511726A6A5A	0500a07914A19	-0497396A7072	0491189a1036	0491C2113C9A0	0491267A51684
0512900000000		0511804A4071	0500a0U813390	-0497666B8657	0491A1C07553	0491C2a1a5287	0491284A2174A
0513000000000		0511896A3750	0500a2654292	-0498052359A6	0501A68A4A46	0491C2A327D190	0491312777743
0513100000000		0511984A3A4A	0500a4281345	-0498518759A8	0491A49a19A6	0491C16A0594	0491336320C4
0513200000000		0512071974A8	0500A5A84A76	-0499BAC00072	0491C28A1888	0491C1A5278A2	0491361C78F3
0513300000000		0512194A0751	0500a6C94190	-0499790211A6	0491C2423A4616	0491C79AC9012	0491348424914
0513400000000		0512244E606A	0500A4584A01	-5010945248A	0491C3177A7A	0491C1279063	0491a76A0A8
0513500000000		0512324A3A2A	0500A4[illegible]	-5011[illegible]	0491C3[illegible]	0491C1[illegible]	0491[illegible]
0513600000000		0513411824A1	0500A9202A02	-5012981[illegible]	0491C1[illegible]	0491C1771513A	0491478941418
0513700000000		0513423177A0	0500A[illegible]	-5013A8[illegible]	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0513800000000		0513573A2A0E	0500A[illegible]	-5013[illegible]	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0513900000000		0513682184A1	0500A8821A70	-5017181[illegible]	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0514000000000		0513724124A5	0500A0A19A75	-5019267[illegible]	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0514100000000		0513804194A7	0500A0168195	-5[illegible]	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0514200000000		0513877A6A0E	0500A17024A1	-5024A128130	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0514300000000		0513944780A0	0500A0770A15	-5028A025A40	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0514400000000		0514015A0974	0500AAC719145	-5032138A06A	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0514500000000		0514079A1A6A	0500A26224475	-5037078A95A	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0514600000000		0514148194A9	0500A619A193	-5043114773I	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0514700000000		0514198A4A3E	0500A9837A49	-5050823790A	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0514800000000		0514247778A8	0500A[illegible]	-5[illegible]	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0514900000000		0514[illegible]	0500A1814000	-5072A358A41	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0514A00000000		0514339A0A9A	0500A[illegible]	-5[illegible]	0491C1[illegible]	0491C1[illegible]	0491[illegible]
0514B00000000		0514399A2A3E	0500A34777A3	-5111759256	0491C1[illegible]	0491C1[illegible]	0491[illegible]

ξ	α	ϕ_1	ϕ_1'	ϕ_1''	ψ_1	ψ_1'	ψ_1''
[illegible]							

ξ	α	ϕ_{II}	ϕ'_{II}	ϕ''_{II}	ψ_{II}	ψ'_{II}	ψ''_{II}
[illegible]							

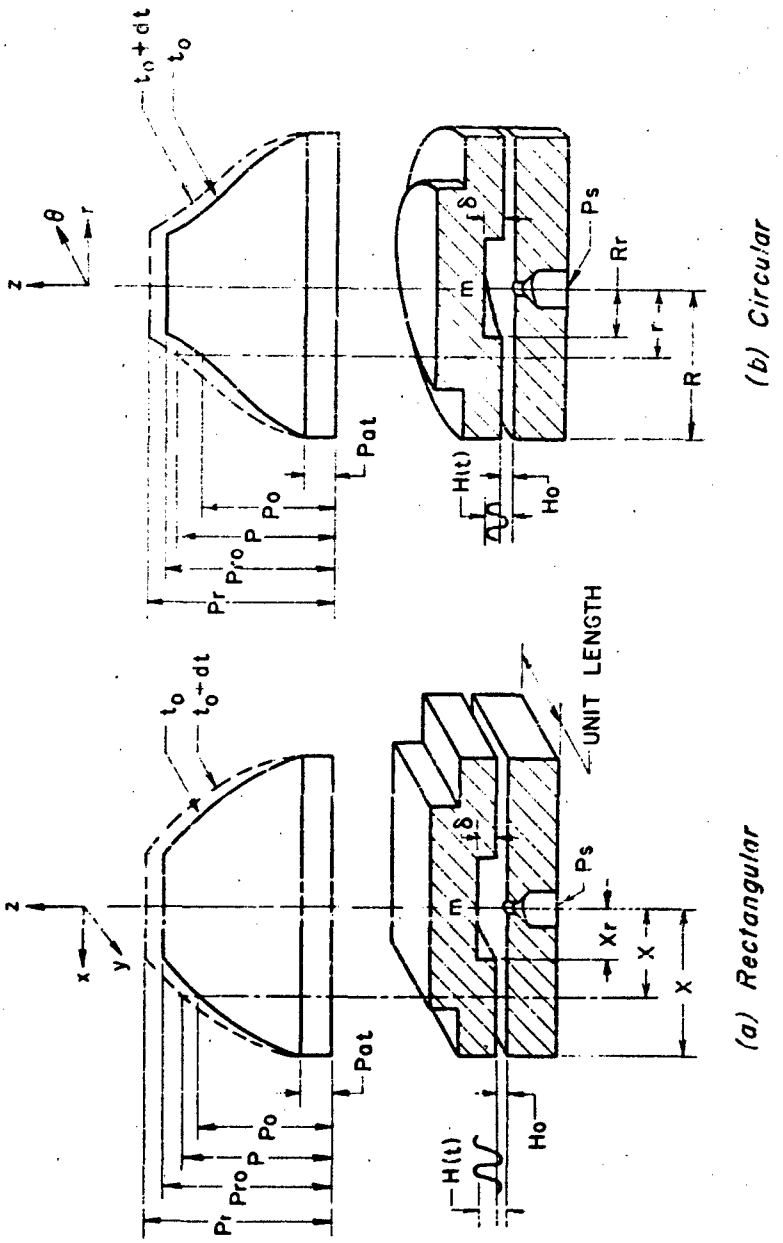
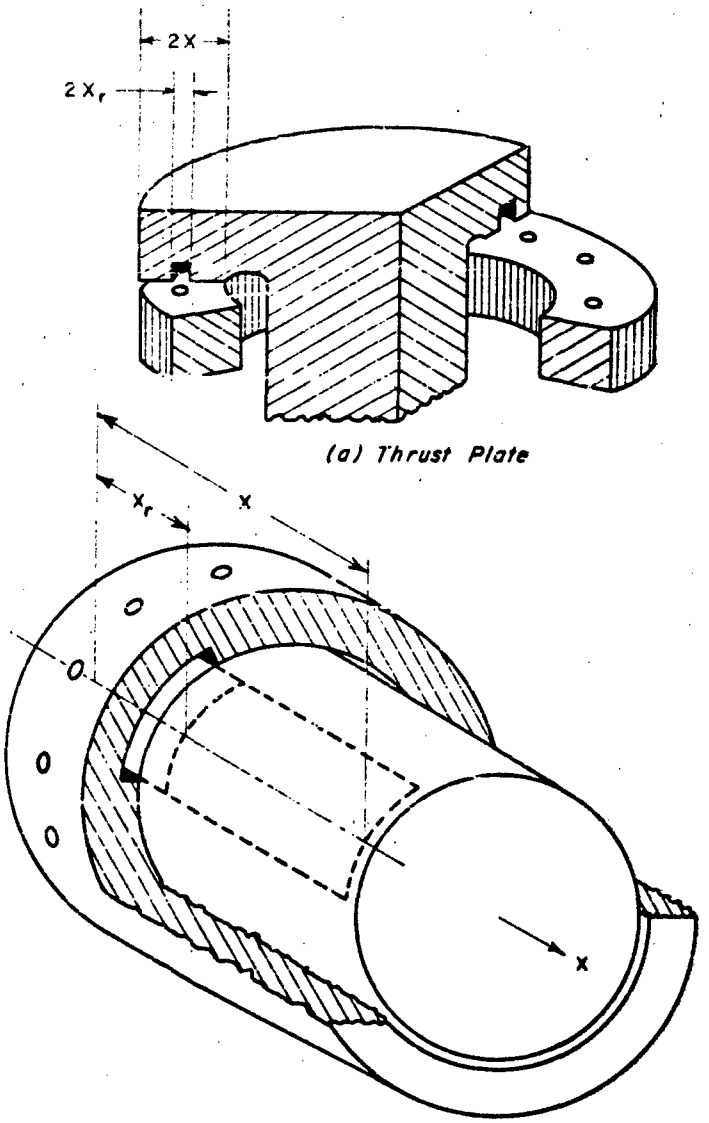


FIG. 1. SCHEMATIC DIAGRAM OF BEARING GEOMETRIES



(a) Thrust Plate

(b) 'Equivalent' Slot in Journal

FIG. 2 APPROXIMATIONS TO RECTANGULAR GEOMETRY

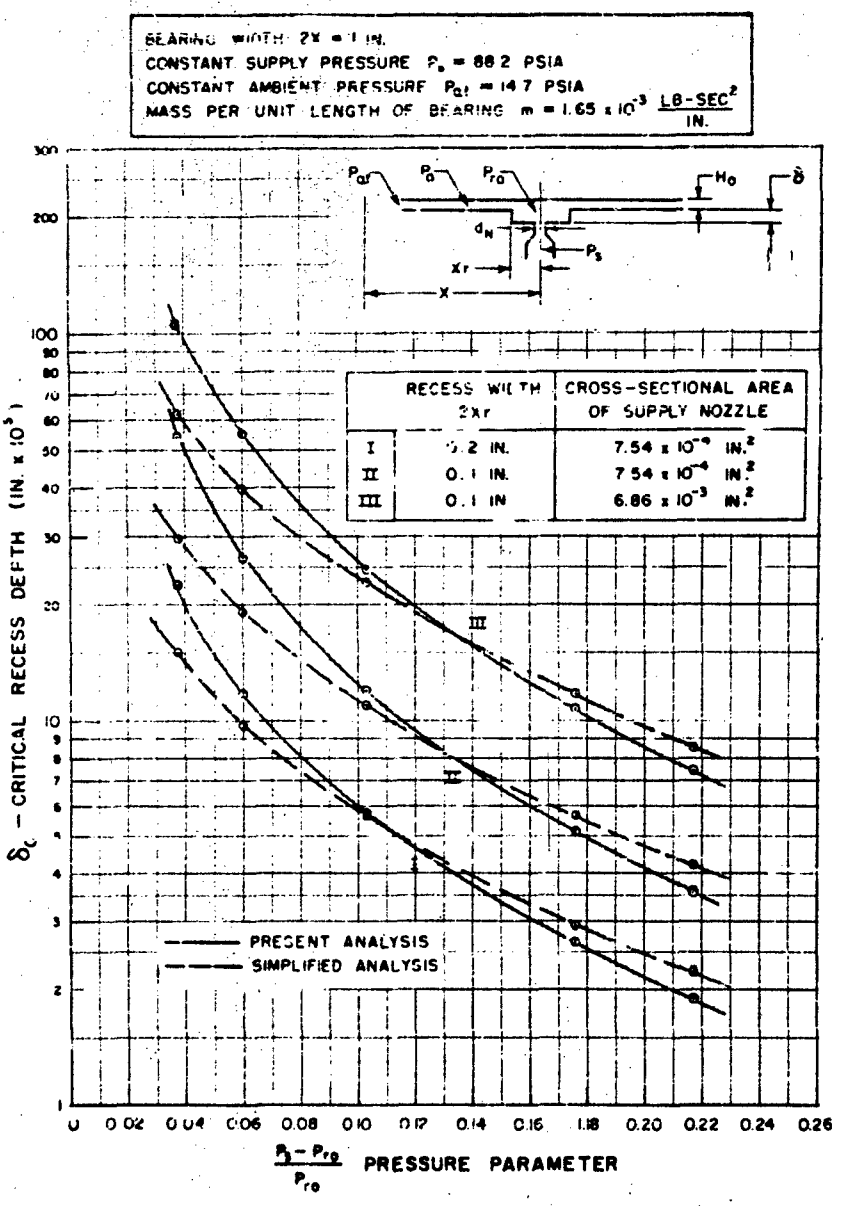


FIG. 9 CRITICAL RECESS DEPTH VS. PRESSURE PARAMETER (CALCULATED)

(Rectangular Geometry)

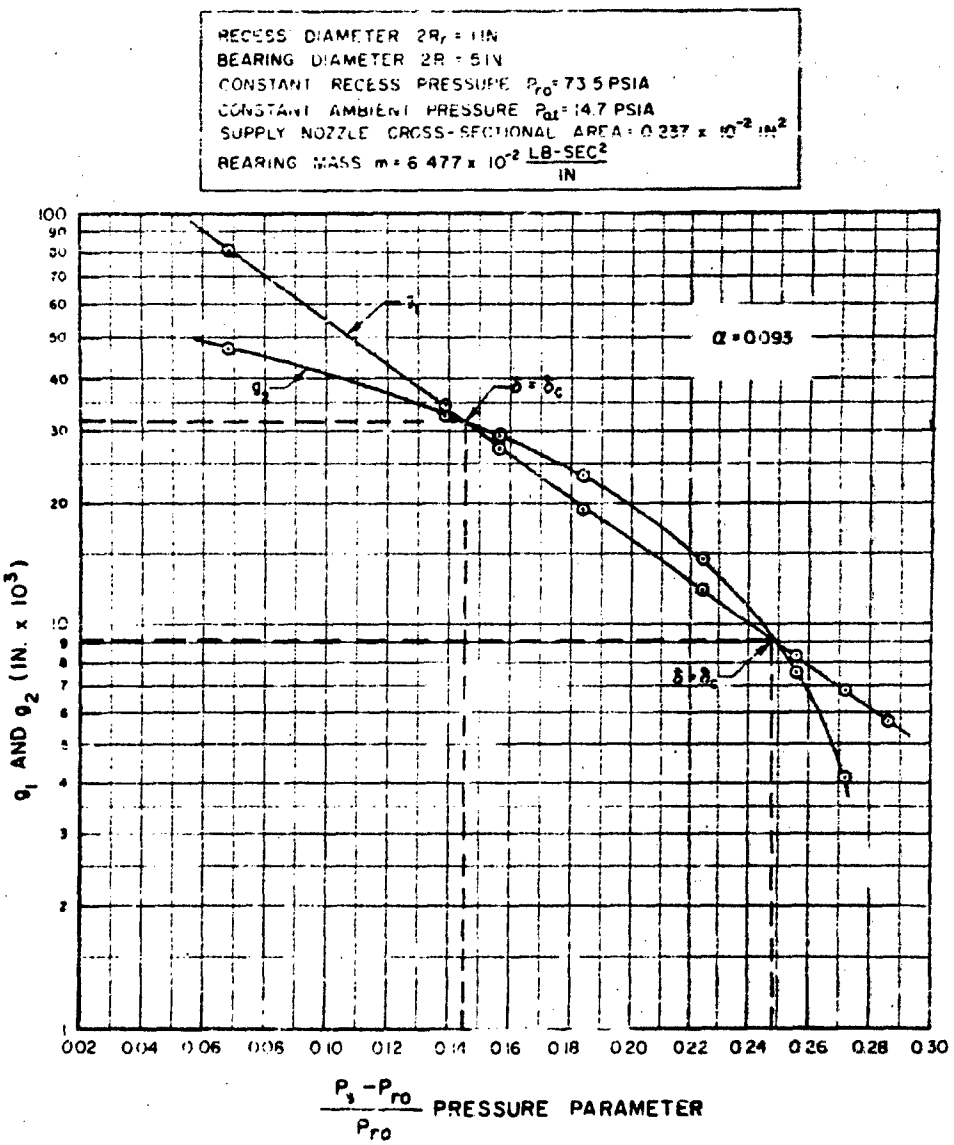


FIG 4 DETERMINATION OF CRITICAL RECESS DEPTH (EQUATION 43b)

(Circular Geometry)

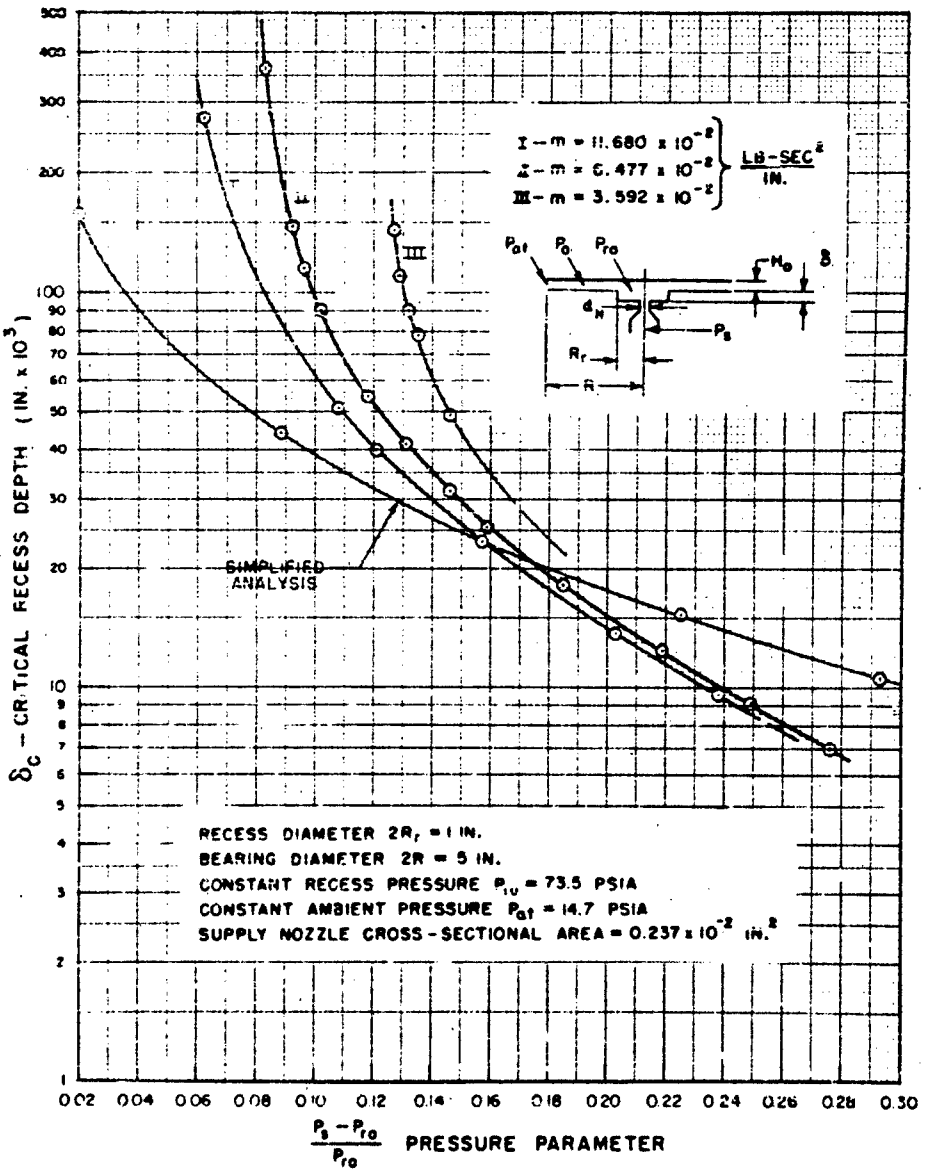


FIG 5 CRITICAL RECESS DEPTH vs PRESSURE PARAMETER (CALCULATED)

(Circular Geometry)

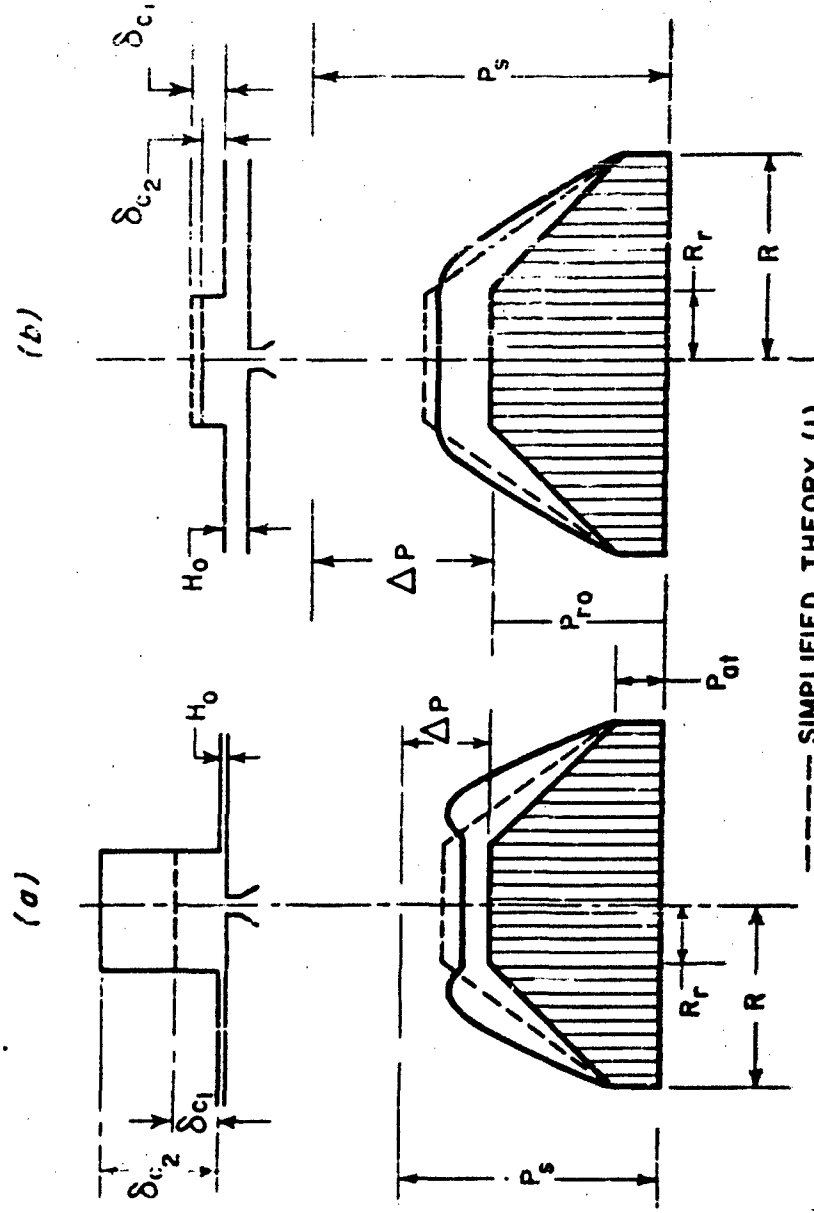
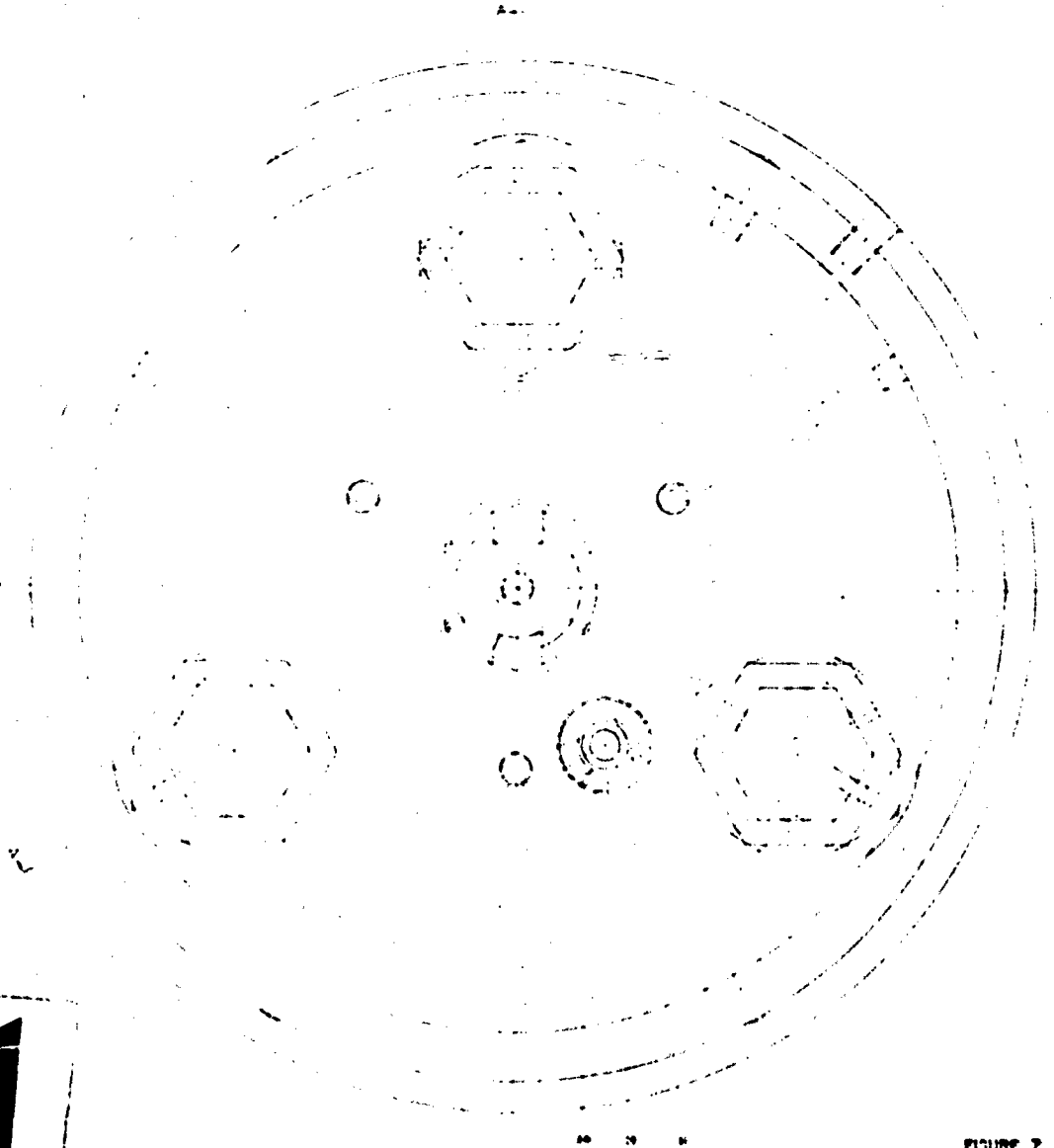


FIG. 6 SCHEMATIC DIAGRAM OF DYNAMIC PRESSURE PROFILES WITH REFERENCE TO PRESENT AND SIMPLIFIED ANALYSES

PC NO	DESCRIPTION	DWG NO
1	UPPER THRUST PLATE	A2049-B-01
2	UPPER TO ADJ SCREW	A2049-B-02
3	UPPER TO LOCK BOLT	A2049-B-03
4	AIR GAUGE ORIFICE	A2049-A-04
5	AIR GAUGE METERING ORIFICE	A2049-A-05
6	AIR GAUGE METERING ORIFICE	A2049-A-06
7	AIR GAUGE ORIFICE RETAINER	A2049-A-07
8	SUPPLY NOZZLE	A2049-A-08
9	SUPPLY NOZZLE	A2049-A-09
10	SUPPLY NOZZLE	A2049-A-10
11	SUPPLY NOZZLE RETAINER	A2049-A-11
12	WASHER NO 1	A2049-B-12

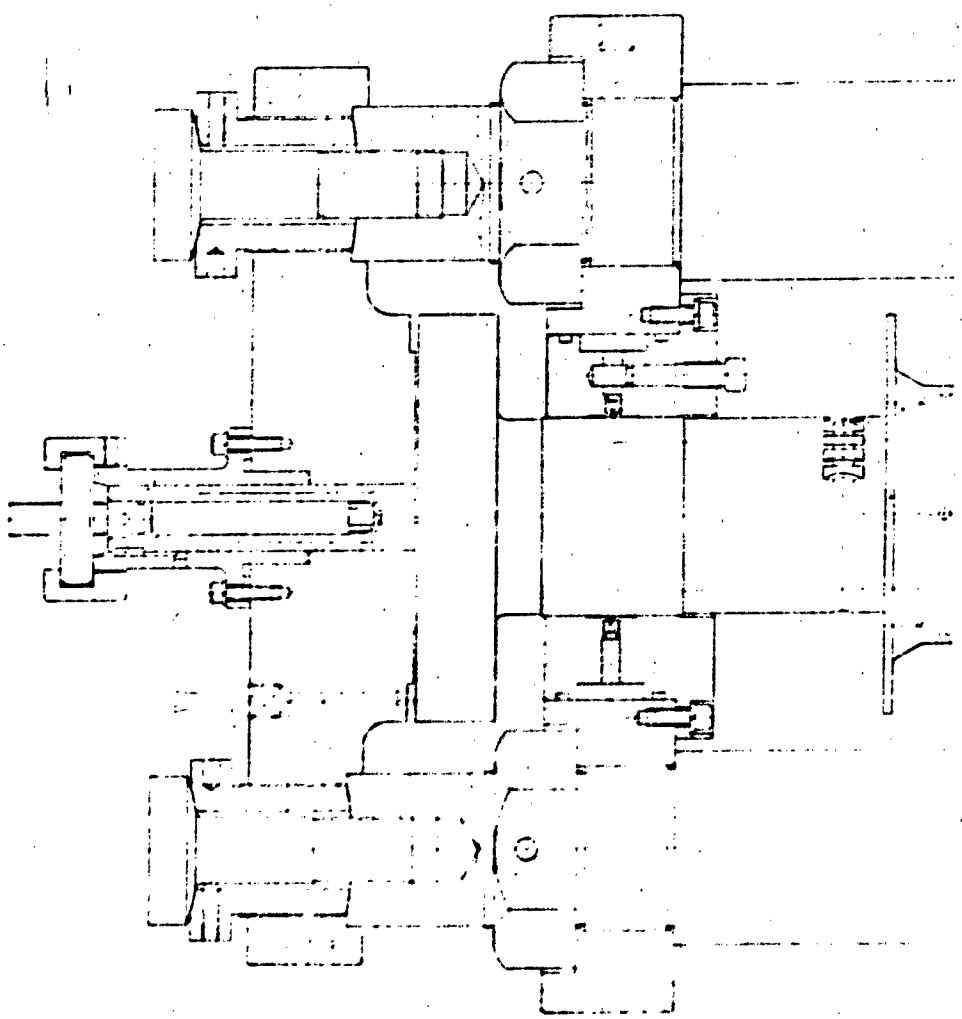


1

FIGURE 7

AIR BEARING STABILITY TEST R

TEST BEARING ASSEMBL



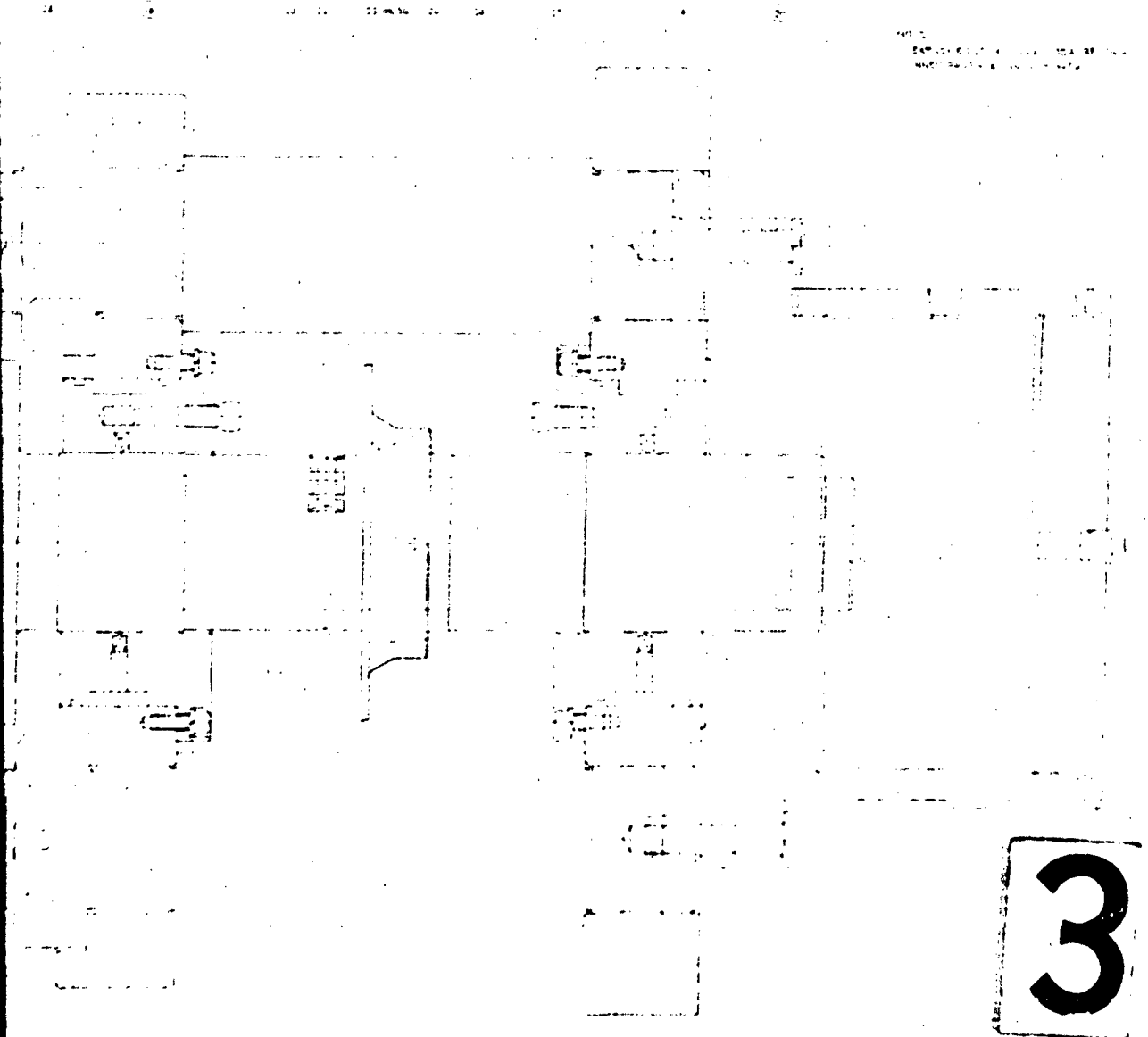
2

FIGURE 7

AIR BEARING STABILITY TEST RIG

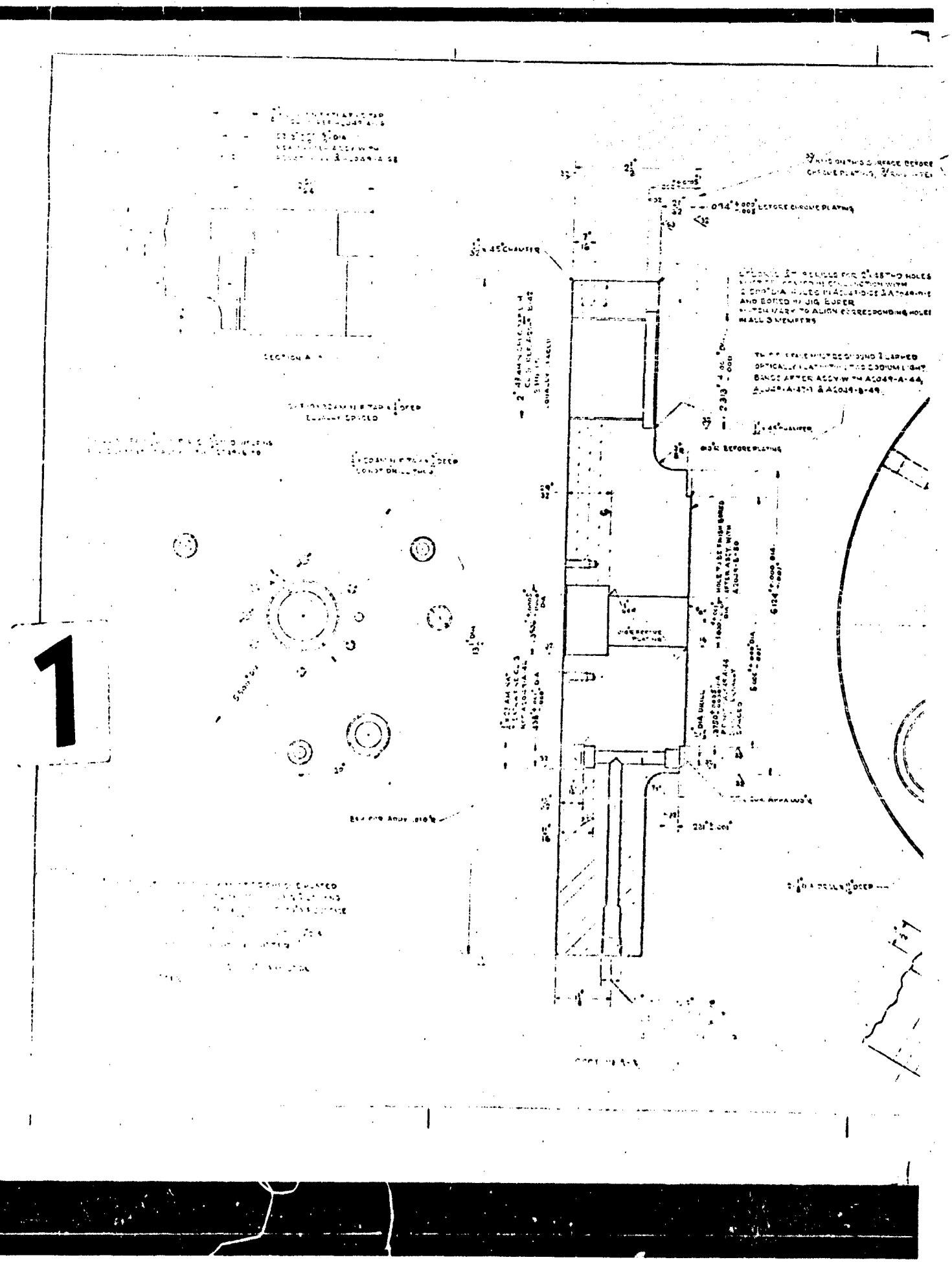
TEST BEARING ASSEMBLY

SECTION A-A

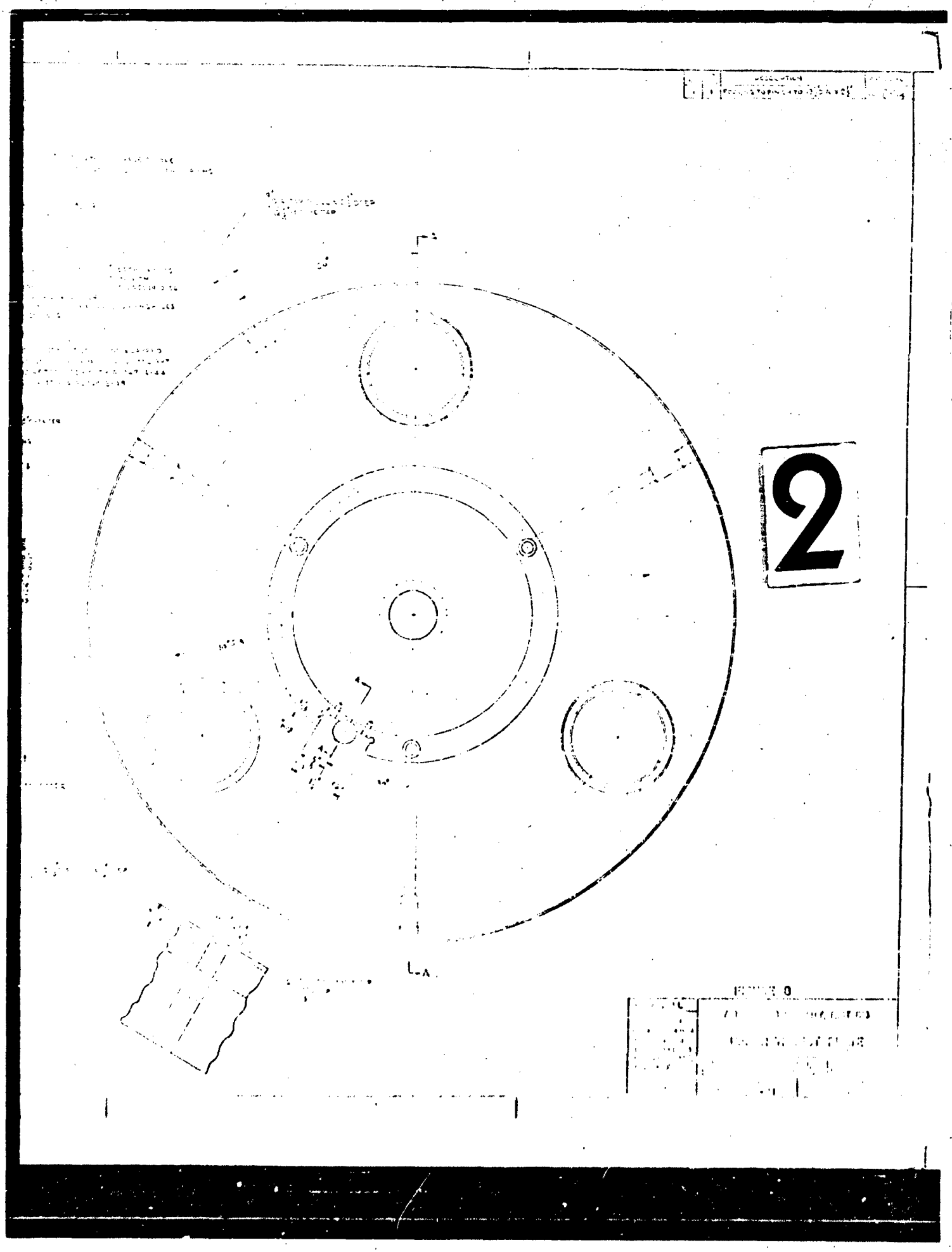


3

SECTION A-A



1


2

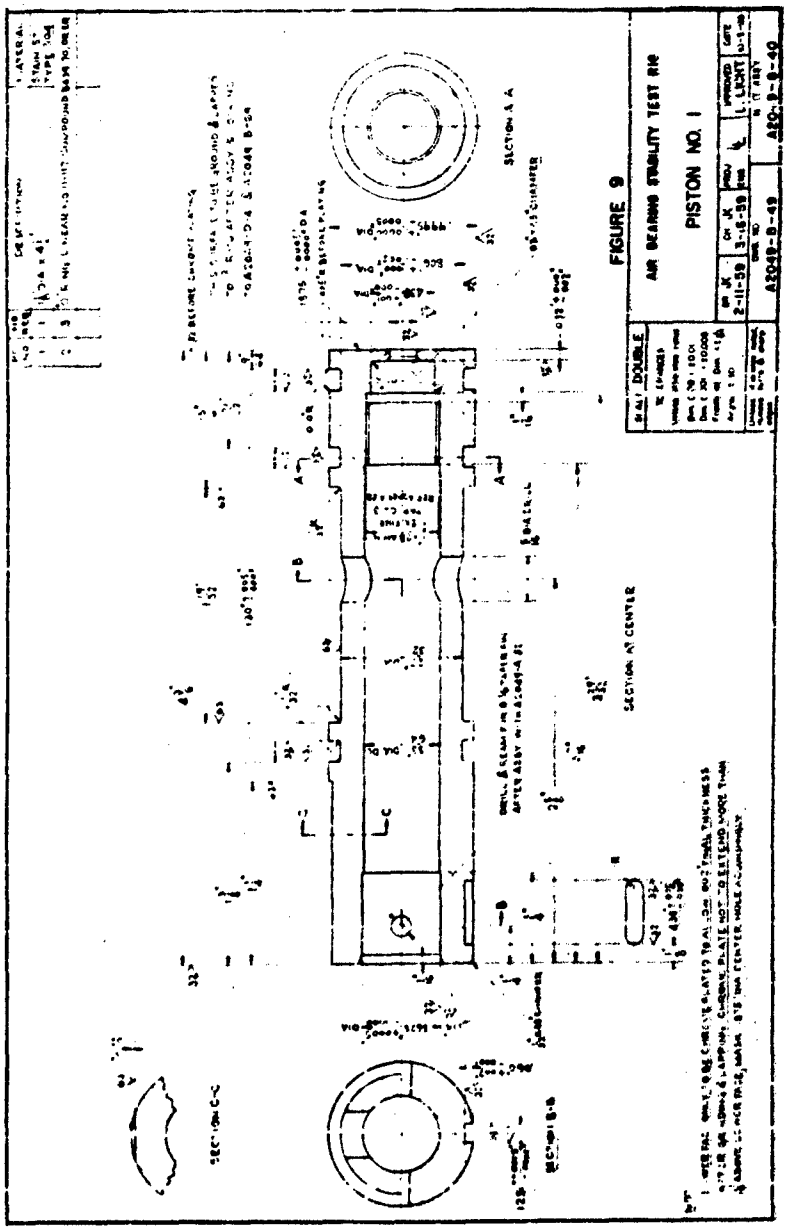


FIGURE 9
AIR BEARING STABILITY TEST RIG
PISTON NO. 1
A2049-B-49
SECTION A-A
SECTION AT CENTER
SECTION B-B
SECTION C-C

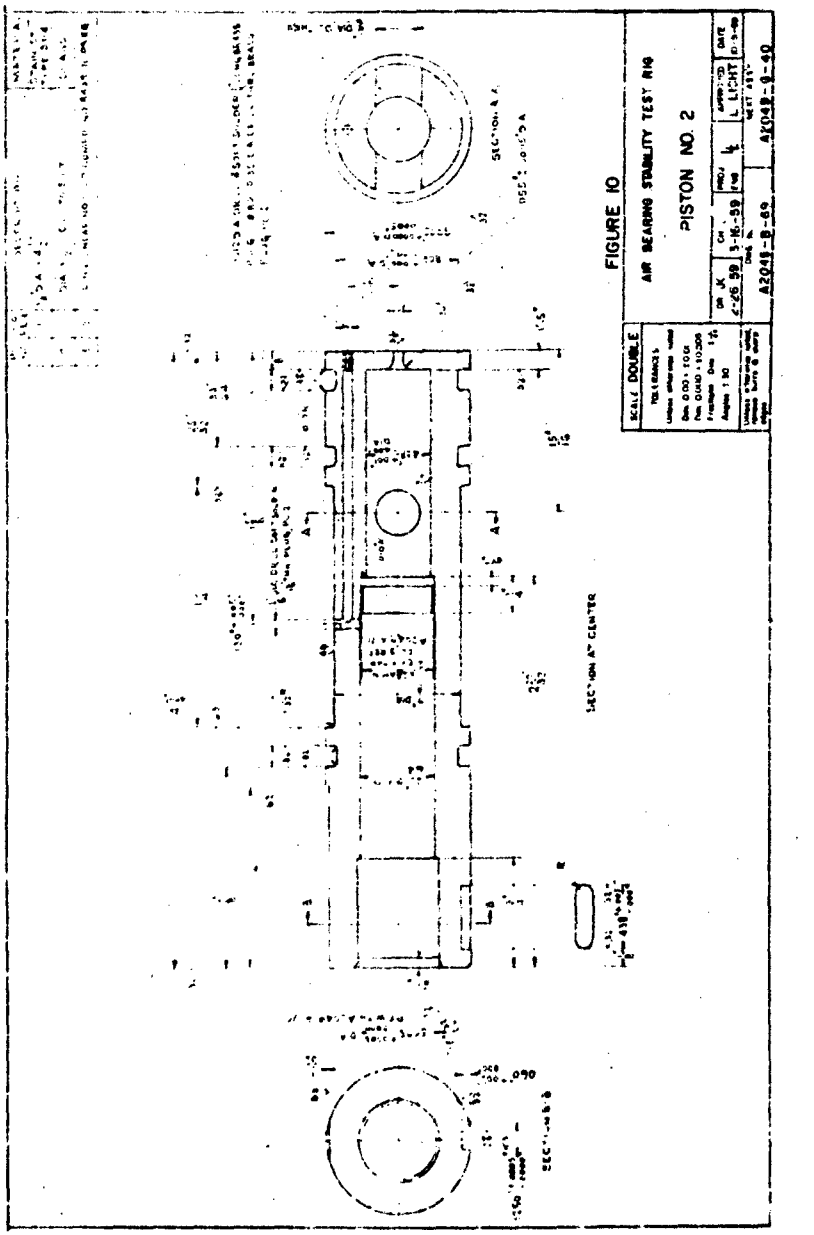


FIGURE 10
AIR BEARING STABILITY TEST RIG
PISTON NO. 2
A2049-0-40
SECTION A-A
SECTION AT CENTER

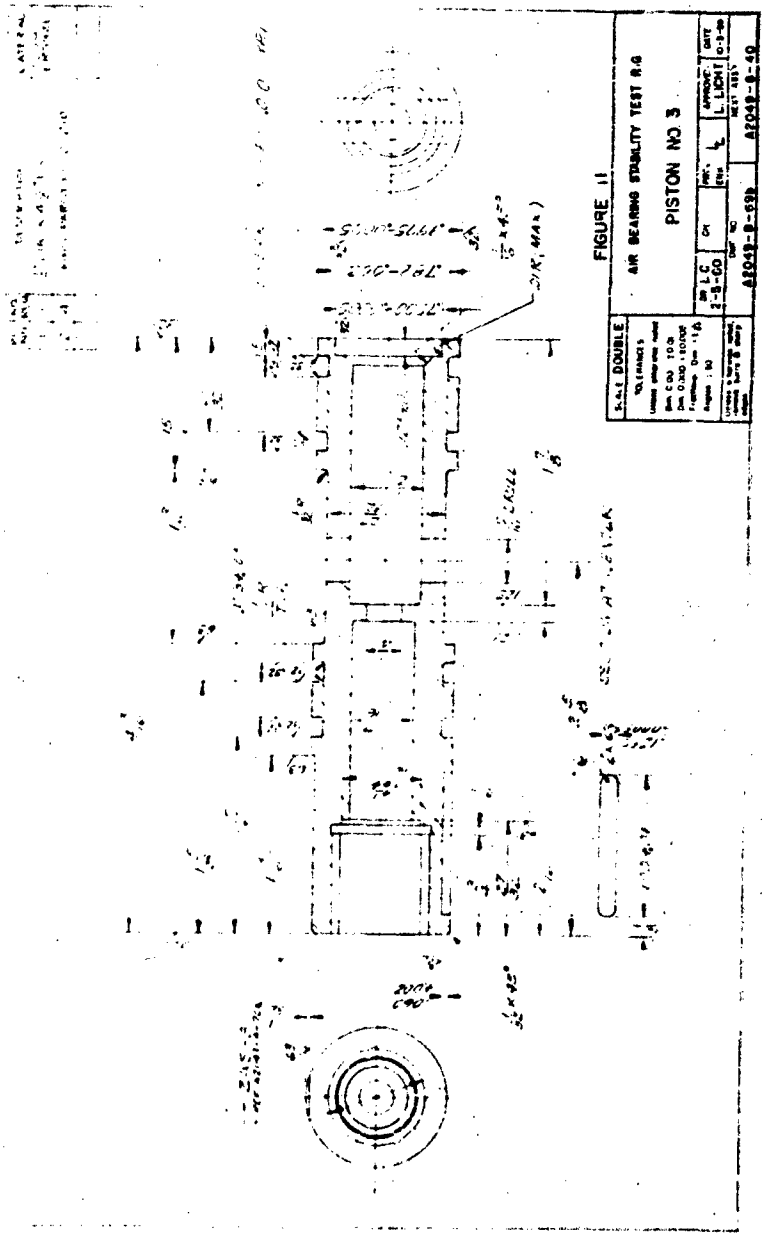
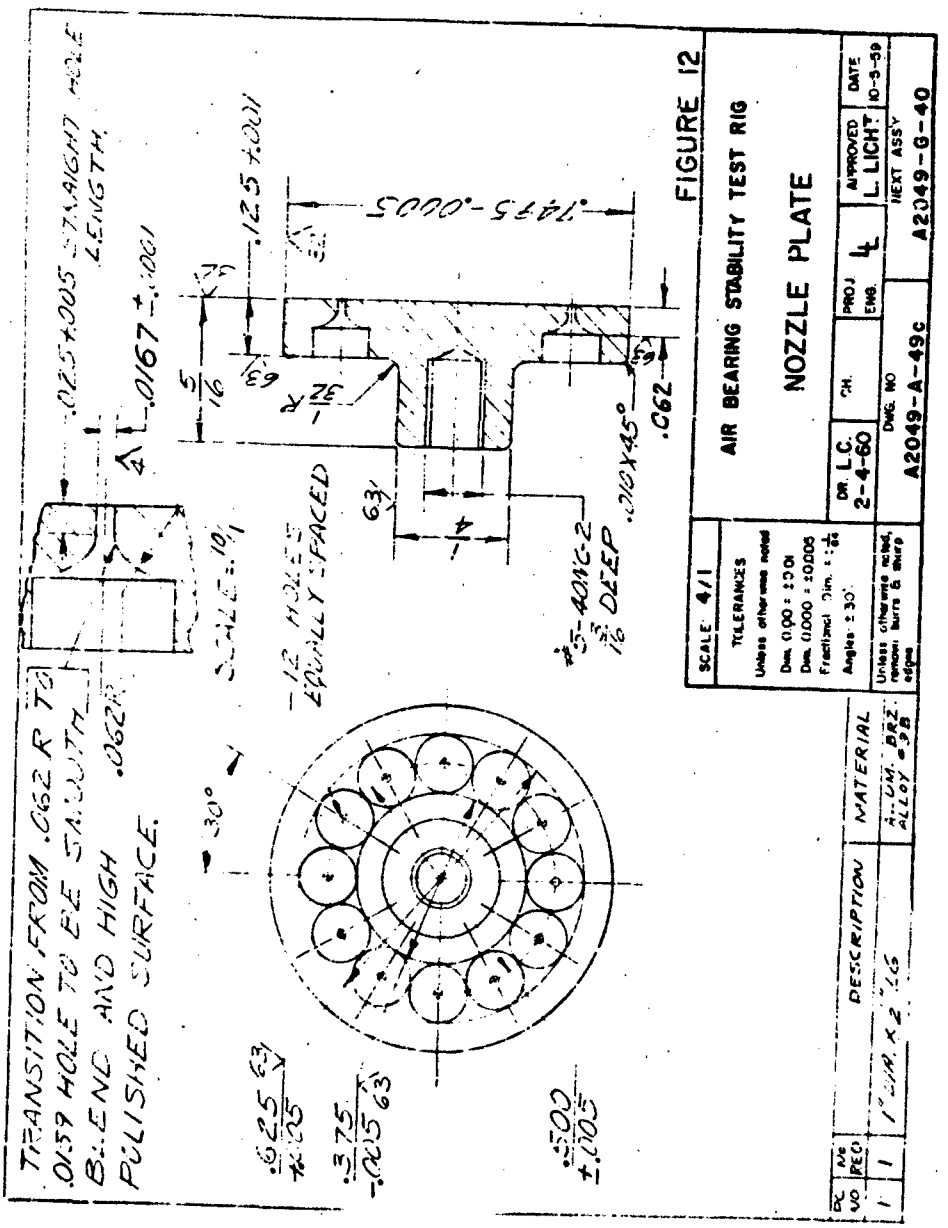
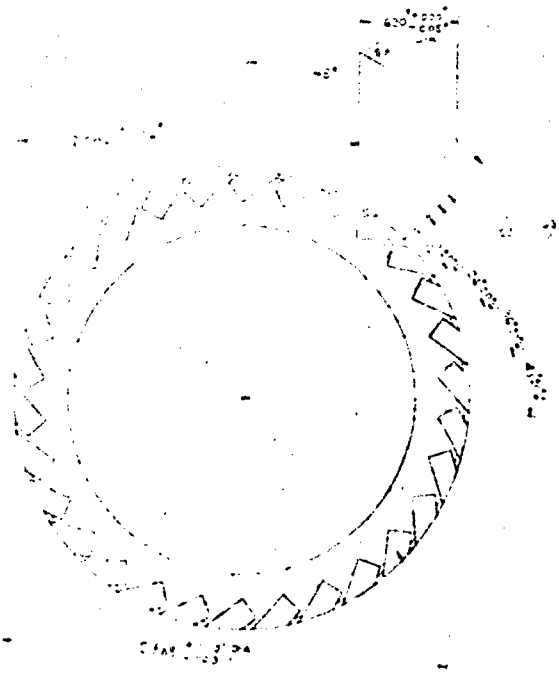
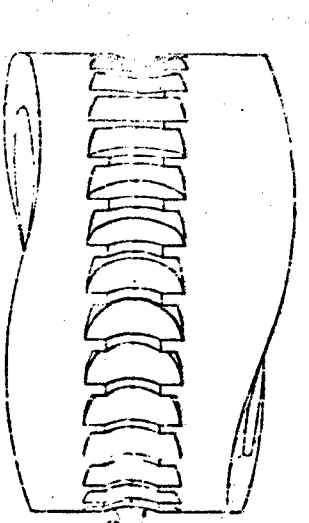


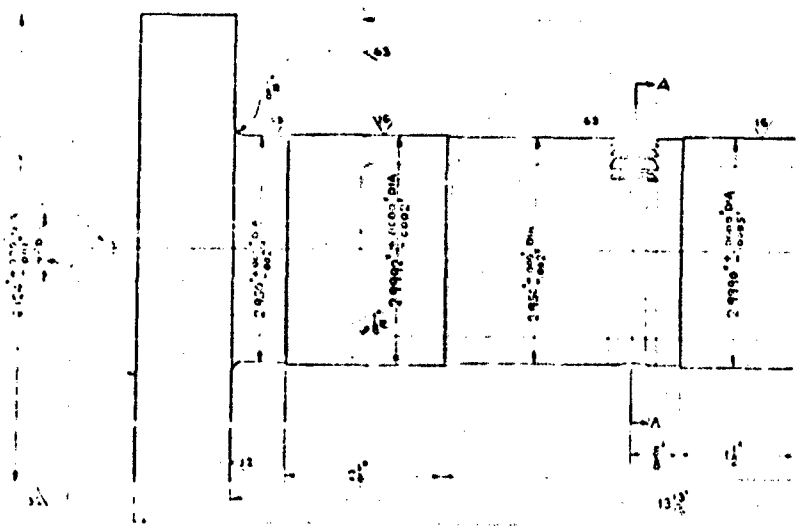
FIGURE 11
AIR BEARING STABILITY TEST RIG
PISTON NO 3
A2049-B-501
A2049-B-50


TRANSITION FROM .062 R TO .059 HOLE TO BE SMOOTH BLEND AND HIGH POLISHED SURFACE.
.062
SCALE: 10/1
12 HOLES EQUALLY SPACED
#5-40UNC-2B .16 DEEP
.30X45°
.062
.1875-.0005
.125±.001
.0167±.0001
.025±.005 STRAIGHT HOLE LENGTH
30°
.625 63 ±.125
.375 -.125 63
.500 ±.005
FIGURE 12
AIR BEARING STABILITY TEST RIG
NOZZLE PLATE
SCALE 4/1
TOLERANCES
Unless otherwise noted
Dec. 0.00 ± .01
Dec. 0.000 ± .0005
Fractional Dim. ± 1/64
Angles ± 30'
Unless otherwise noted, remove burrs & sharp edges
DR. L.C. 2-4-60
CH.
PROJ ENG. L
APPROVED L. LICHT
DATE 10-3-59
DWG. NO A2049-A-49c
NEXT ASS'Y A2049-G-40
DESCRIPTION
MATERIAL
ALUM. BRZ. ALLOY #2B
NO REQ 1
DWG NO 1

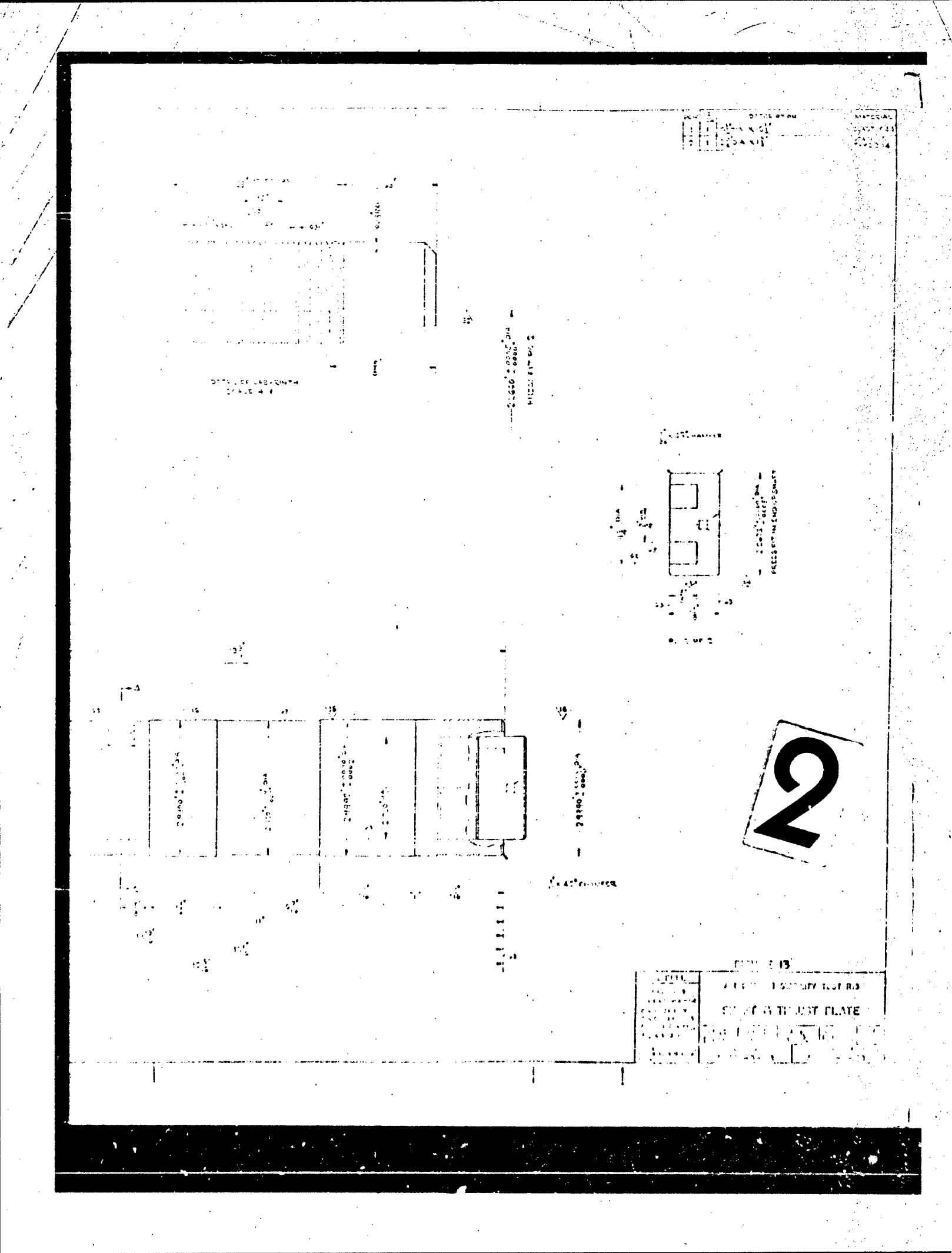


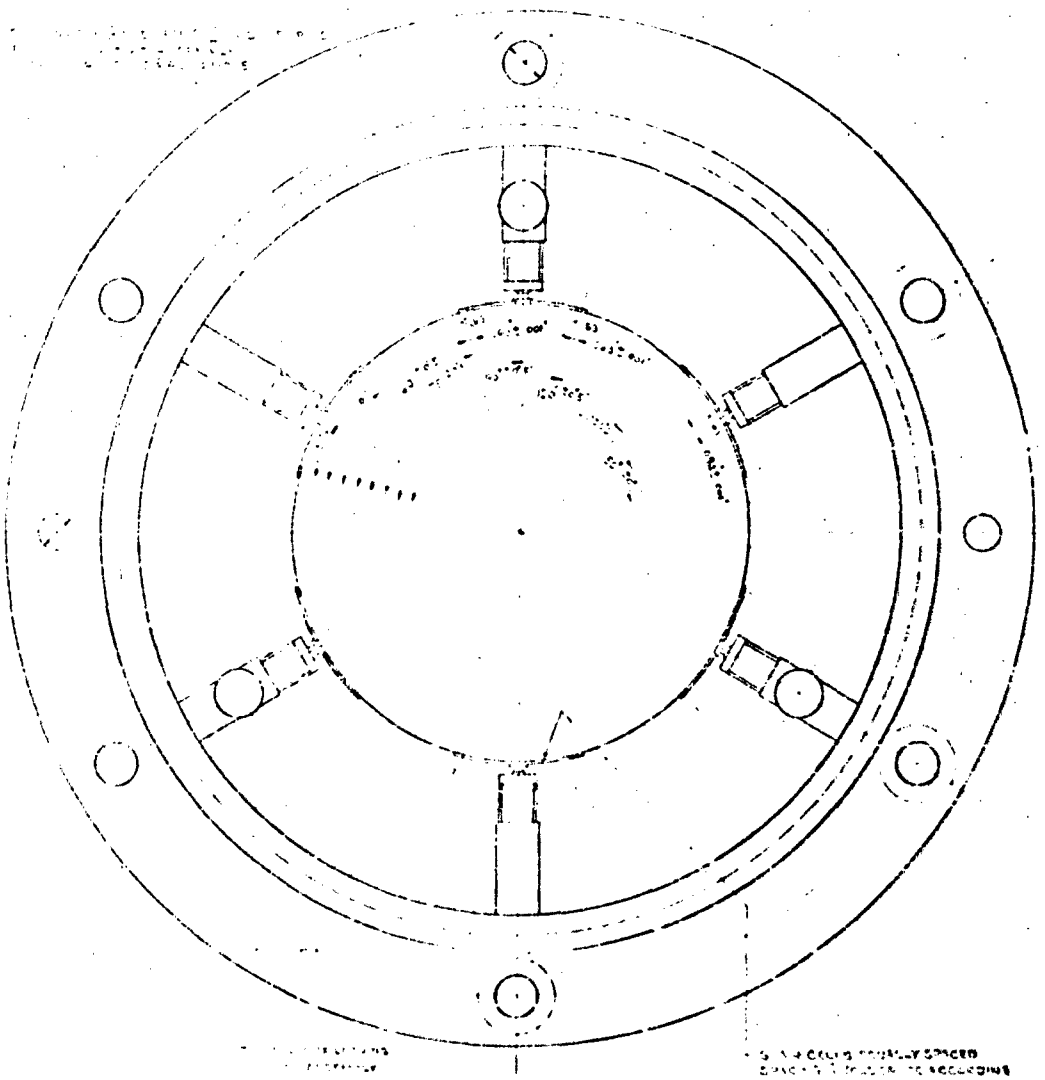


SECT ON A-A



1





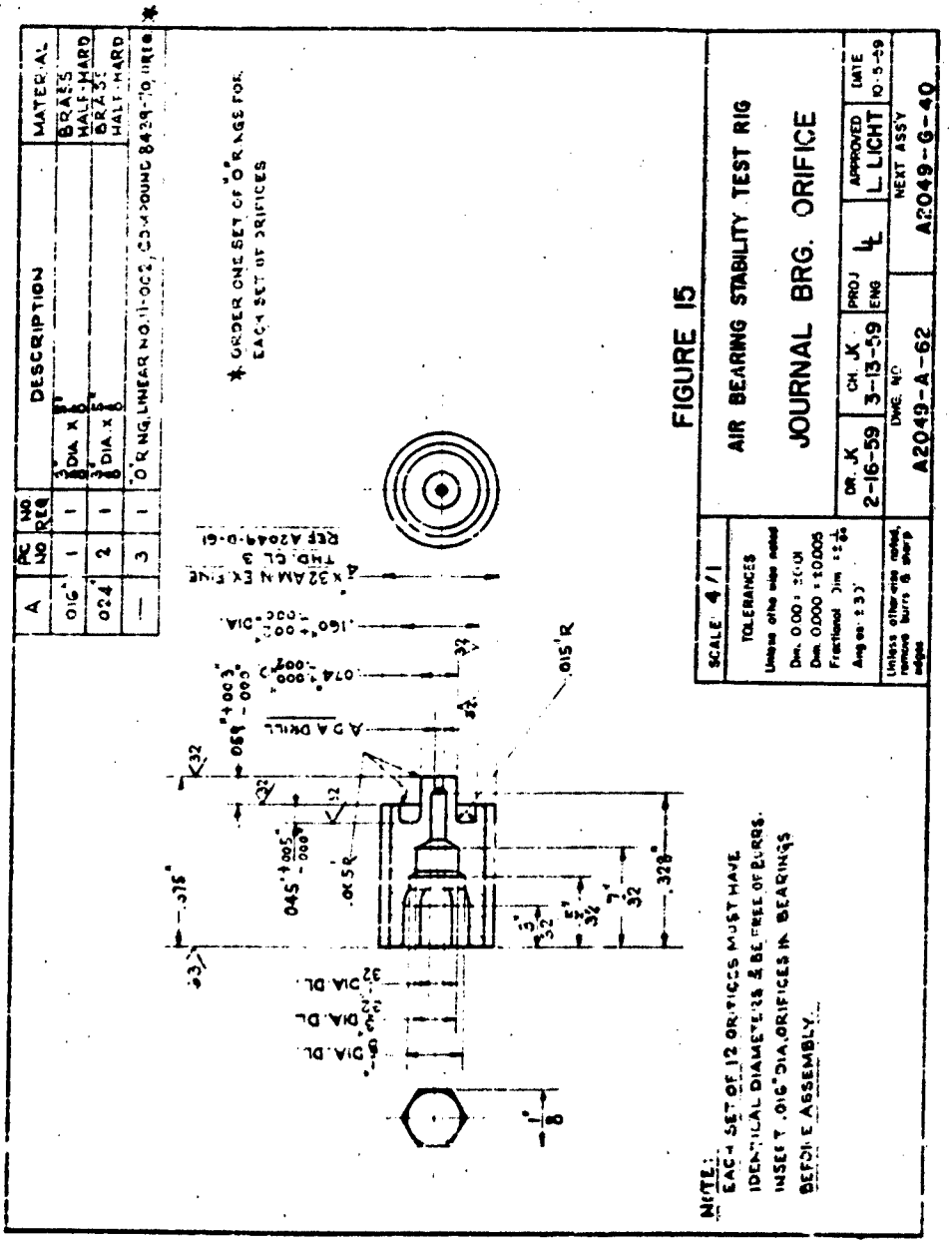
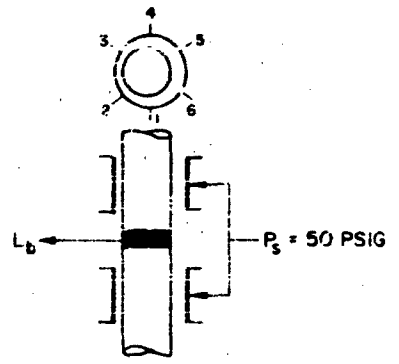


FIGURE 15
AIR BEARING STABILITY TEST RIG
JOURNAL BRG. ORIFICE
A2049-A-62
A2049-G-40
DR. JK 2-16-59
CH. JK 3-13-59
APPROVED L. LICHT
SCALE 4/1
TOLERANCES
DESCRIPTION
MATERIAL
BRASS HALF-HARD
BRZ. ST HALF-HARD
O RING, LINEAR NO. 11-002, COMPOUND 8429-70 (REF)
ORDER ONE SET OF O RINGS FOR EACH SET OF ORIFICES
NOTE:
EACH SET OF 12 ORIFICES MUST HAVE IDENTICAL DIAMETERS & BE FREE OF BURRS. INSERT .016" DIA ORIFICES IN BEARINGS BEFORE ASSEMBLY.

NOMINAL BEARING DIMENSIONS

BEARING DIAM = 3 IN
BEARING LENGTH = 2 IN
DIAM CLEARANCE = 0.0014 IN.

SIX EQUALLY SPACED
PRESSURE PADS AND
0.016 IN DIAMETER SUPPLY
ORIFICE RESTRICTORS



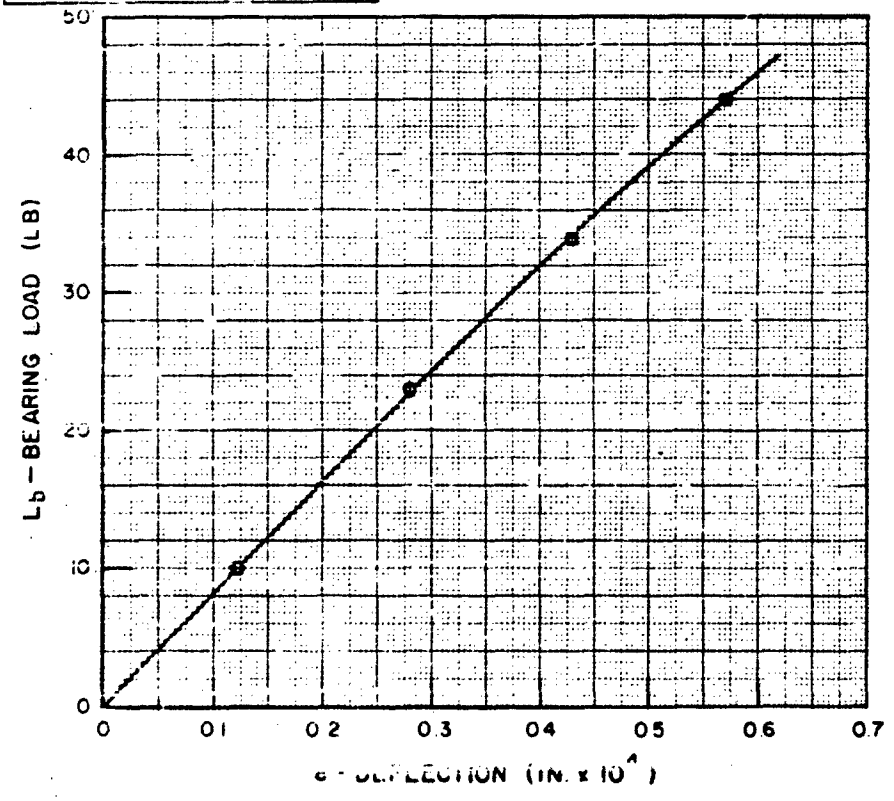


FIG 16 LOAD-DEFLECTION CHARACTERISTIC OF GUIDE BEARINGS

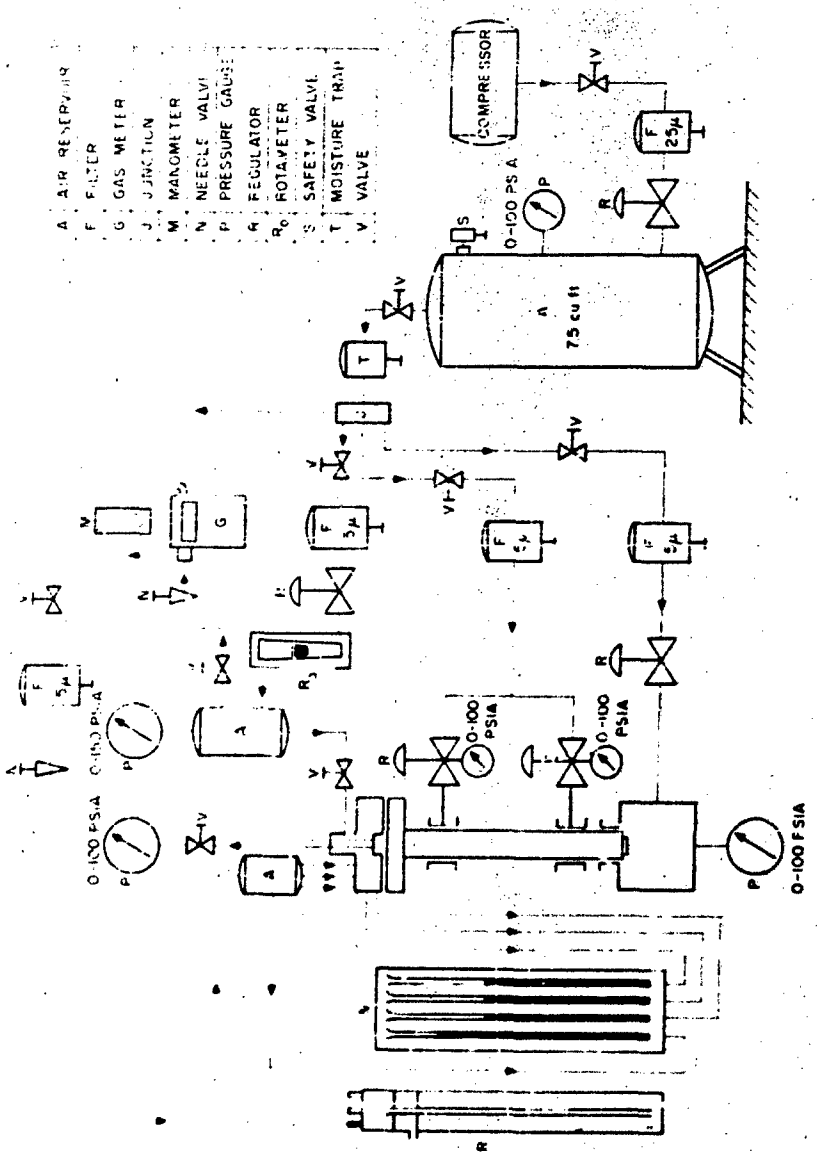


FIG. 17. SCHEMATIC DIAGRAM OF AIR SUPPLY SYSTEM

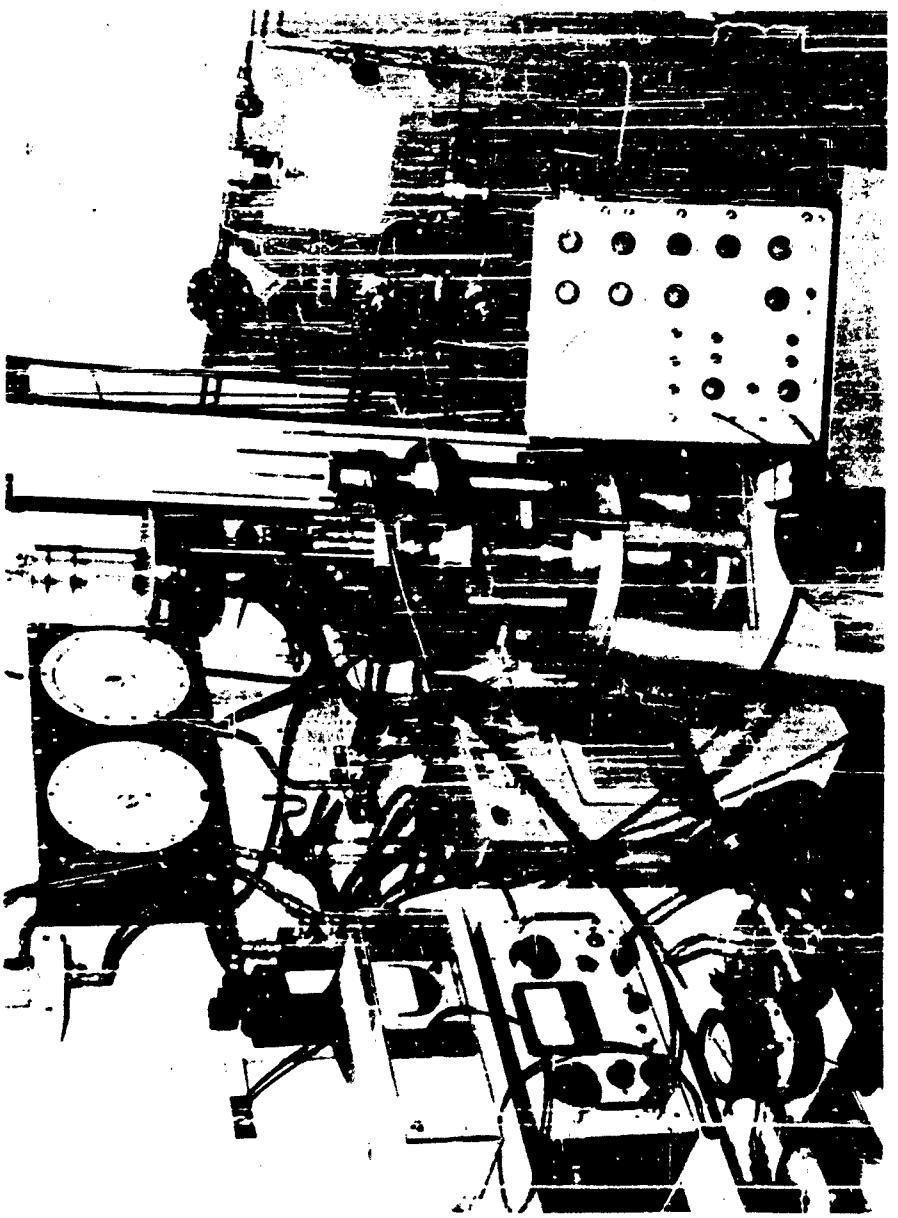


FIG. 18 VIEW OF TEST APPARATUS, INSTRUMENTS AND AIR SUPPLY SYSTEM

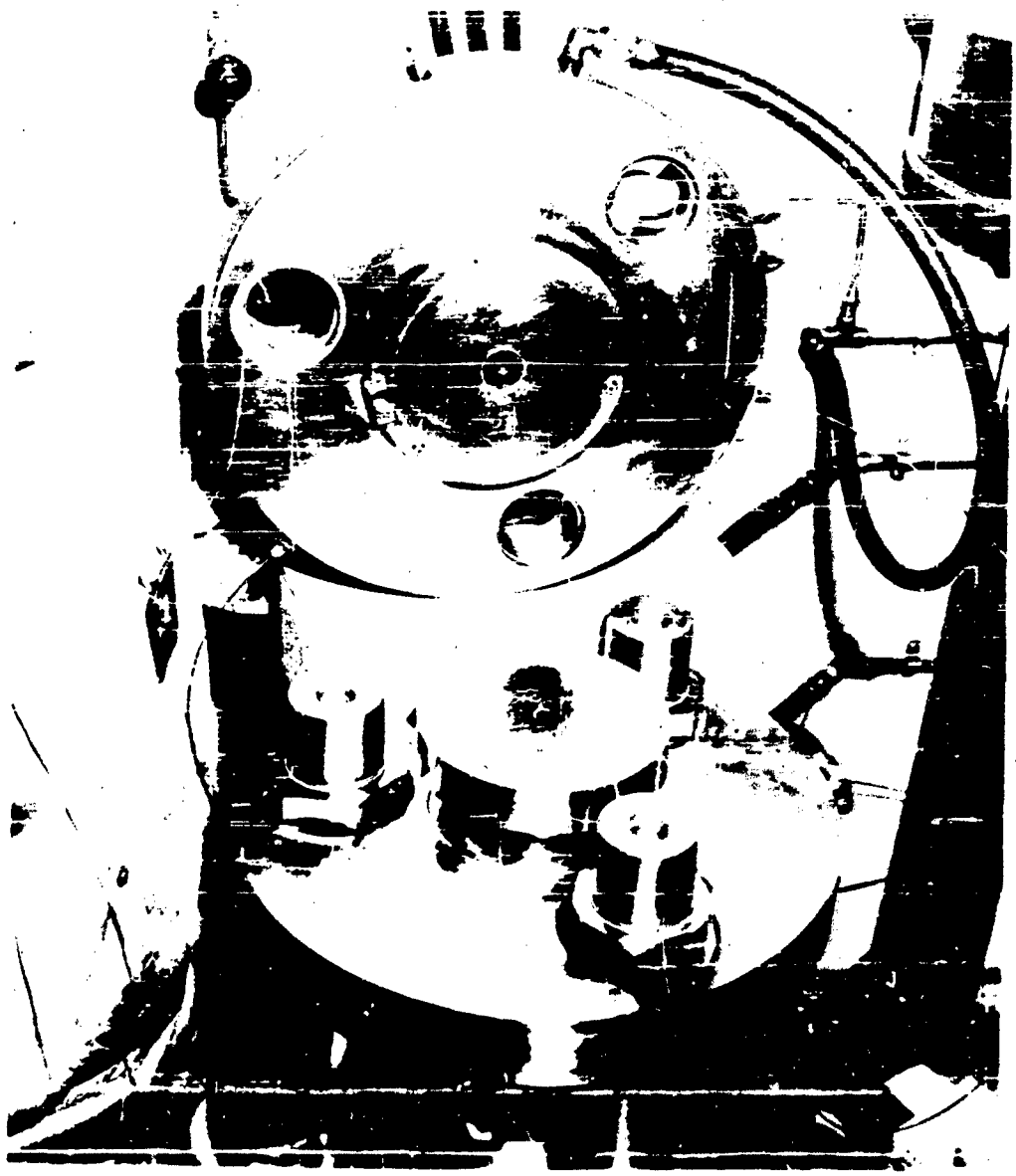


FIG 19. VIEW OF BEARING THRUST PLATES (1)

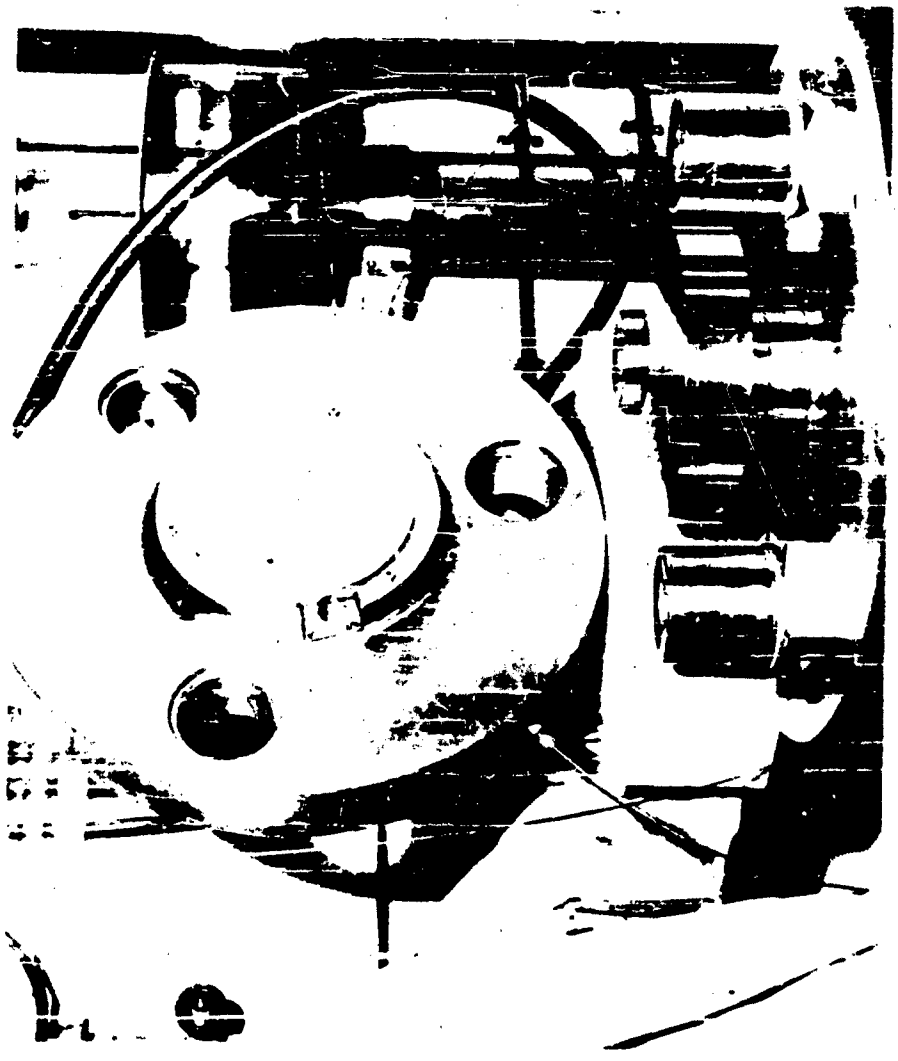


FIG. 20. VIEW OF BEARING THRUST PLATES (2)

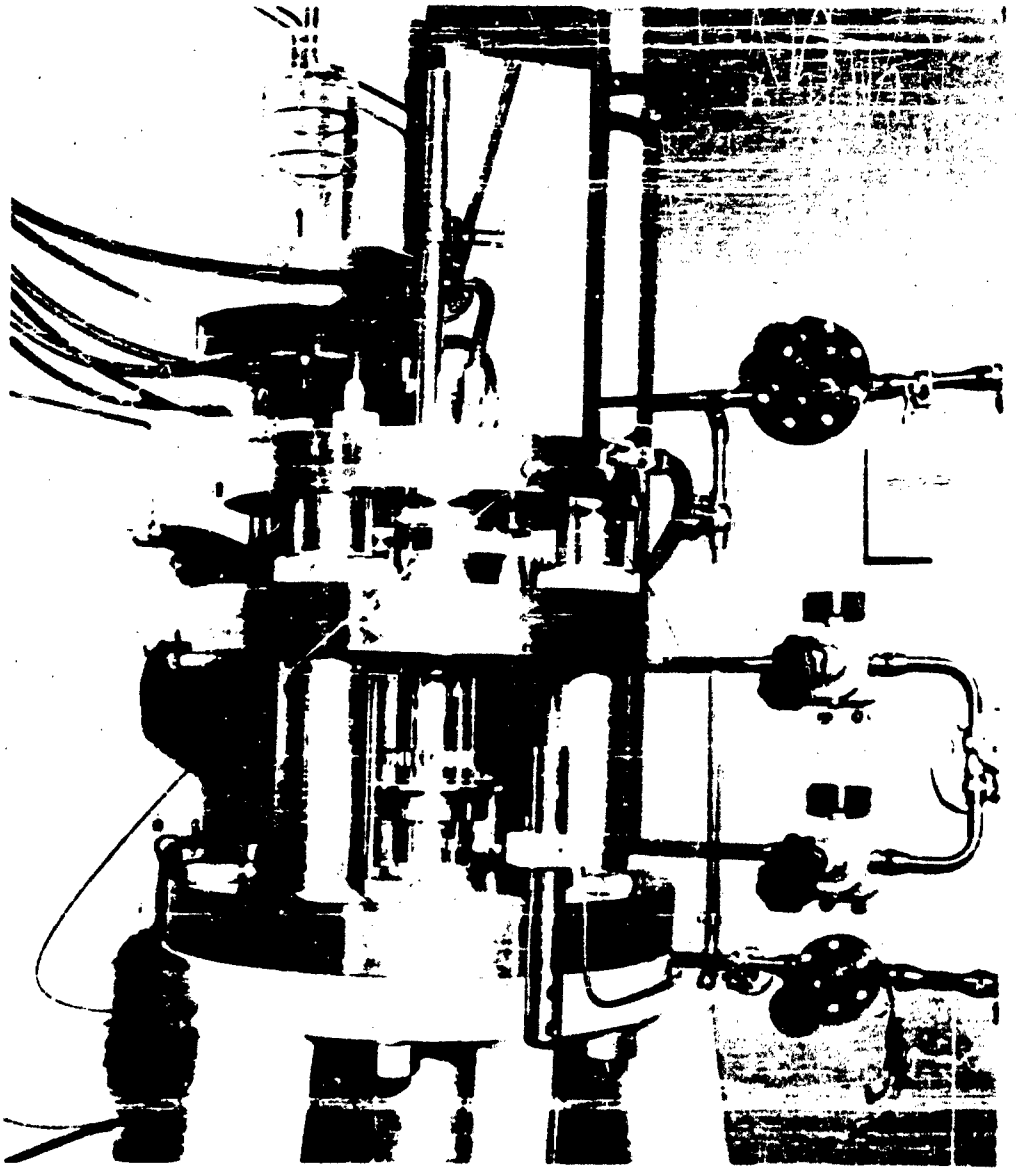


FIG 21. CLOSE VIEW OF TEST BEARING ASSEMBLY

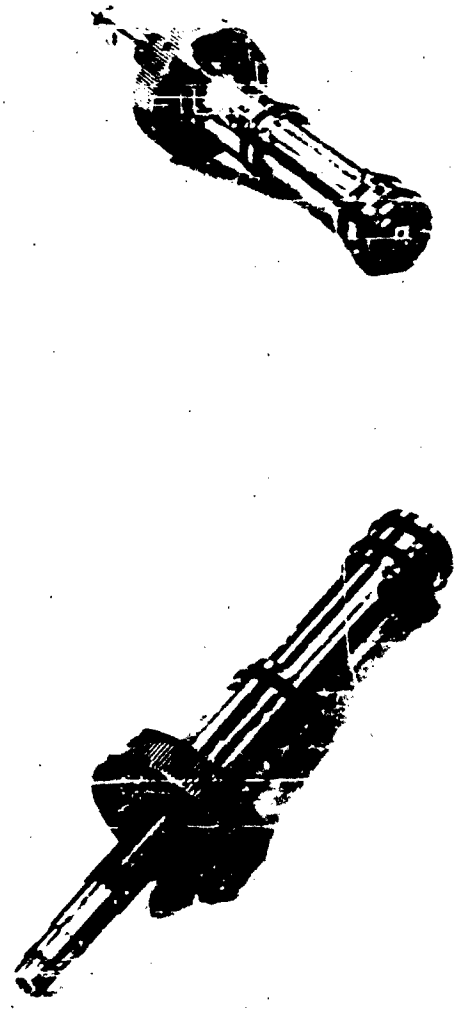
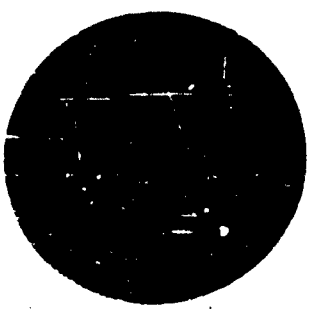


FIG 22. MODIFIED PISTON NO 2 - SECONDARY RECESS



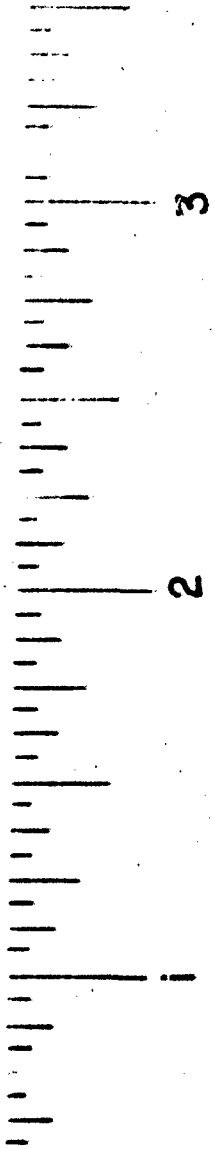


FIG. 23. NOZZLE PLATES FOR PISTON NO. 3

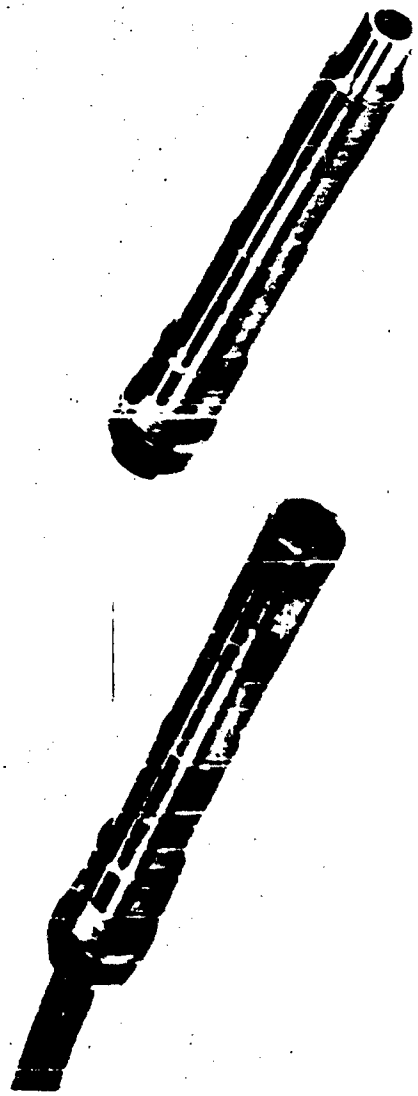


FIG. 24. PRIMARY CAPACITANCE PROBES AND PROBE HOLDERS

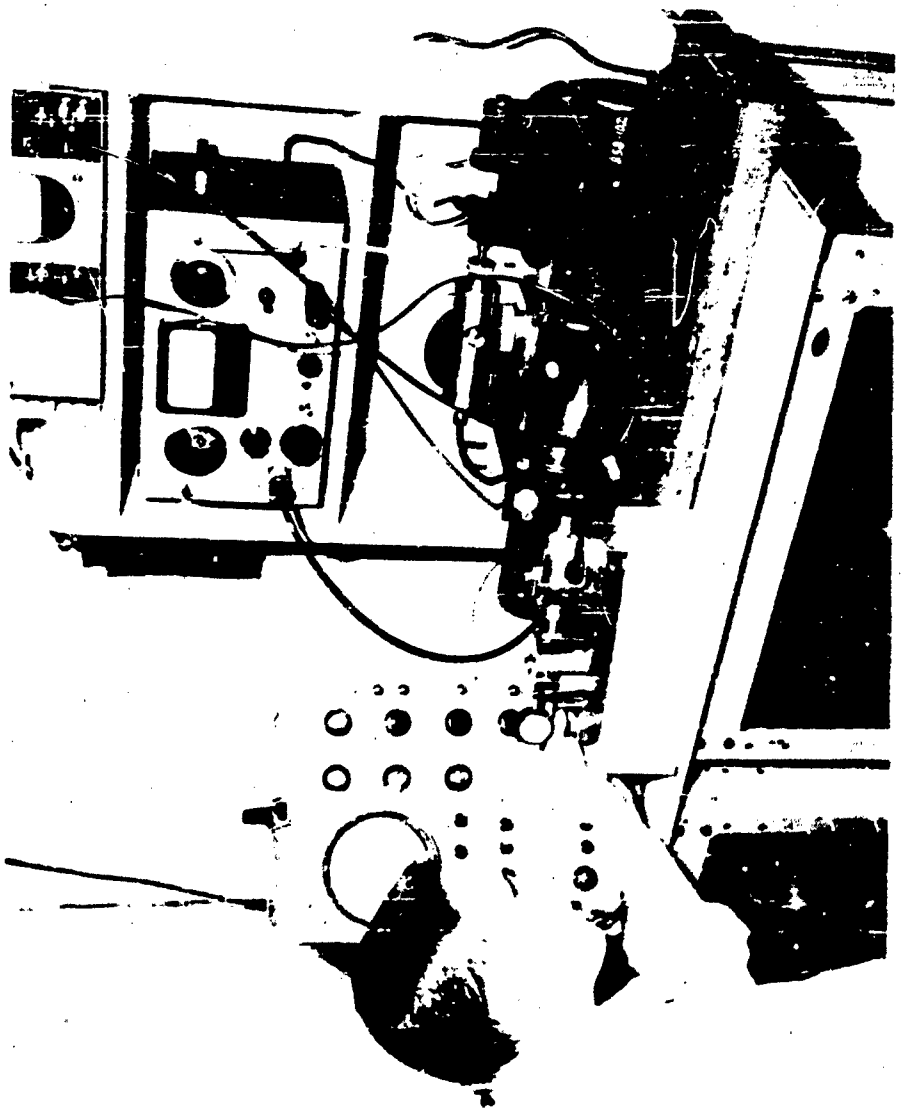


FIG. 25. CALIBRATION FIXTURES AND AUXILIARY PROBE

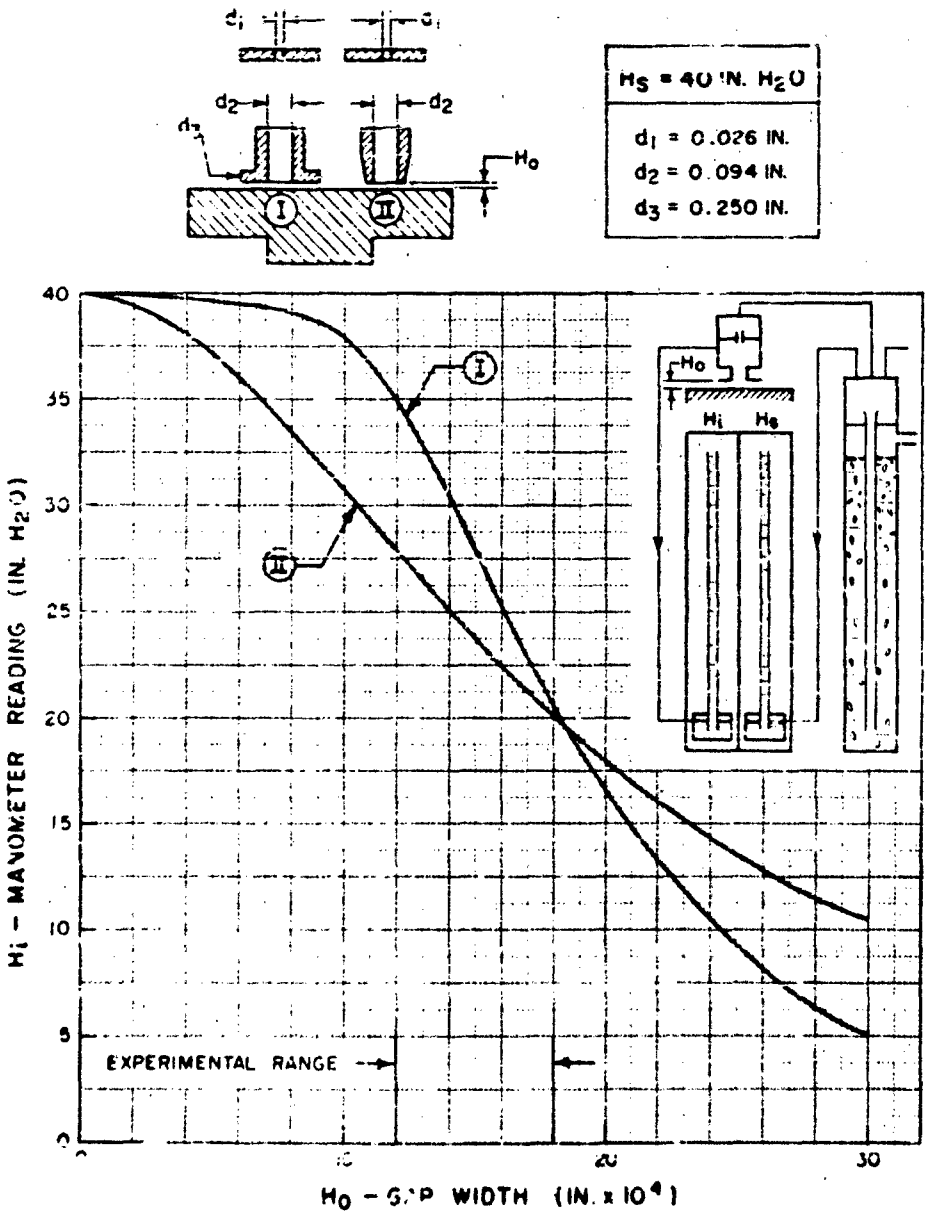


FIG 26 CALCULATED (APPROXIMATE) AIR GAUGE CHARACTERISTICS

SUPPLY ORIFICE DIAM.	d_1 = 0.026"
SUPPLY ORIFICE LENGTH	l_1 = 0.060"
PROBE INSIDE DIAM	d_2 = 0.094"
PROBE OUTSIDE DIAM	d_3 = 0.250"
PROBE LENGTH	l_2 = 0.385"
SUPPLY HEAD	H_s = 39.2 IN H_2O
BAROMETRIC PRESSURE	P_{at} = 14.72 PSIA
ROOM TEMPERATURE	T_a = 74°-76°F

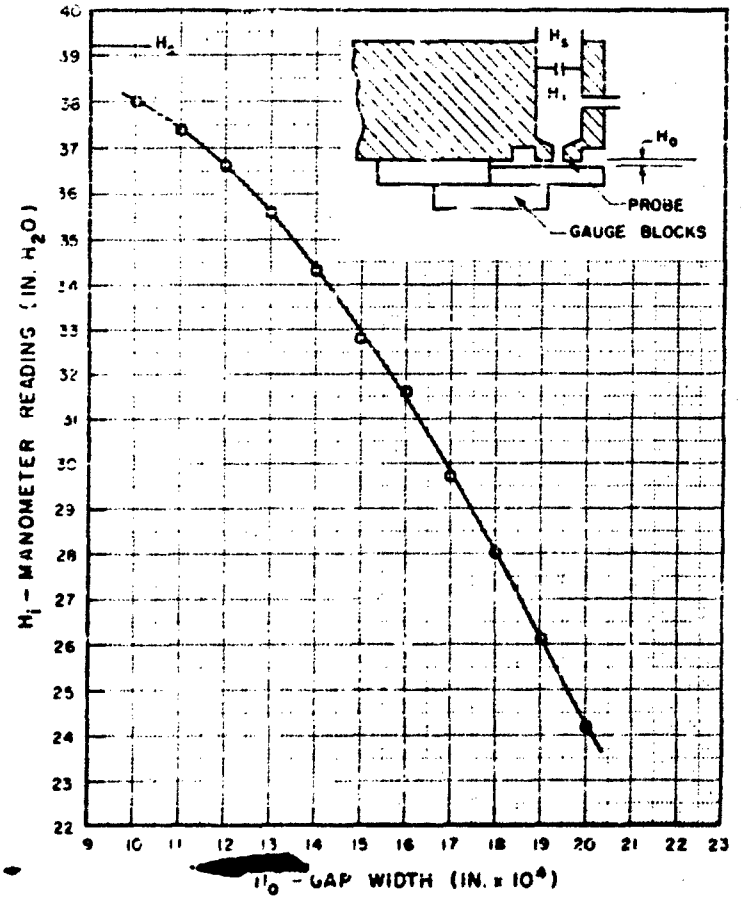


FIG. 27 CALIBRATION CURVE
(Air Gauge No 1)

SUPPLY ORIFICE DIAM	d_1 = 0.026"
SUPPLY ORIFICE LENGTH	l_1 = 0.030"
PROBE INSIDE DIAM	d_2 = 0.094"
PROBE OUTSIDE DIAM	d_3 = 0.250"
PROBE LENGTH	l_2 = 0.385"
SUPPLY HEAD	H_s = 39.2 IN H_2O
BAROMETRIC PRESSURE	P_{a1} = 14.72 PSIA
ROOM TEMPERATURE	T_0 = 74°-76°F

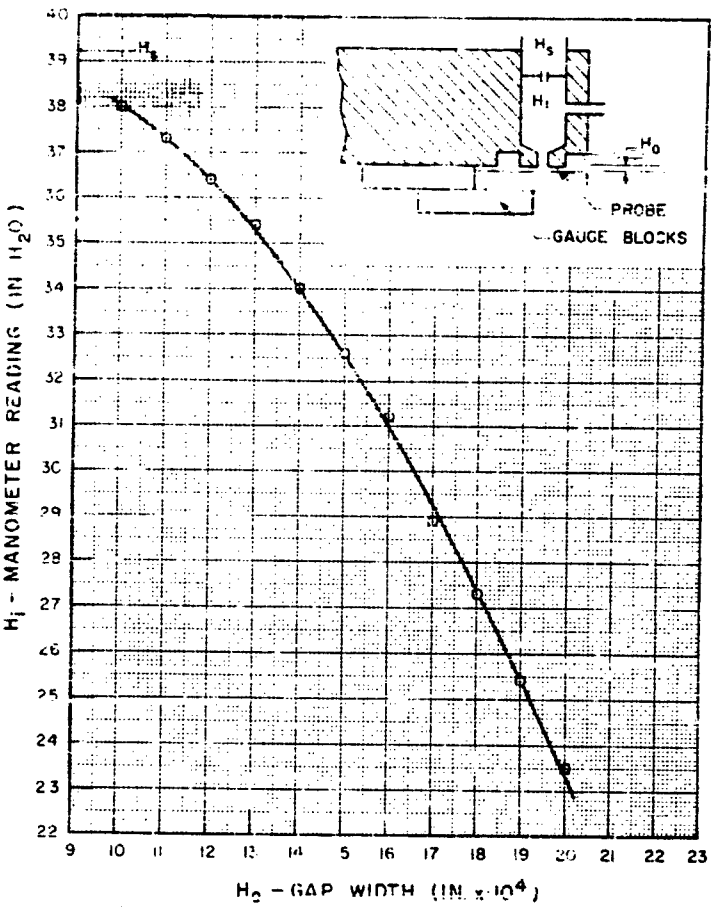


FIG. 26. CALIBRATION CURVE

(Air Gauge No 2.)

SUPPLY ORIFICE DIAM.	d_1 = 0.026"
SUPPLY ORIFICE LENGTH	l_1 = 0.060"
PROBE INSIDE DIAM.	d_2 = 0.094"
PROBE OUTSIDE DIAM.	d_3 = 0.250"
PROBE LENGTH	l_2 = 0.485"
SUPPLY GAP	H_s = 39.2 IN H_2O
BAROMETRIC PRESSURE	P_a = 14.72 PSIA
ROOM TEMPERATURE	T_0 = 74°-76°F

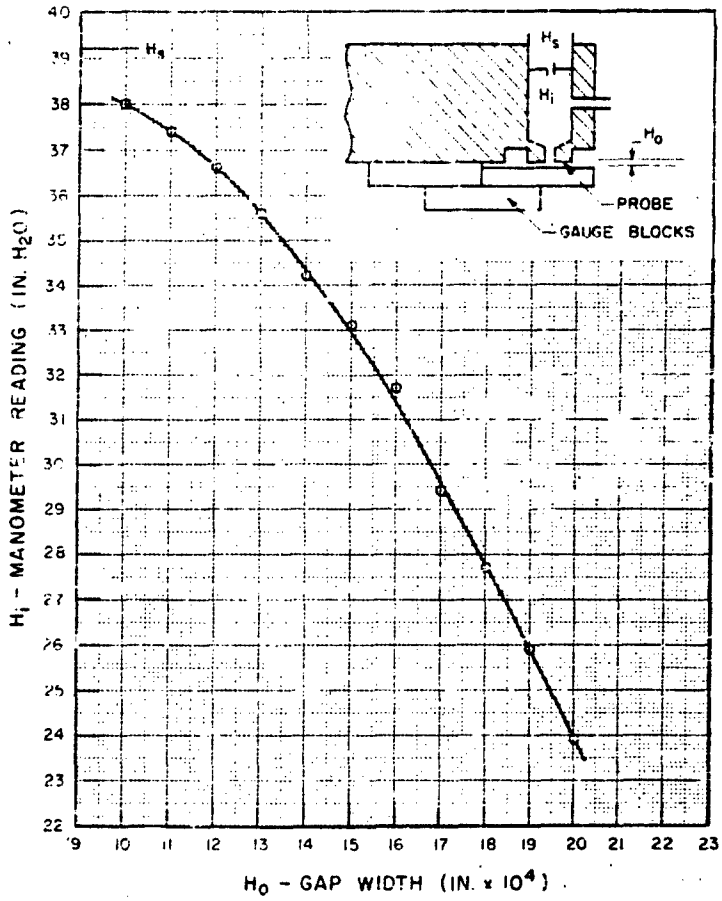


FIG. 29. CALIBRATION CURVE

(Air Gauge No. 3)

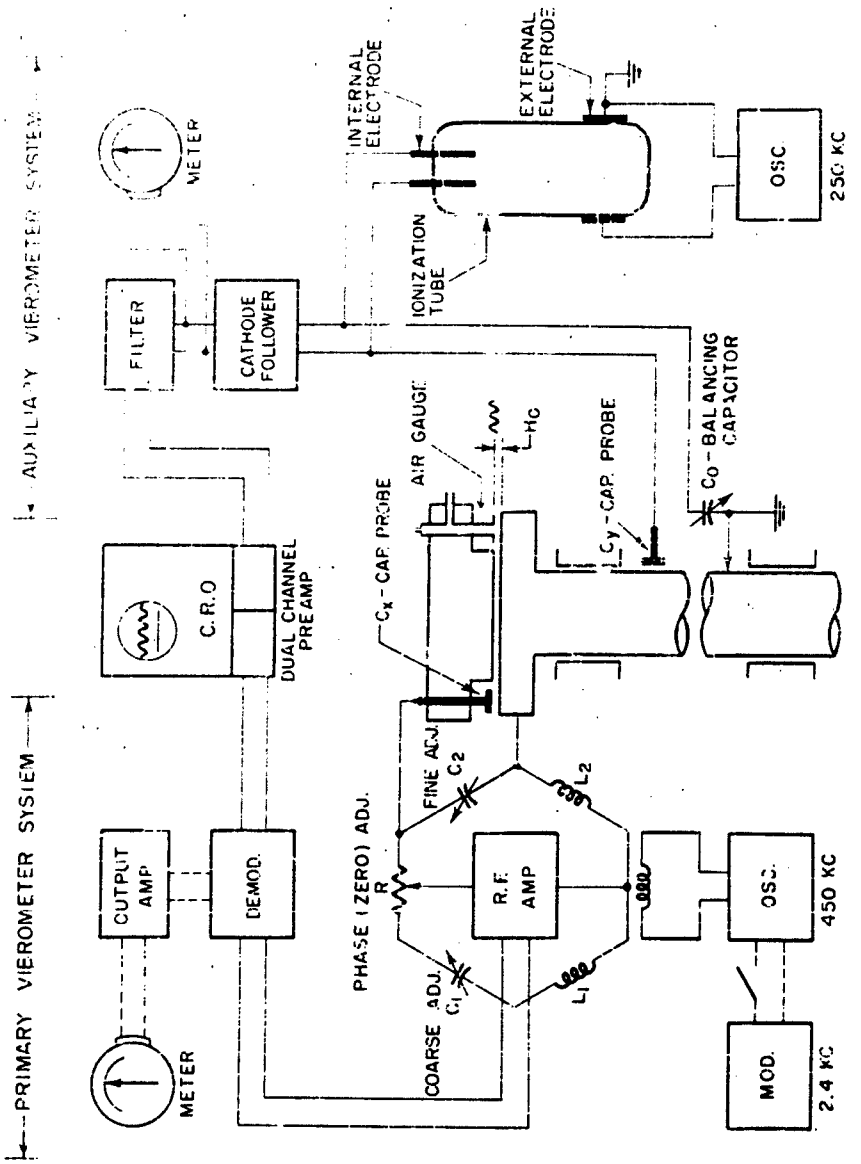


FIG. 30 SCHEMATIC DIAGRAM OF INSTRUMENTS FOR VIBRATION DETECTION AND MEASUREMENT

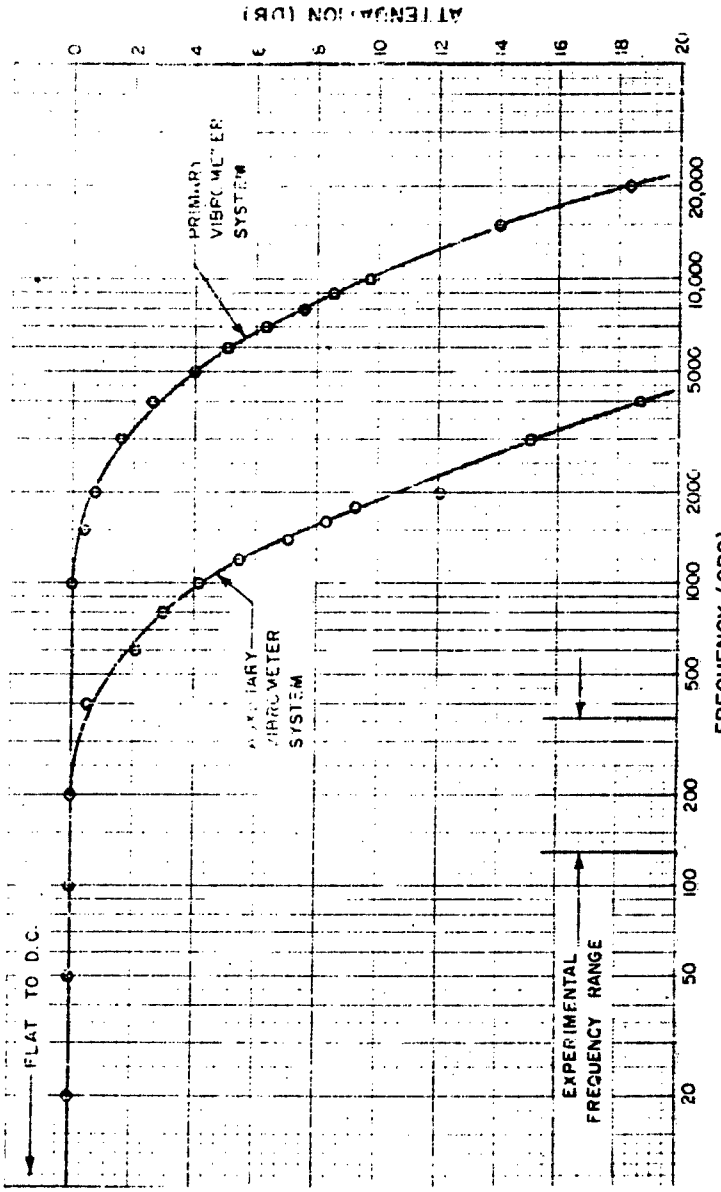


FIG. 31. FREQUENCY RESPONSE OF VIBRATION INSTRUMENTS

PROBE DIMENSION: 0.25 IN. DIAM
INITIAL (NULL) OFFSET: 4.35×10^{-3} IN.
SENSITIVITY: 1.05 V / 10^{-5} IN. AT NULL POINT

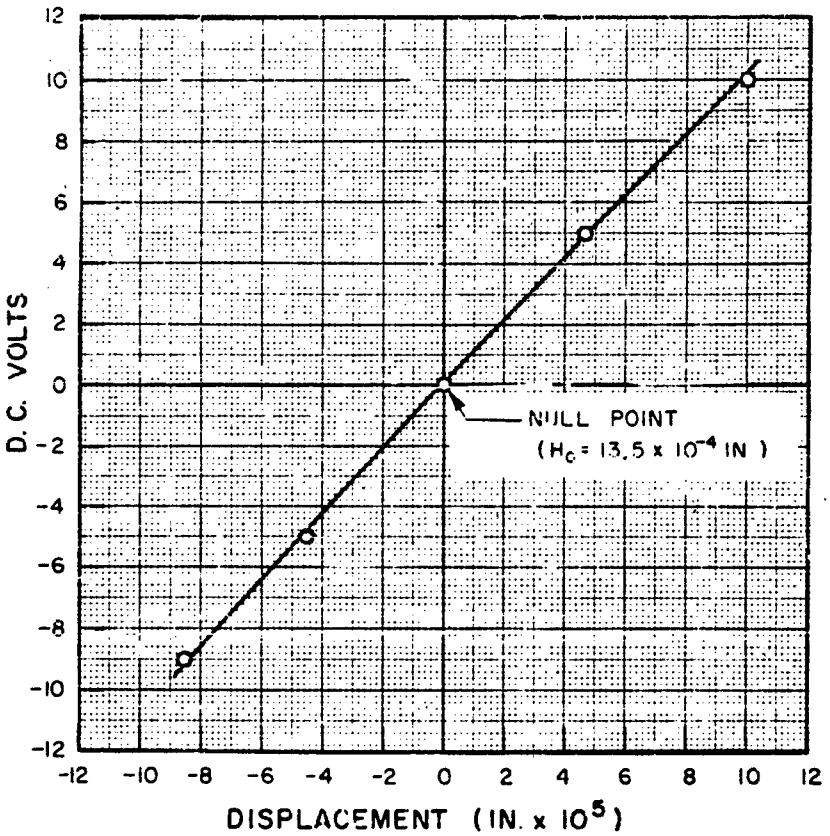


FIG. 32. CALIBRATION CURVE I.
(Primary Capacitance Probe)

PROBE DIMENSION:	0.25 IN. DIAM
INITIAL (NULL) OFFSET:	4.50×10^{-3} IN.
SENSITIVITY:	0.95 V / 10^{-5} IN. AT NULL POINT

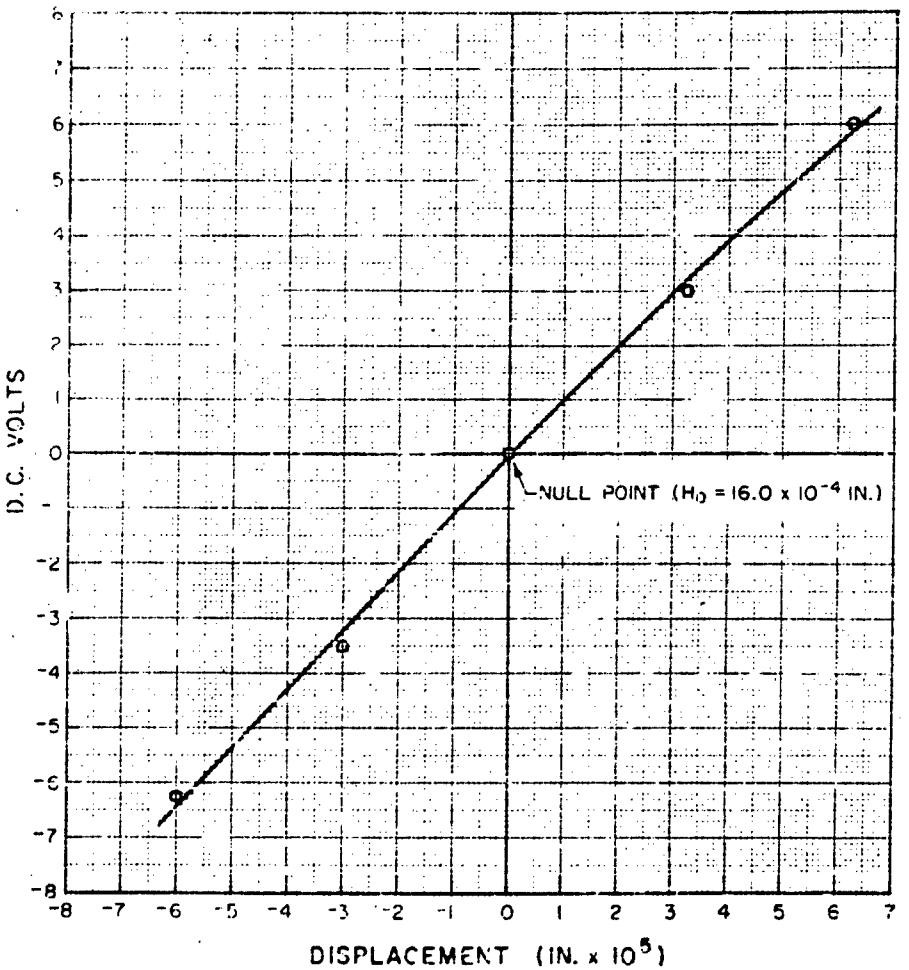


FIG. 33. CALIBRATION CURVE 2

(Primary Capacitance Probe)

PROBE DIMENSION :	0.25 IN DIAM
INITIAL (NULL) OFFSET :	4.80×10^{-3} IN.
SENSITIVITY : 0.95 V/10^{-5} IN	AT NULL POINT

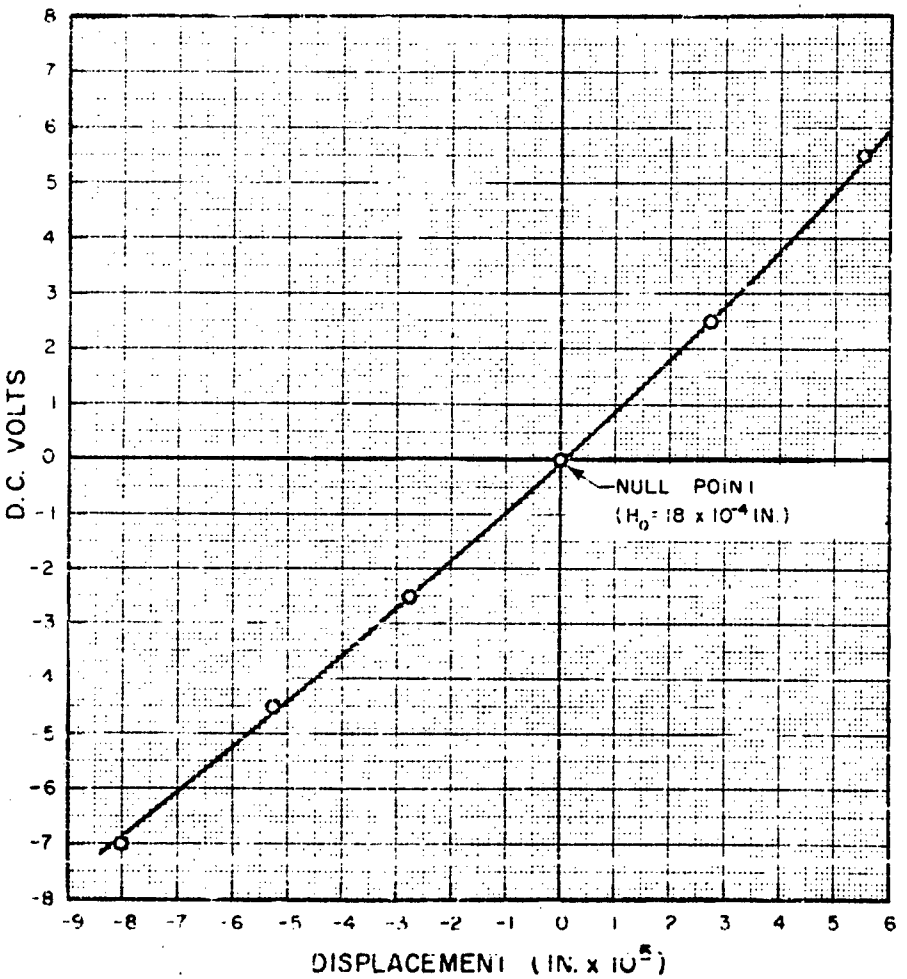


FIG. 34. CALIBRATION CURVE 3

(Primary Capacitance Probe)

PROBE DIMENSION: $\frac{1}{4}'' \times \frac{7}{8}''$
INITIAL (NULL) OFFSET: 1.3×10^{-3} IN.
SENSITIVITY: 0.02V/10^{-5} IN. AT NULL POINT

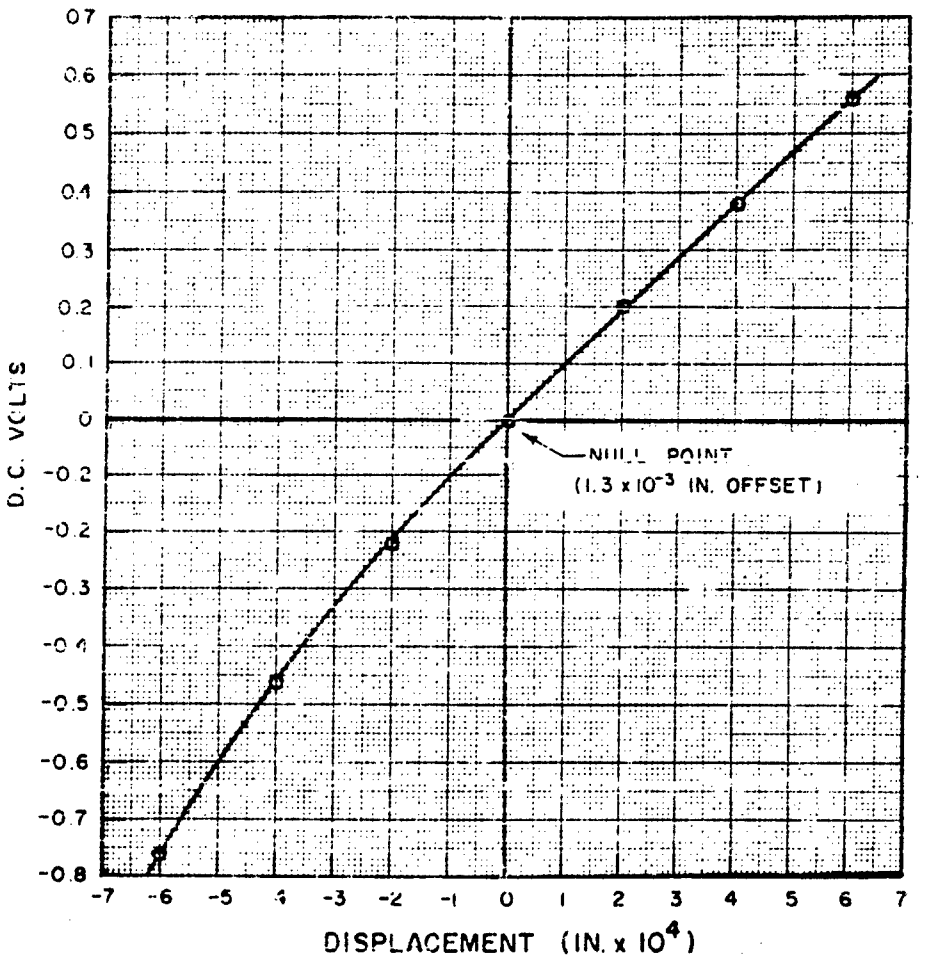
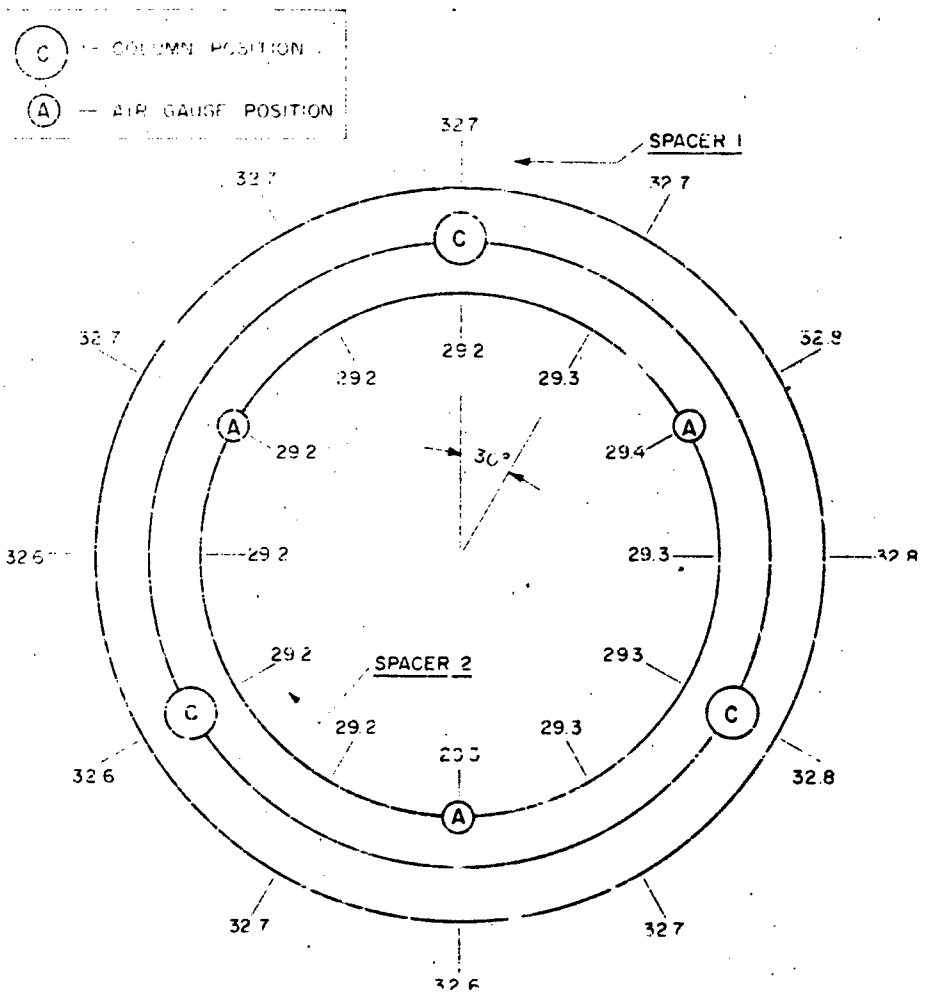


FIG. 35. CALIBRATION CURVE
(Auxiliary Capacitance Probe)



NUMBERS REFER TO MANOMETER READINGS IN INCHES H_2O (SEE FIG 27)

FIG. 36 RESULTS OF THRUST PLATE ALIGNMENT TEST

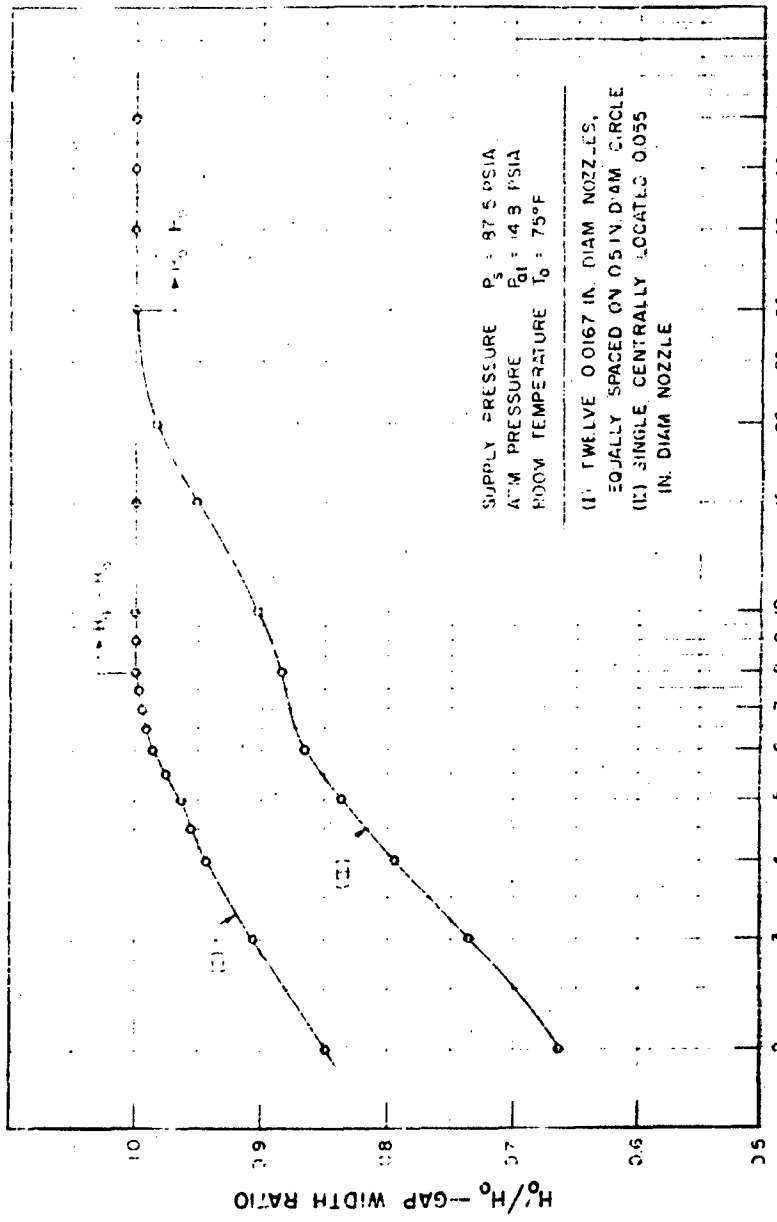


FIG. 37. EFFECT OF DECREASE IN RECESS DEPTH ON BEARING GAP WIDTH

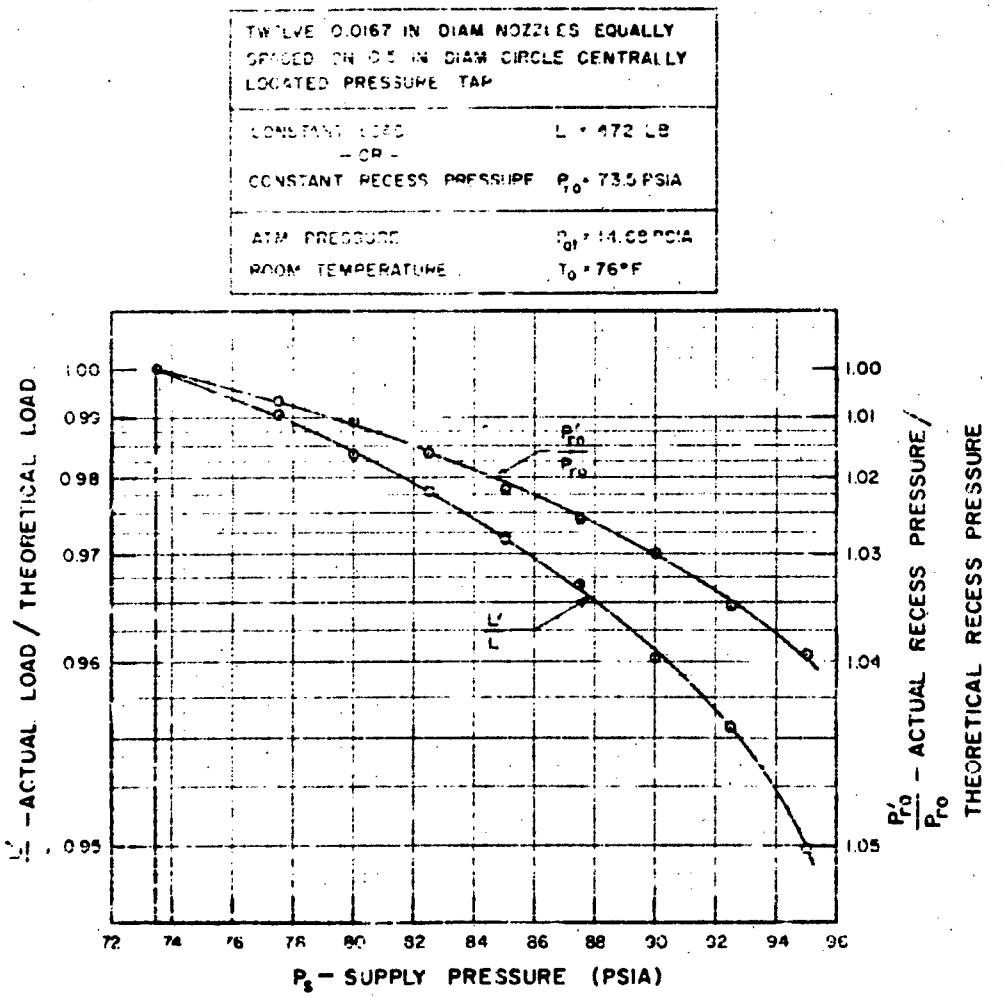


FIG. 38. VARIATION OF LOAD CAPACITY AND/OR RECESS PRESSURE WITH SUPPLY PRESSURE

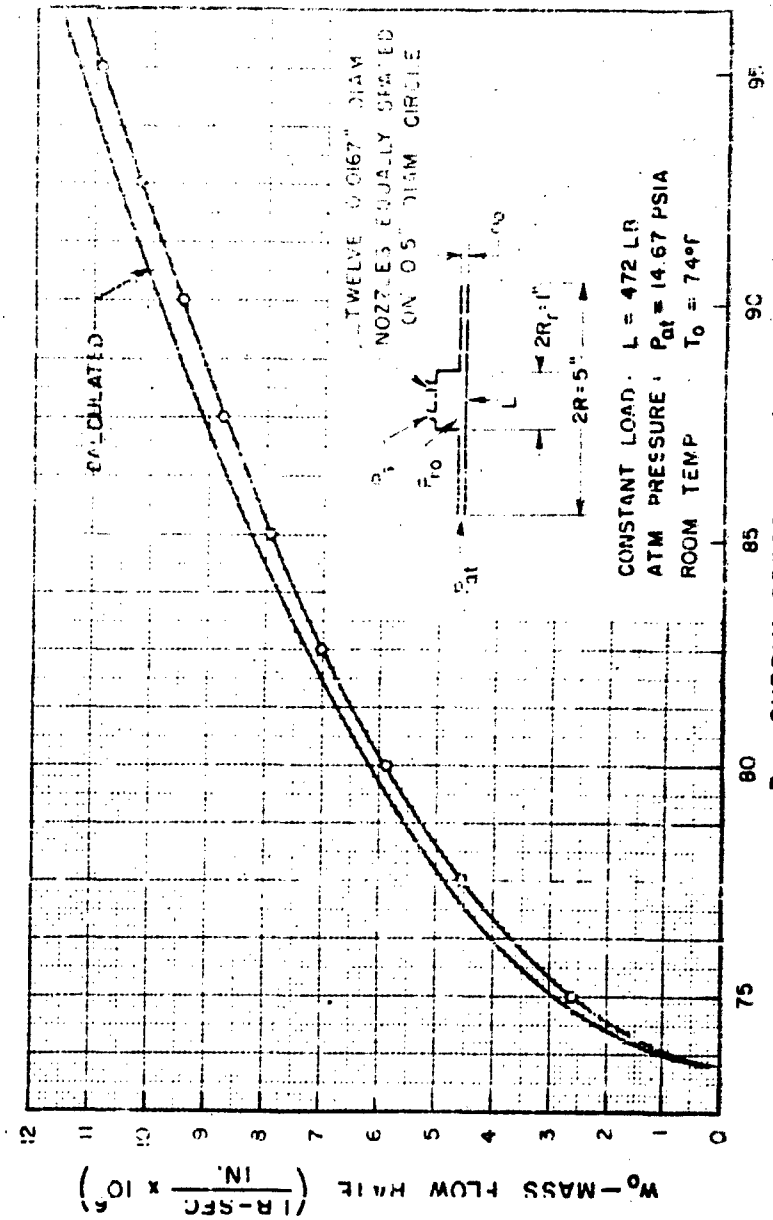


FIG. 39 BEARING MASS FLOW RATE vs. SUPPLY PRESSURE (CONSTANT LOAD = 472 LB)

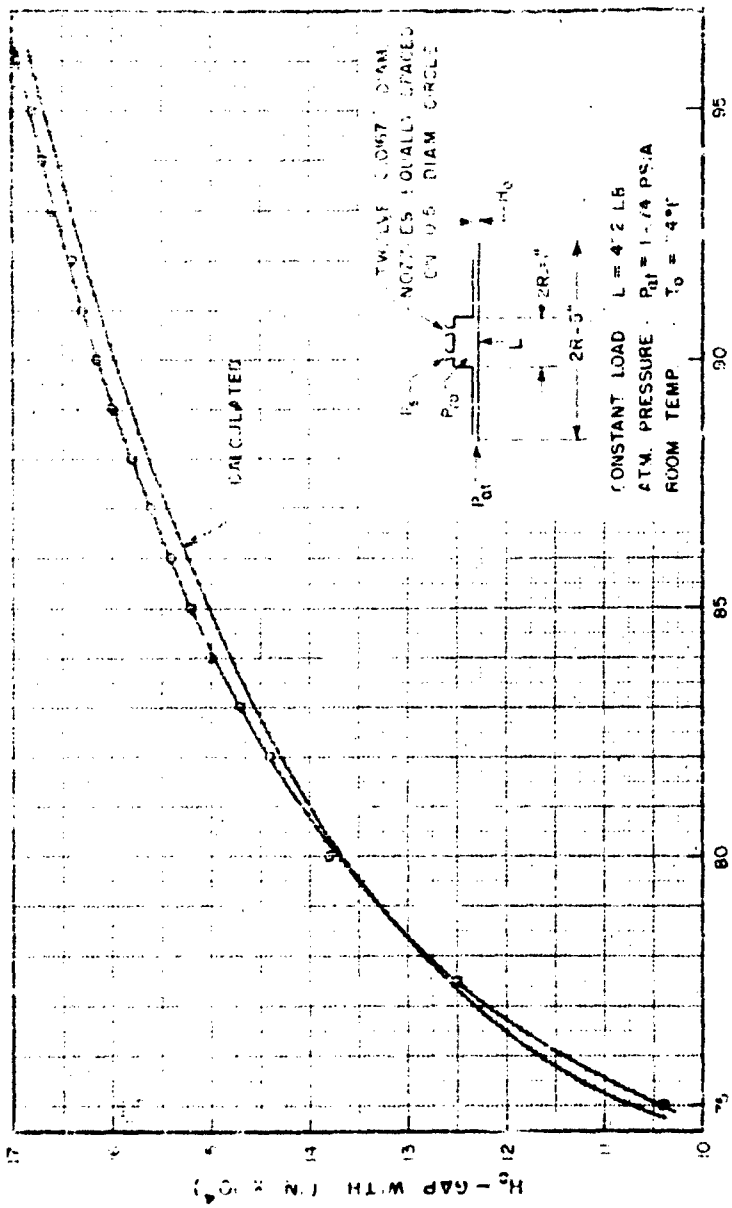


FIG. 40. BEARING GAP WIDTH vs SUPPLY PRESSURE (CONSTANT LOAD = 472 LB)

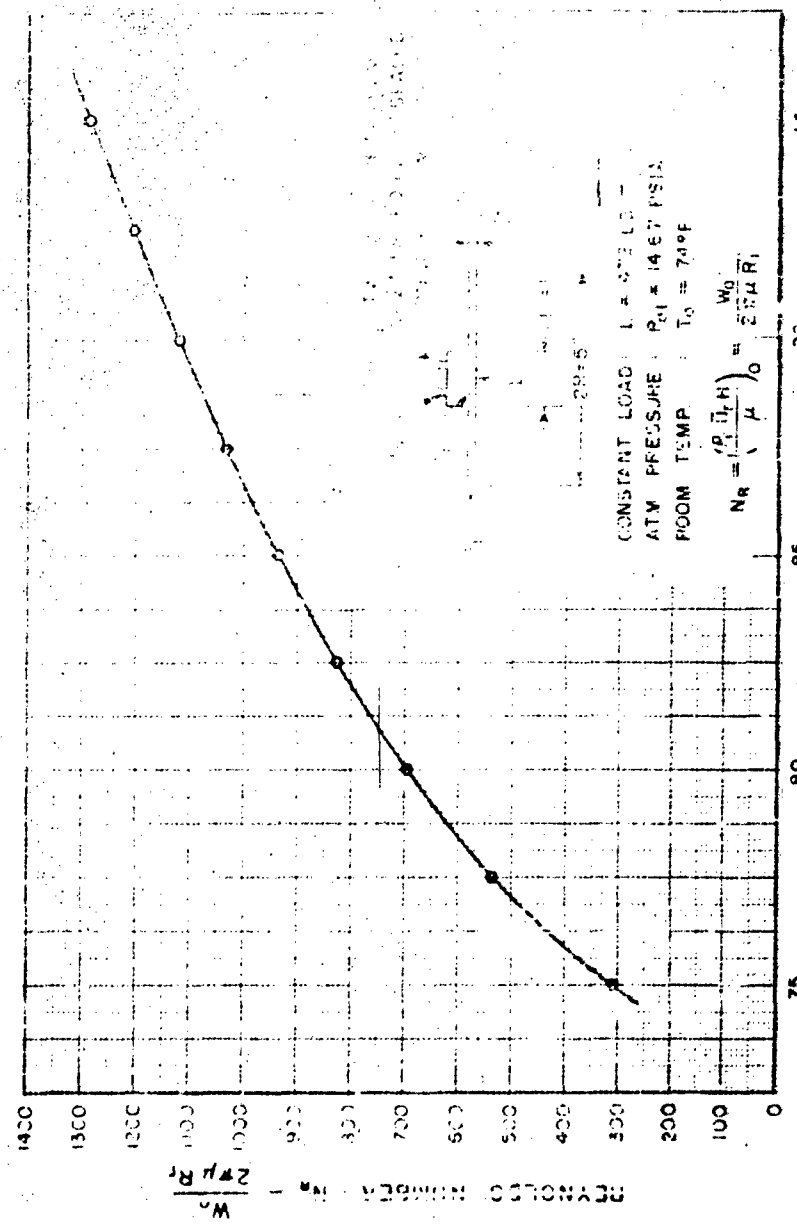


FIG. 41 BEARING REYNOLDS NUMBER vs SUPPLY PRESSURE
(CONSTANT LOAD = 472 LB)

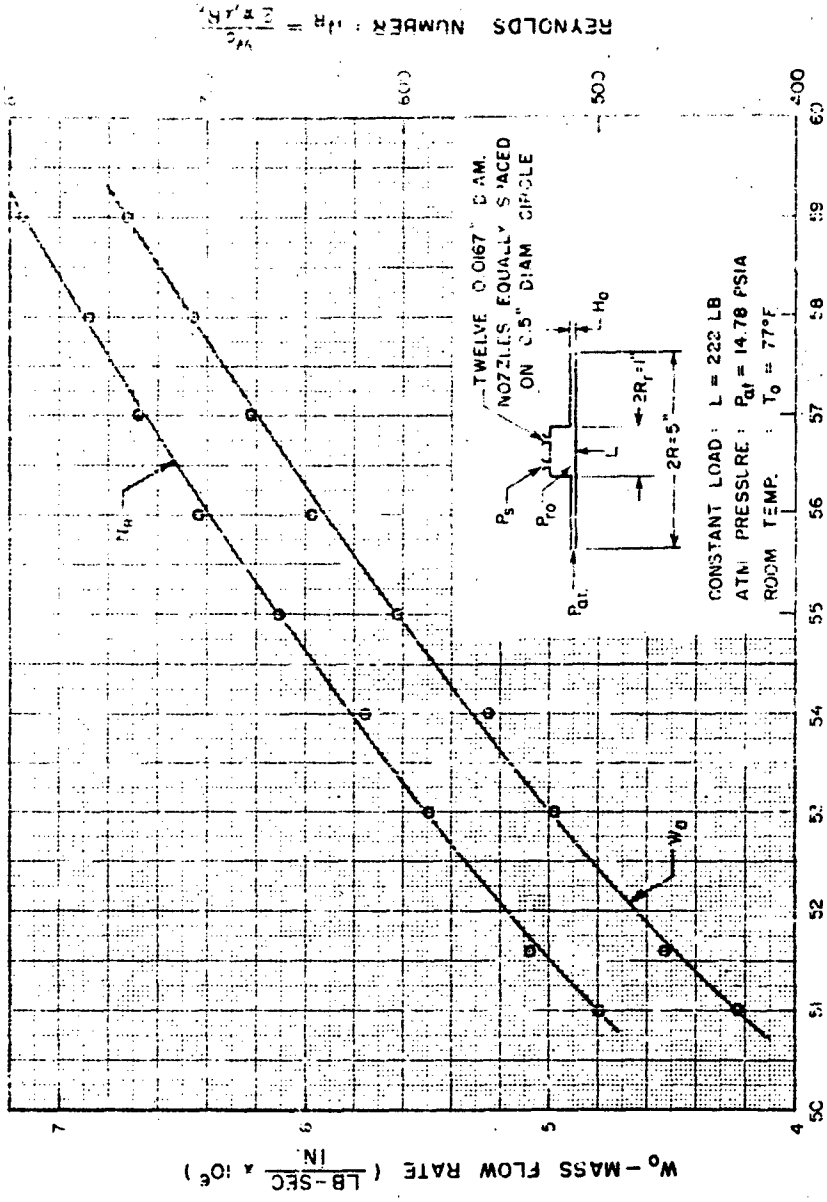


FIG. 42 BEARING MASS FLOW RATE AND REYNOLDS NUMBER VS. SUPPLY PRESSURE (CONSTANT LOAD = 222 LB)

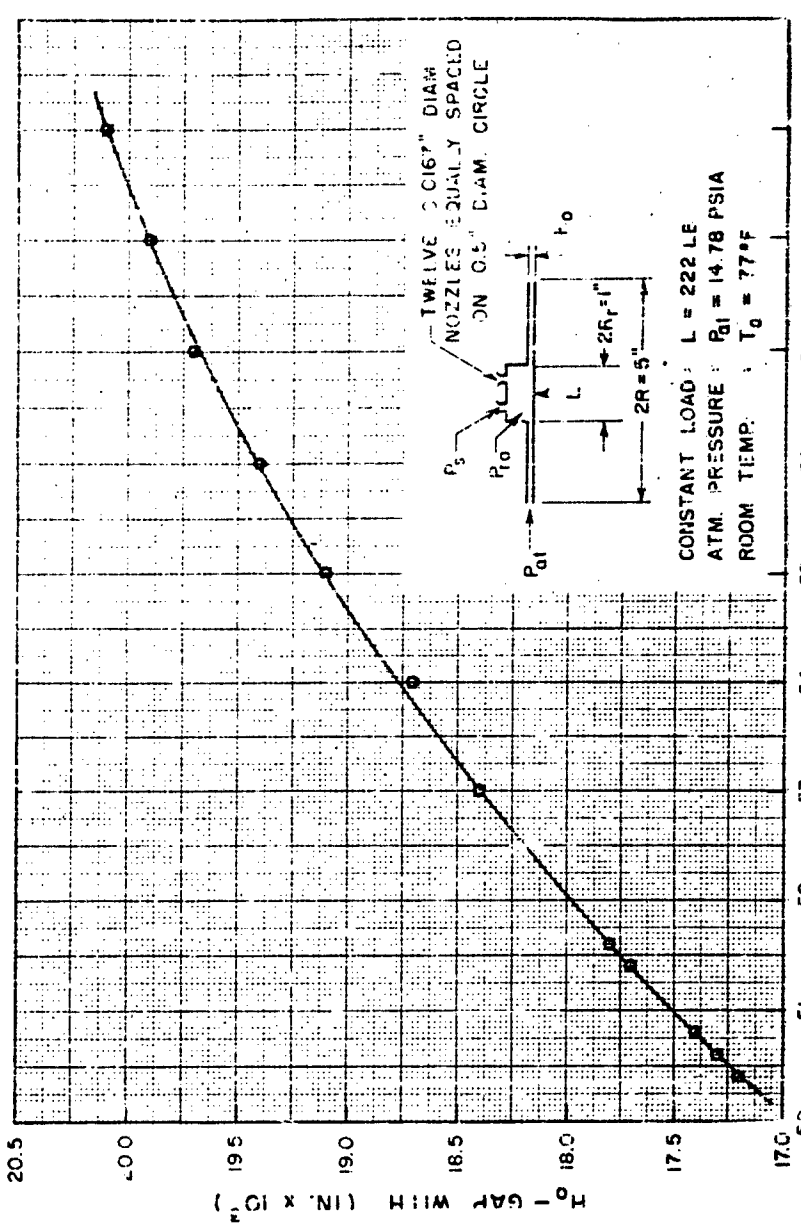


FIG. 43. BEARING GAP WIDTH vs. SUPPLY PRESSURE (CONSTANT LOAD = 222 LB)

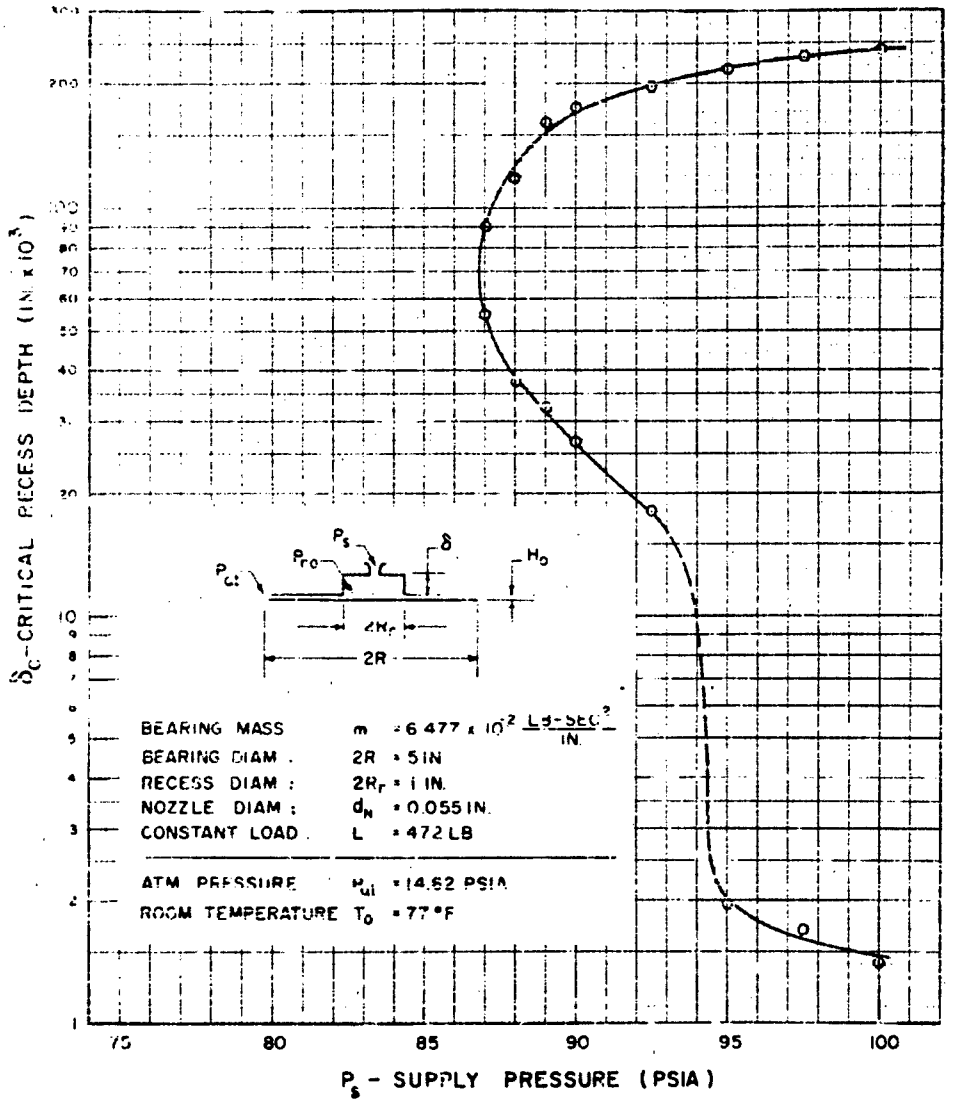


FIG 44 CRITICAL RECESS DEPTH vs. SUPPLY PRESSURE

(Effect of Decrease in Recess Depth for Single Nozzle Supply)

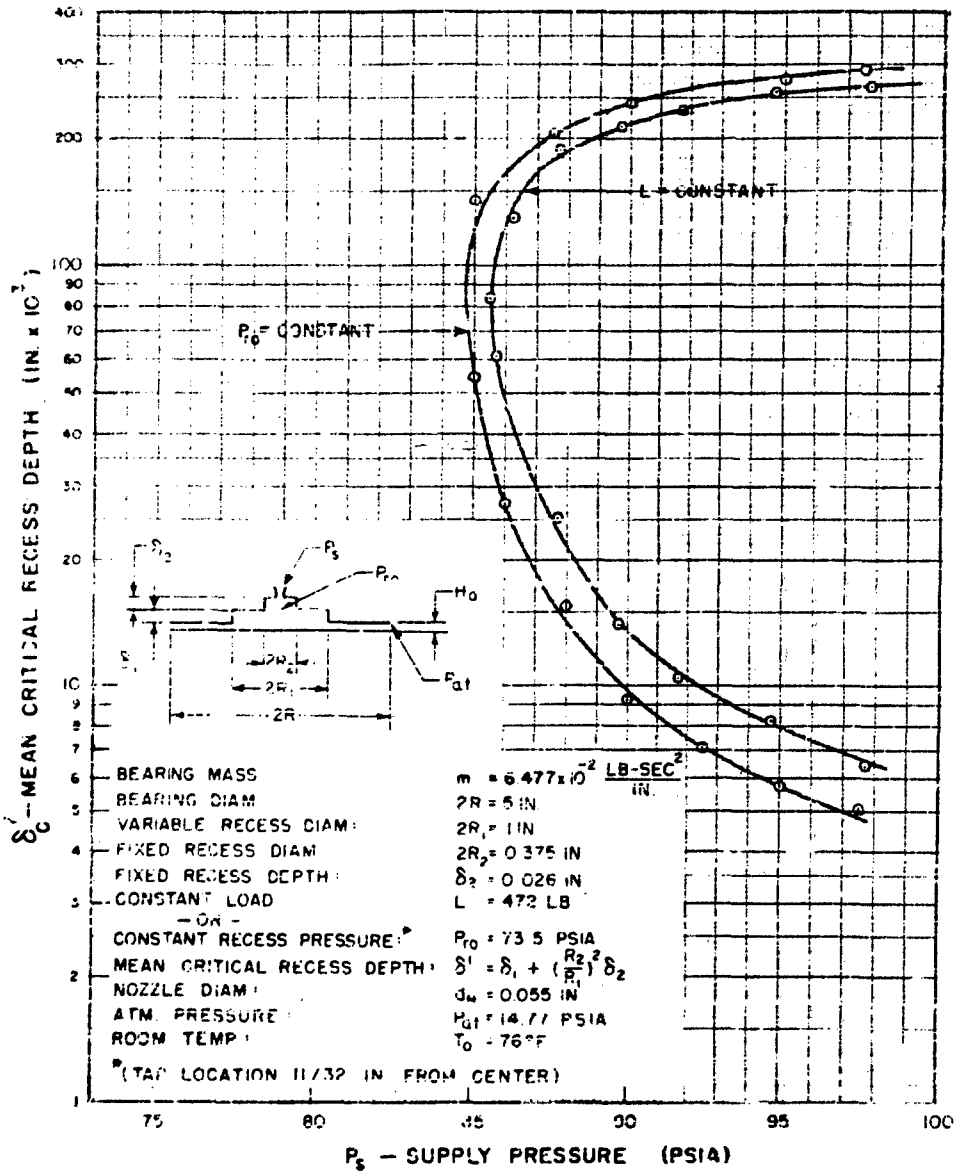


FIG. 45. MEAN CRITICAL RECESS DEPTH vs. SUPPLY PRESSURE

(Results of Constant Load and Constant Recess Pressure Tests)

Modified Piston No. 2 and Single Nozzle Supply

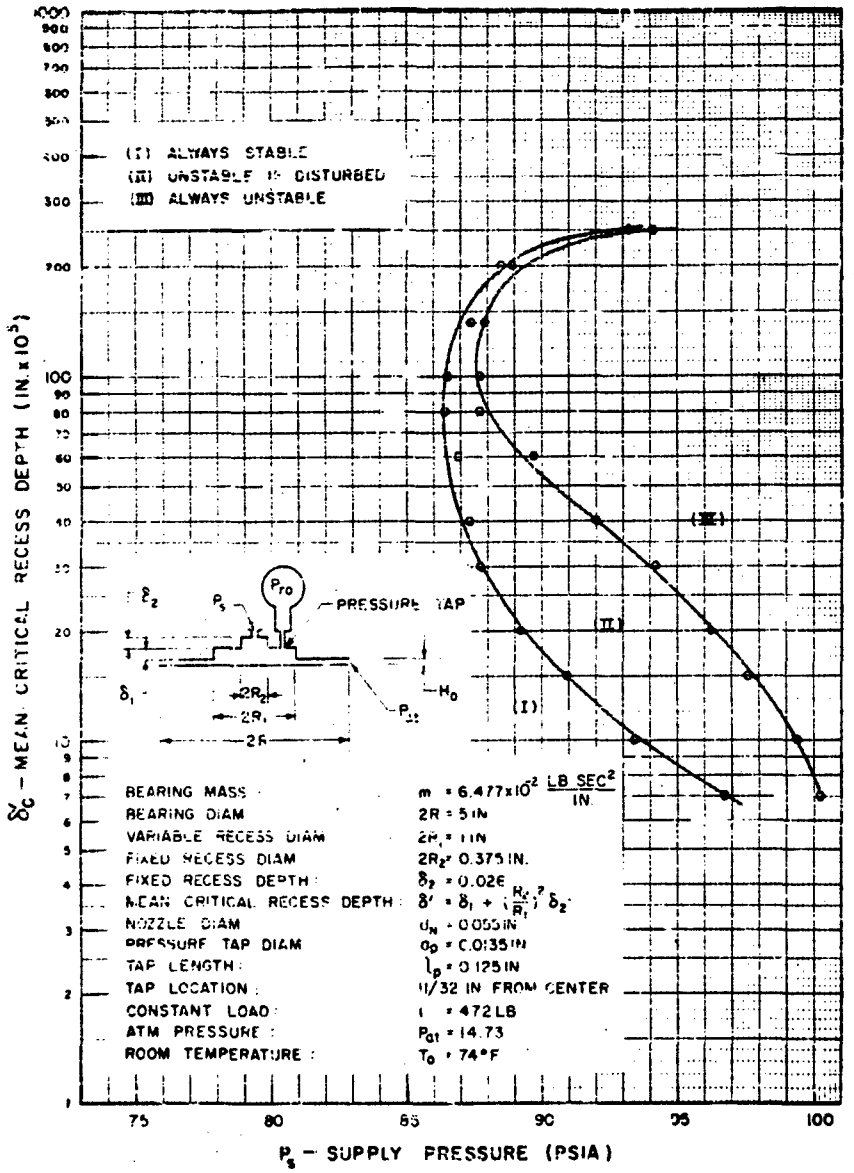


FIG 46 MEAN CRITICAL RECESS DEPTH vs. SUPPLY PRESSURE

(Hysteresis Effect)

Modified Piston No.2, Single Nozzle Supply and Open Pressure Tap

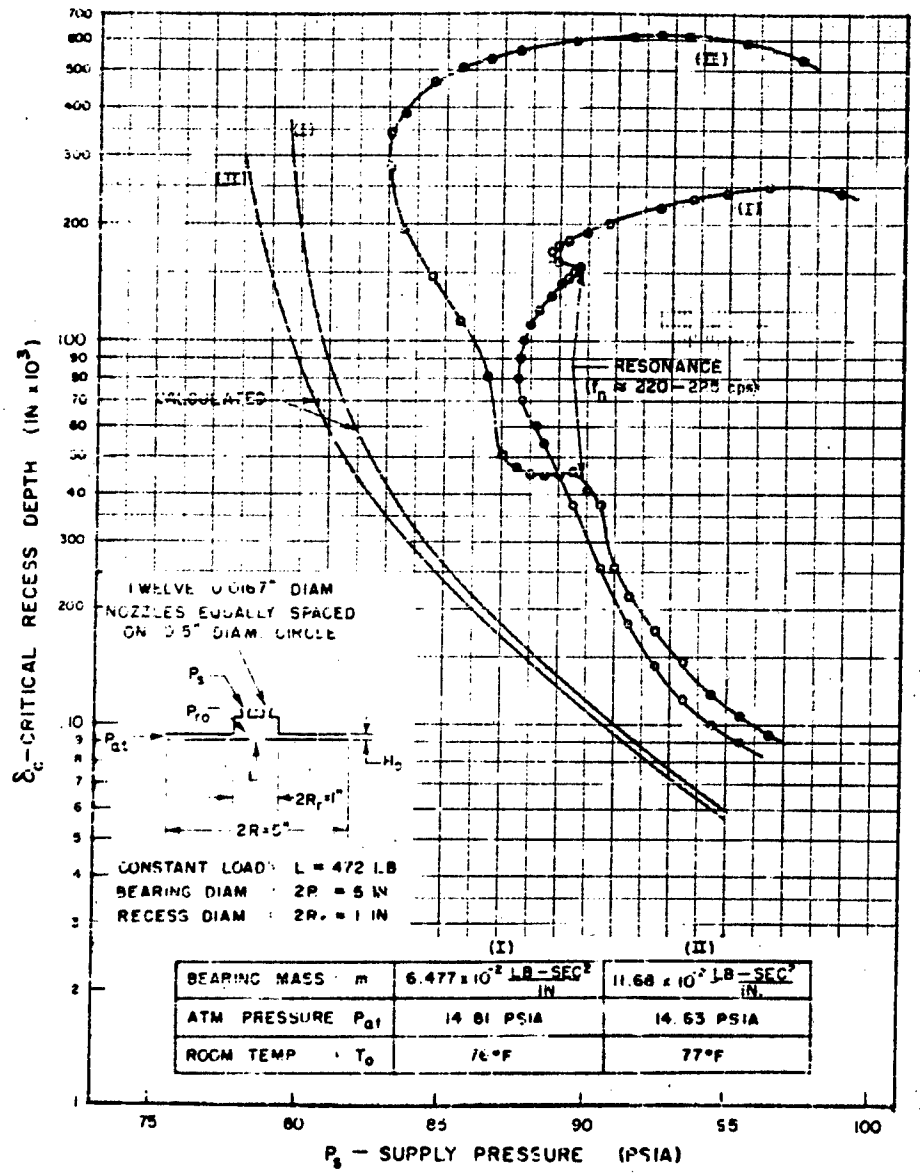


FIG. 47. CRITICAL RECESS DEPTH vs SUPPLY PRESSURE

(Effect of Resonance with Non-isolated Test Stand)

Twelve Nozzle Supply and Piston No 3

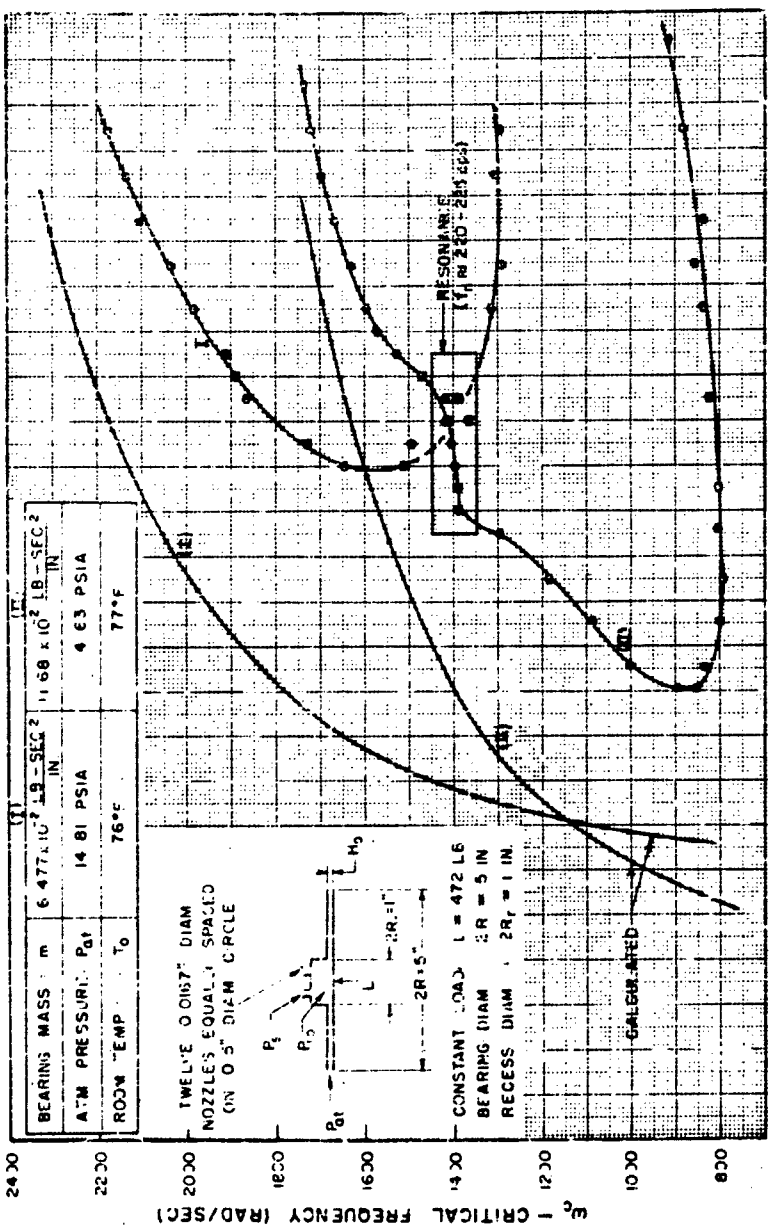


FIG. 48. CRITICAL FREQUENCY vs. SUPPLY PRESSURE (Effect of Resonance with Non-Isolated Test Stand)
Twelve Nozzle Supply and Piston No.3

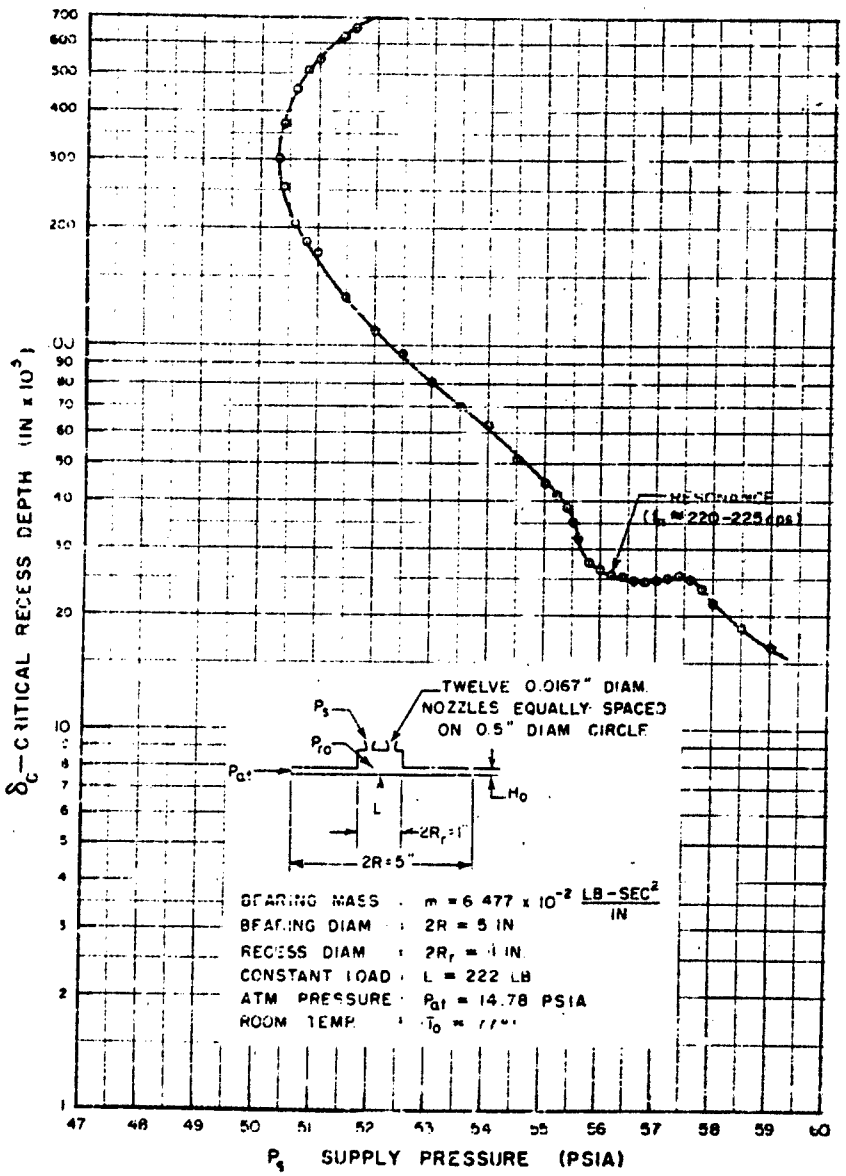
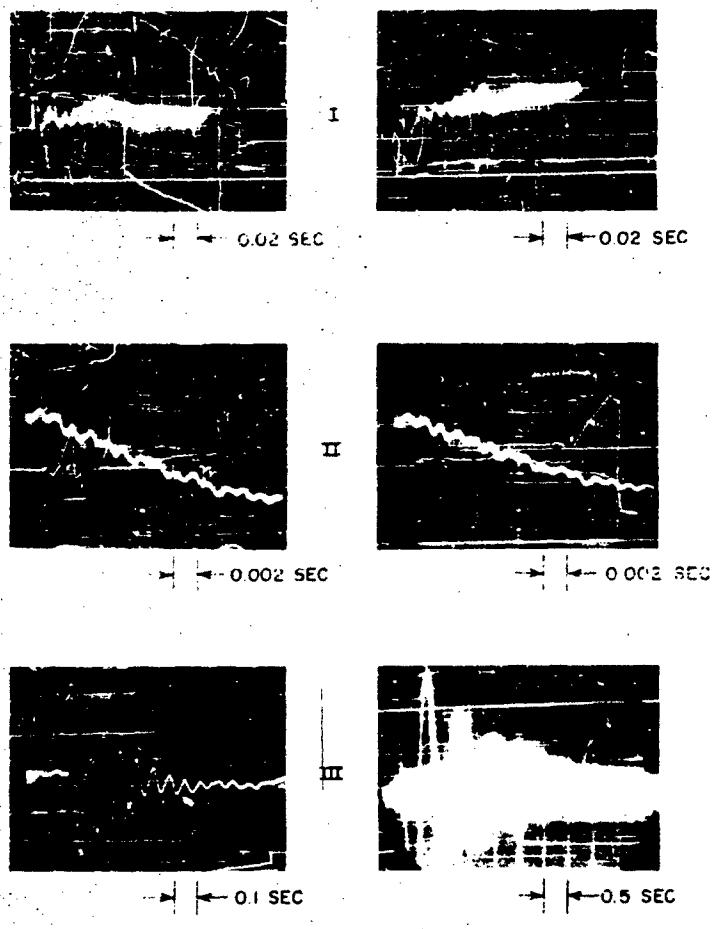


FIG 42 CRITICAL RECESS DEPTH vs. SUPPLY PRESSURE
(Effect of Resonance With Non-Isolated Test Stand)
Reduced, 222 lb Load, Twelve Nozzle Supply
and Piston No. 3



I TEST RIG AND CASE MOUNTED DIRECTLY ON FLOOR (220-225 CPS)
II. UPPER THRUST PLATE SECURED TO COLUMNS (730 CPS)
III TEST RIG AND BASE MOUNTED ON ISOLATORS (15 CPS)

FIG. 50. DETERMINATION OF NATURAL FREQUENCIES OF TEST STAND AND UPPER THRUST PLATE

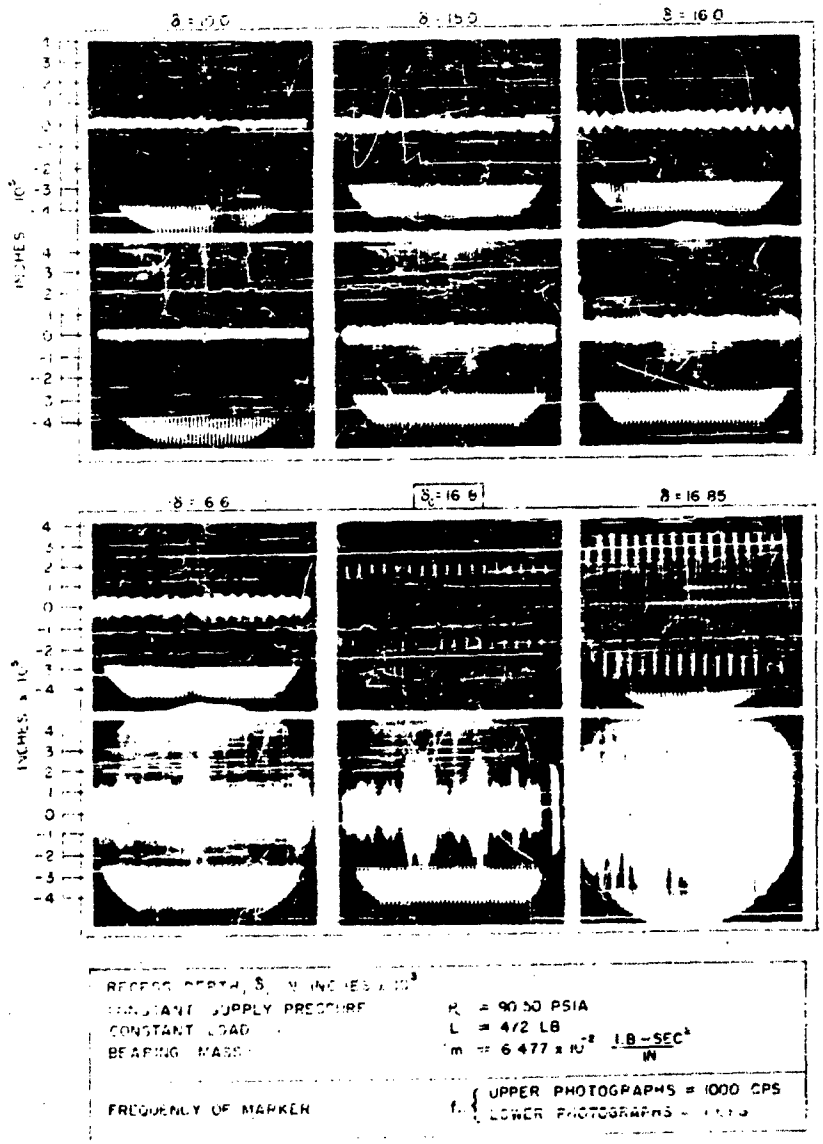


FIG. 51 AMPLITUDE GROWTH OF AUTO-OSCILLATIONS IN APPROACHING STABILITY BOUNDARY (P_s = 90.50 PSIA)

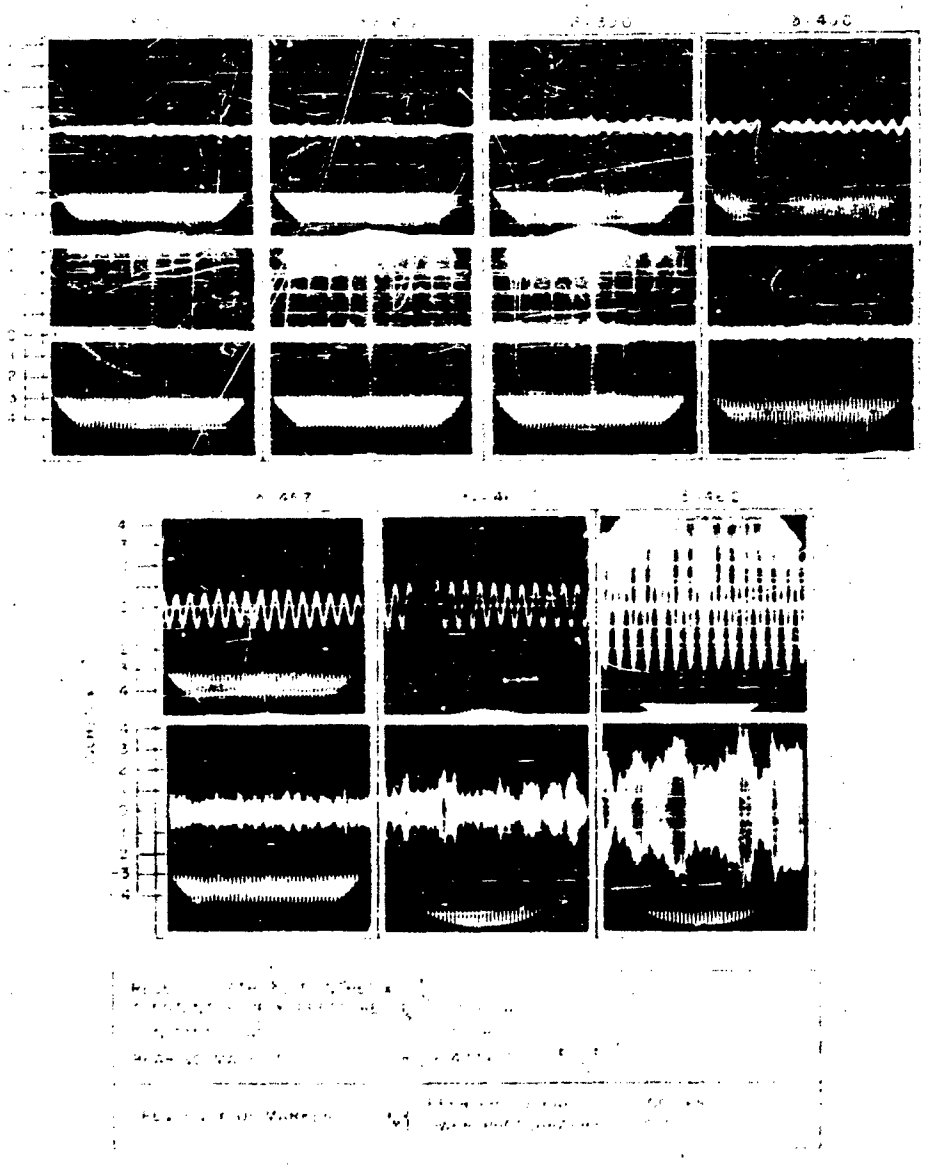


FIG. 52. AMPLITUDE GROWTH OF AUTO-OSCILLATIONS IN APPROACHING STABILITY BOUNDARY (P_s = 87.50 PSIA)

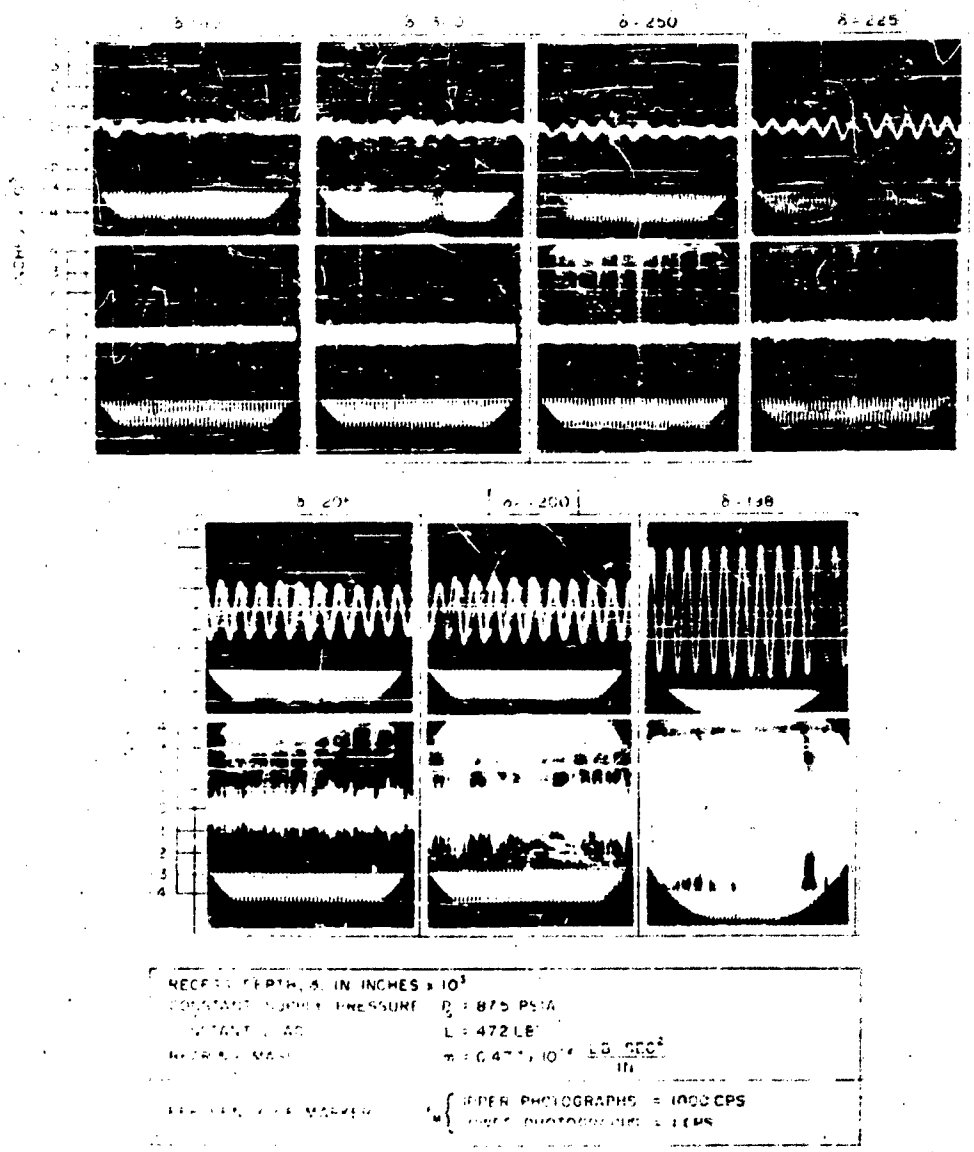


FIG. 53. AMPLITUDE GROWTH OF AUTO OSCILLATIONS IN APPROACHING STABILITY BOUNDARY (P_s = 875 PSIA)

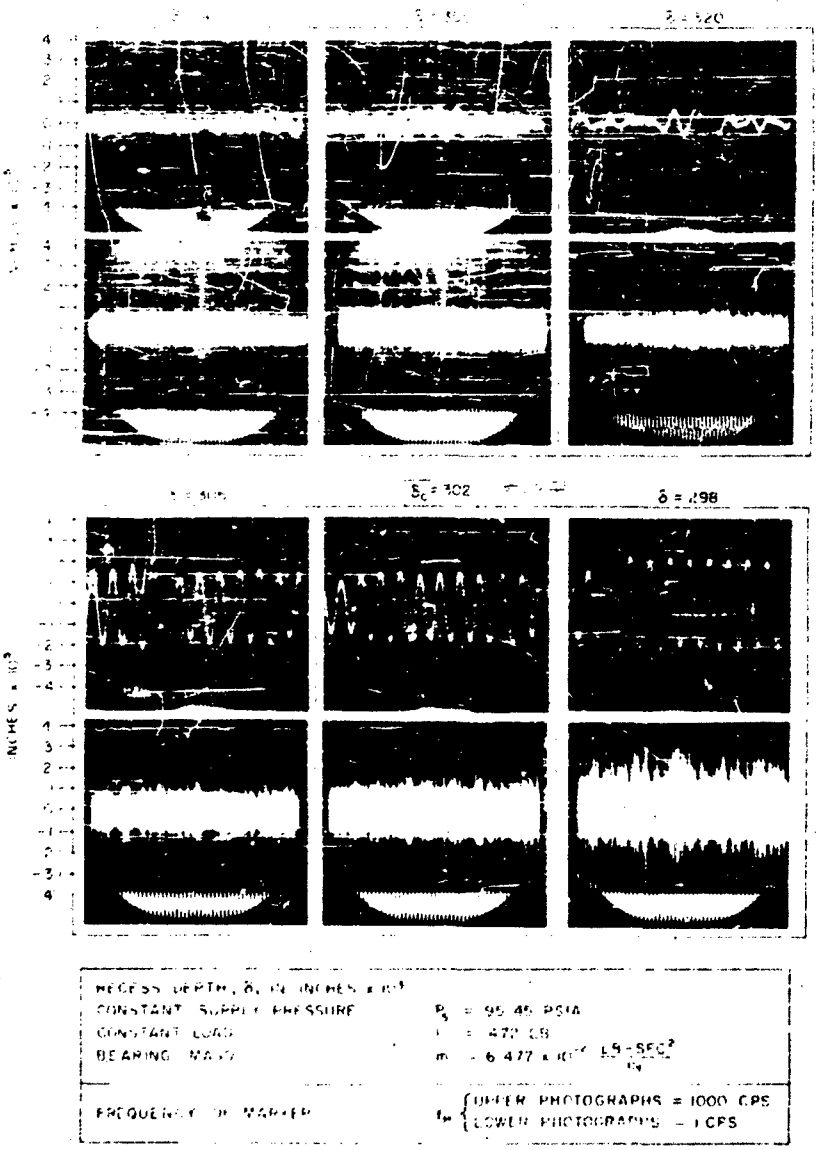
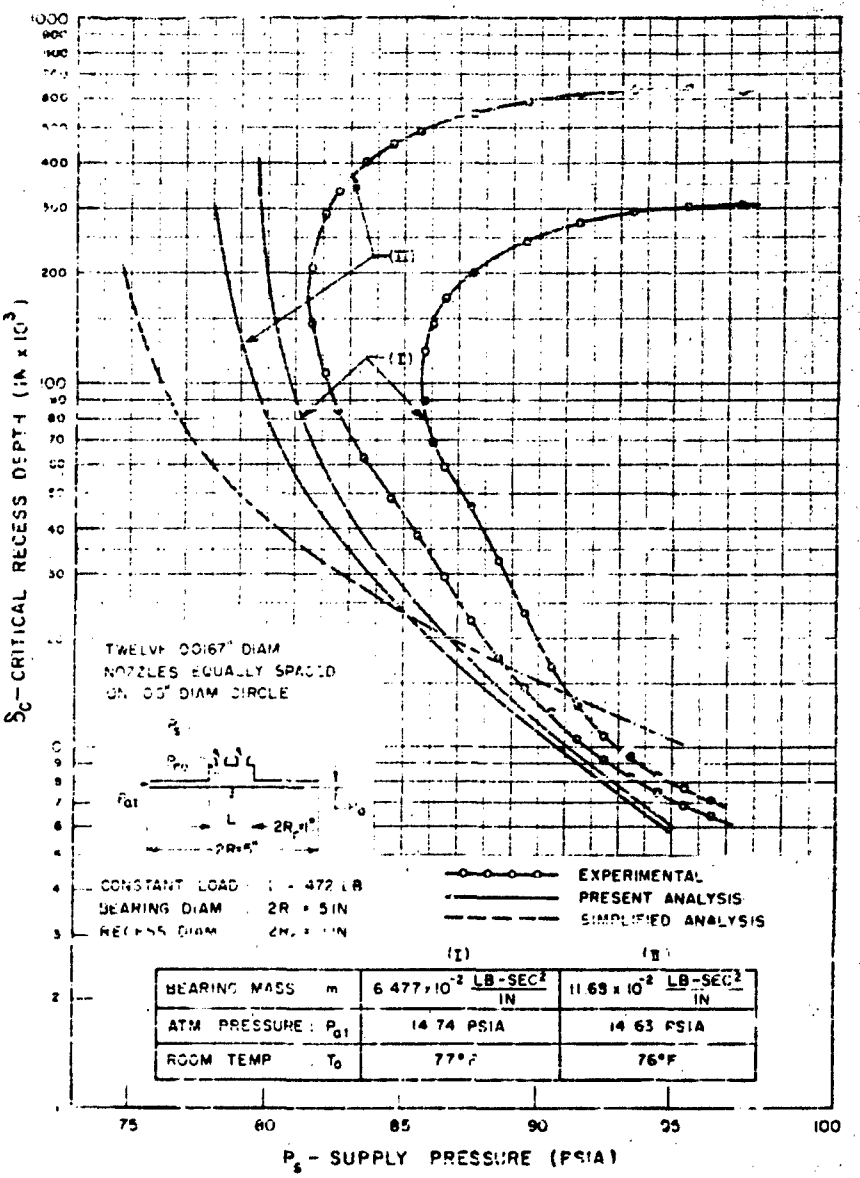


FIG 54 AMPLITUDE GROWTH OF AUTO-OSCILLATIONS IN APPROACHING STABILITY BOUNDARY (P_s = 95.45 PSIA)



		(I)	(II)
BEARING MASS	m	6.477×10^{-2} LB-SEC²/IN	11.69×10^{-2} LB-SEC²/IN
ATM PRESSURE	P_{a1}	14.74 PSIA	14.63 PSIA
ROOM TEMP	T_o	77°F	76°F

FIG. 55 CRITICAL RECESS DEPTH vs. SUPPLY PRESSURE

(Concluding Results)

Twelve Nozzle Supply and Piston No. 3

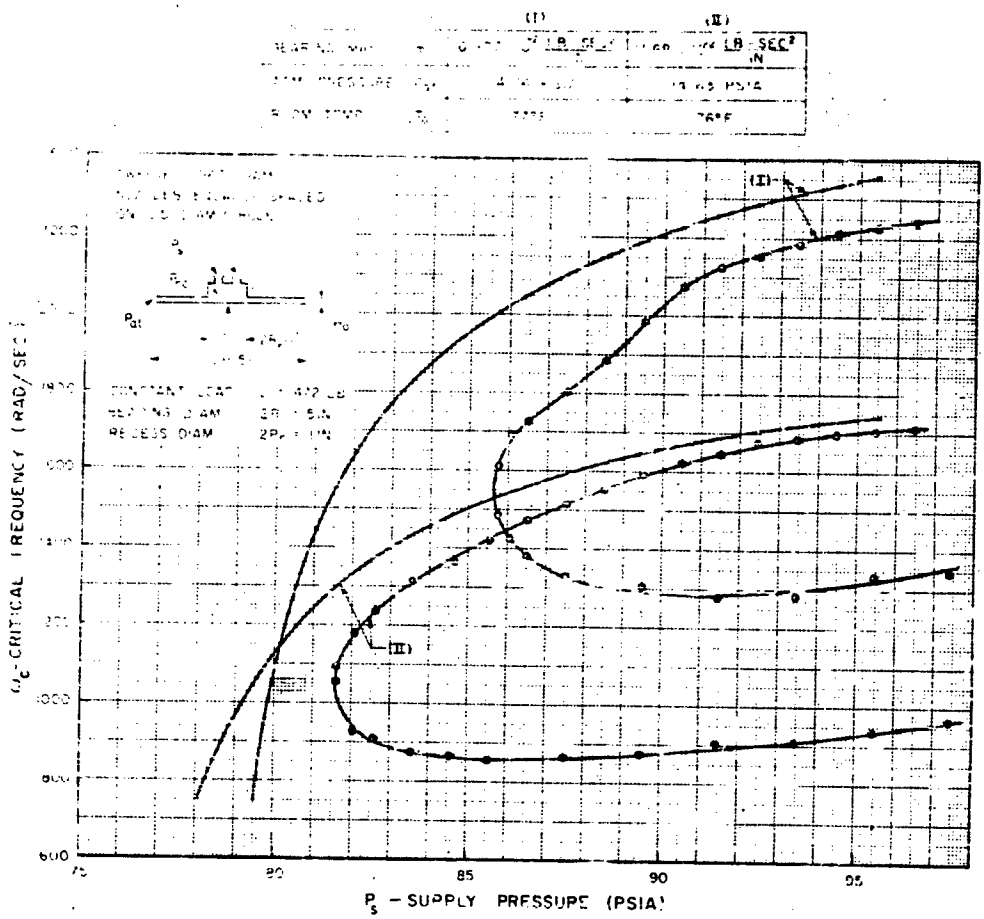


FIG. 56. CRITICAL FREQUENCY vs. SUPPLY PRESSURE

(Concluding Results)

Twelve Nozzle Supply and Piston No. 3

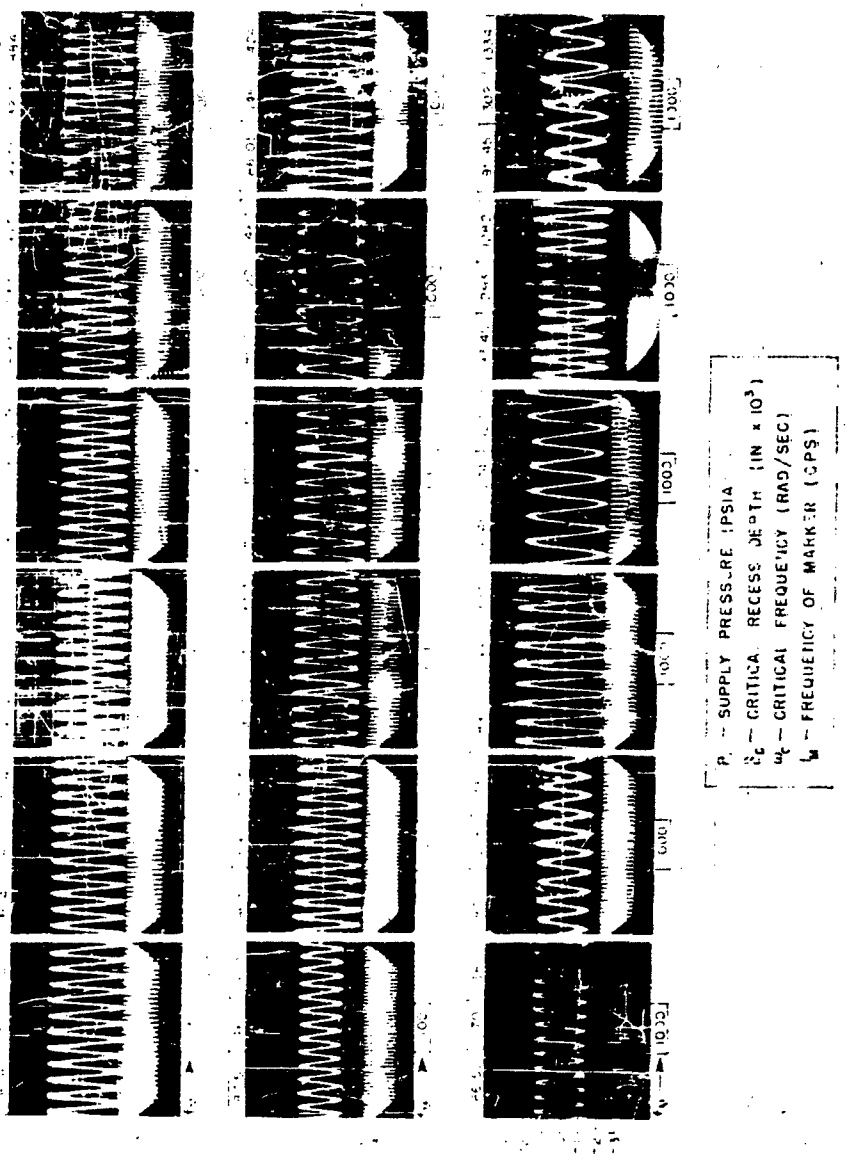
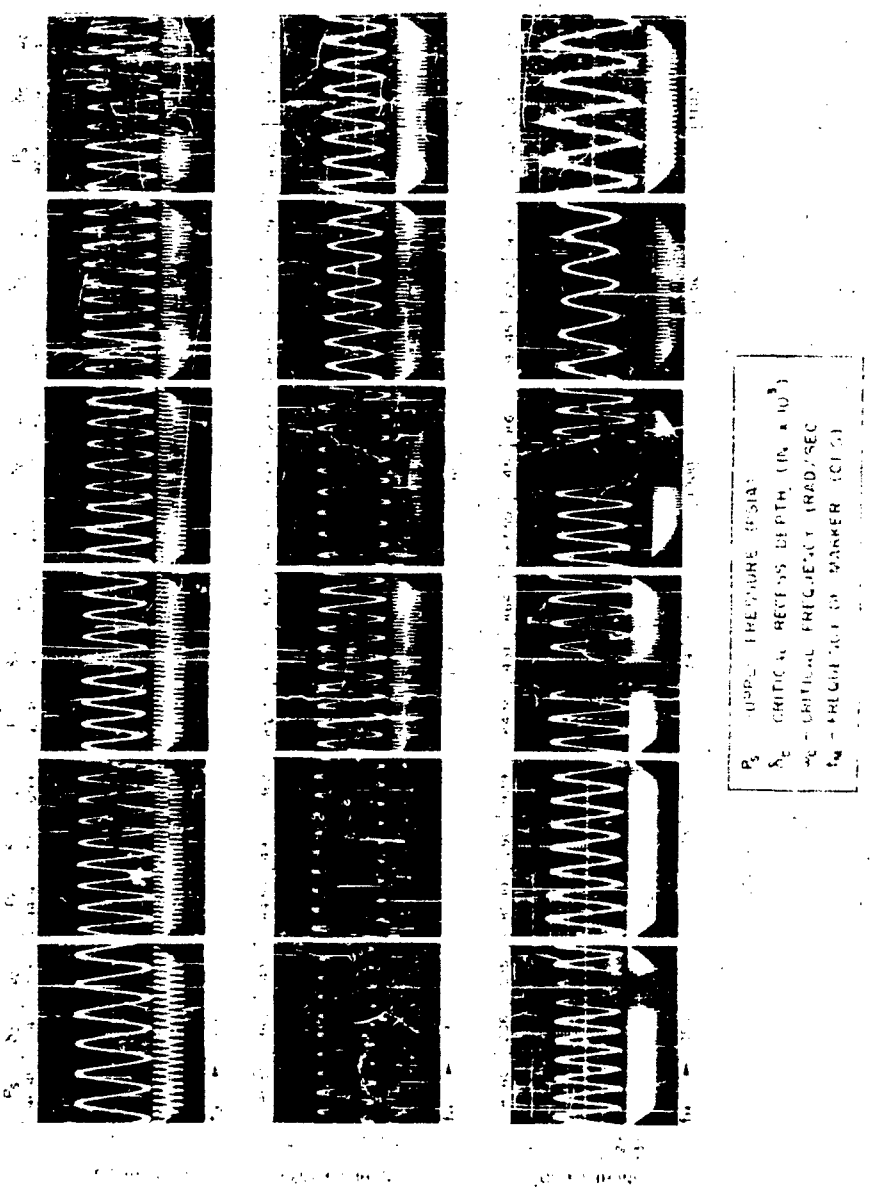
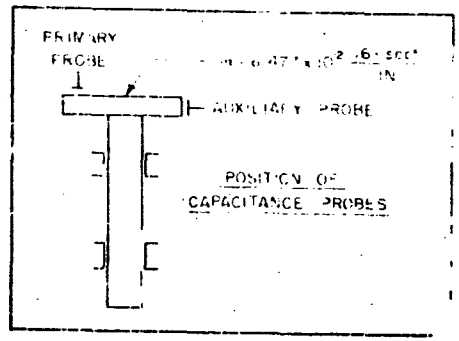


FIG. 57. DETERMINATION OF CRITICAL FREQUENCIES (BEARING MASS = 6.477 x 10^{-2} LB-SEC^2/IN.)



P_3 - SUPPLY PRESSURE (PSIA)
S_c - CRITICAL WHIRL DEPTH ($IN. \times 10^3$)
ω_c - CRITICAL FREQUENCY (RAD/SEC)
t_M - TIME (SEC OR MARKER SEC)

FIG. 58 DETERMINATION OF CRITICAL FREQUENCIES (BEARING MASS = 11.68×10^{-2} LB-SEC^2/IN.)



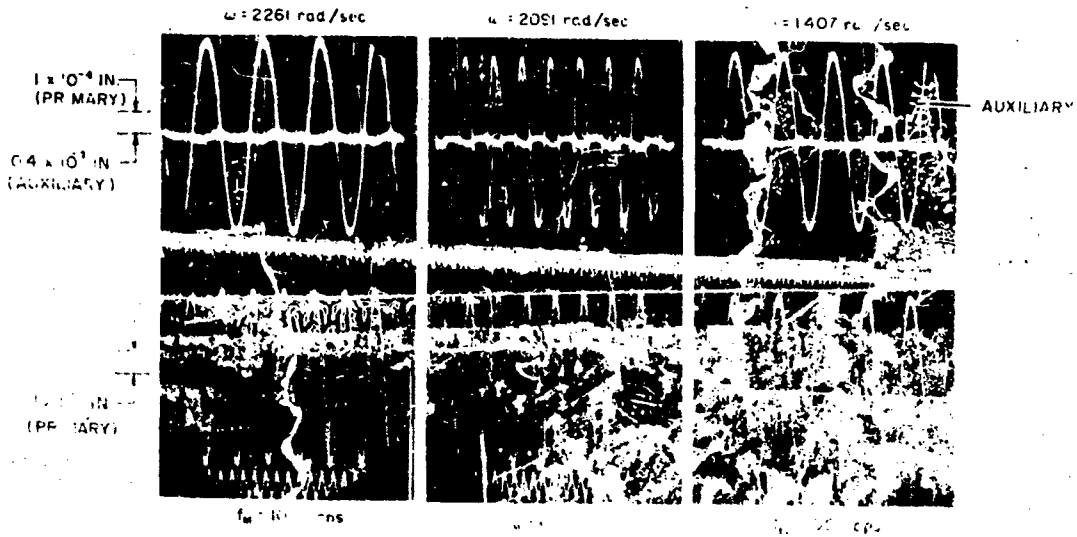


FIG. 59. COMPARISON OF AMPLITUDES OF AXIAL AND PITCHING MODES OF VIBRATION